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Tobias Meng

Majorana Fermion induced selective equal spin Andreev reflections

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Abstract

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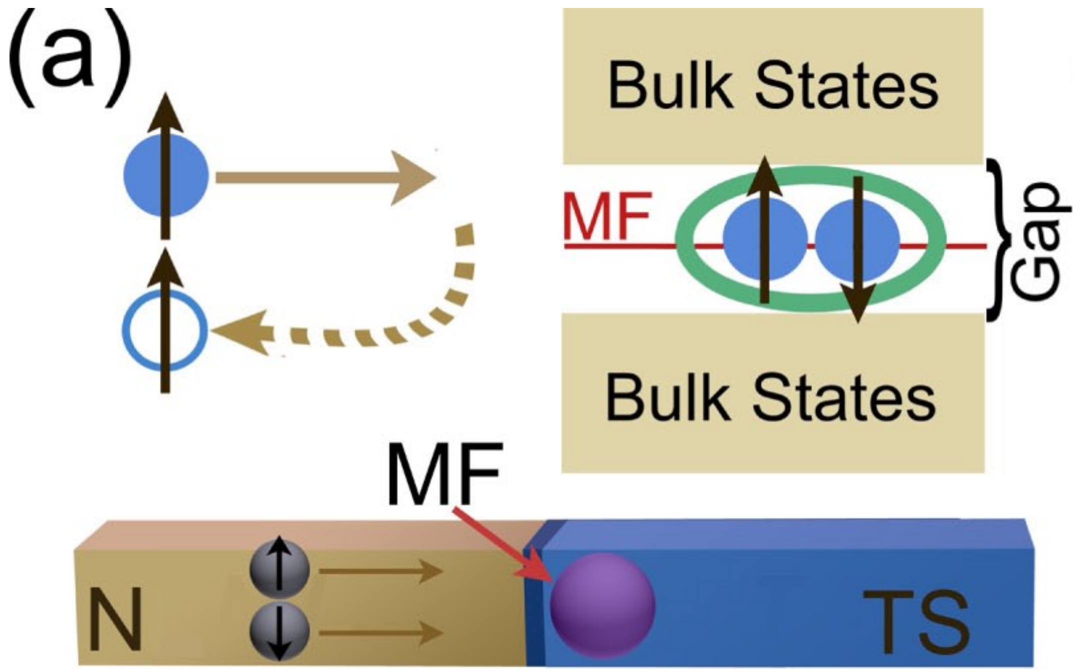
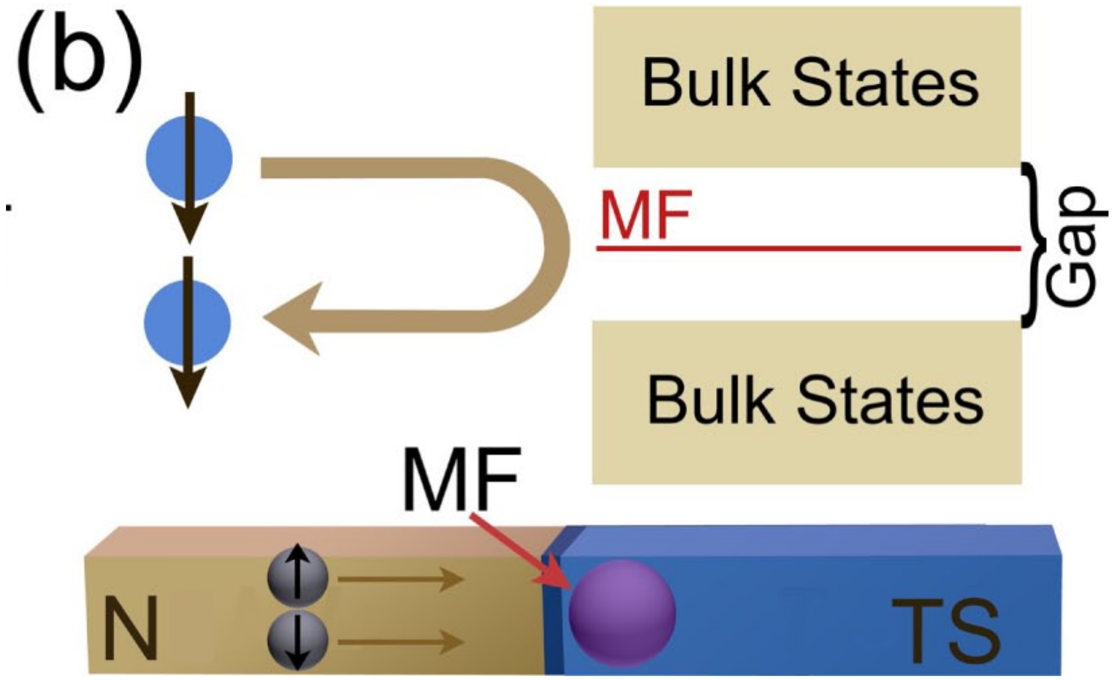
In this work, we find that Majorana fermions induce selective equal spin Andreev reflections (SESARs), in which incoming electrons with certain spin polarization in the lead are reflected as counter propagating holes with the same spin. The spin polarization direction of the electrons of this Andreev reflected channel is selected by the Majorana fermions. Moreover, electrons with opposite spin polarization are always reflected as electrons with unchanged spin. As a result, the charge current in the lead is spin-polarized. Therefore, a topological superconductor which supports Majorana fermions can be used as a novel device to create fully spin-polarized currents in paramagnetic leads. We point out that SESARs can also be used to detect Majorana fermions in topological superconductors.

System and idea in pictures

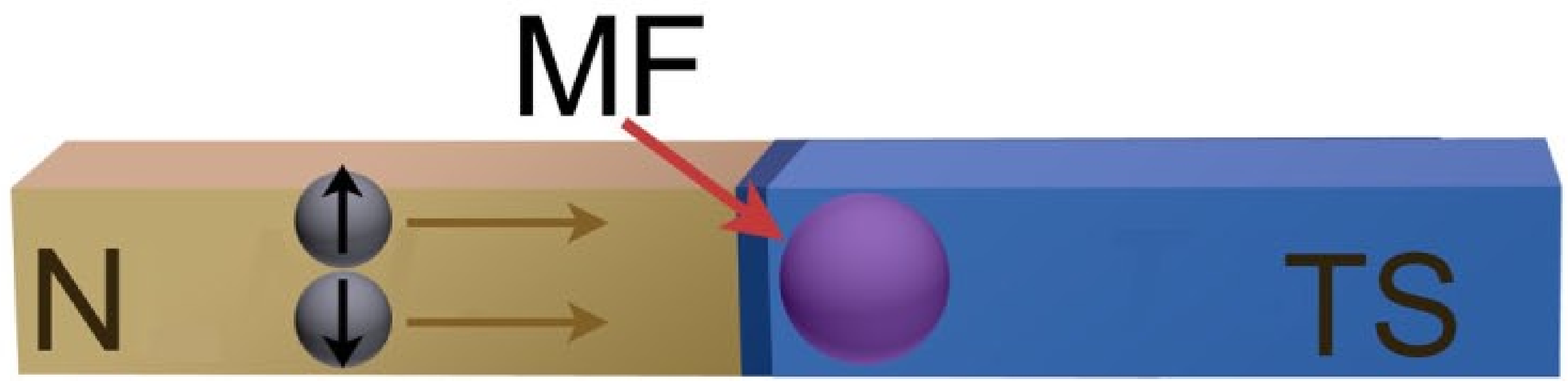


(a) Electron of one spin species:
can be reflected as hole with same spin



(b) Electron of other spin species:
always reflected as an electron of this spin

Effective Hamiltonian for N/TS junction



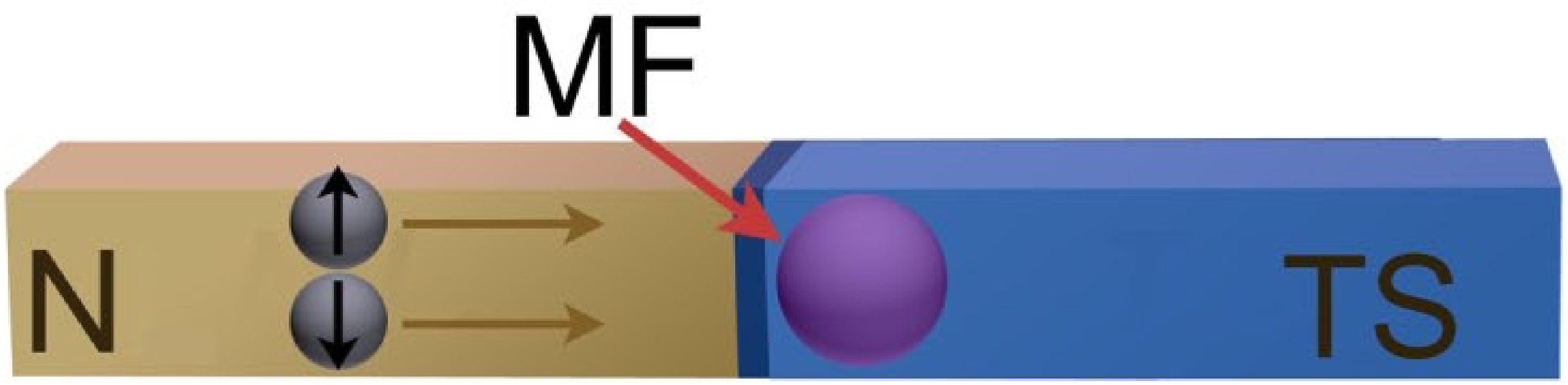
$$H_T = H_L + H_c,$$

$$H_L = -iv_F \sum_{\alpha \in \uparrow/\downarrow} \int_{-\infty}^{+\infty} \psi_\alpha^\dagger(x) \partial_x \psi_\alpha(x) dx,$$

$$H_c = \tilde{t}\gamma[a\psi_\uparrow(0) + b\psi_\downarrow(0) - a^*\psi_\uparrow^\dagger(0) - b^*\psi_\downarrow^\dagger(0)].$$

$\gamma = \gamma^\dagger$ and $\tilde{t} \in \mathbb{R}$ and $|a|^2 + |b|^2 = 1.$

Effective Hamiltonian for N/TS junction



$$H_T = H_L + H_c,$$

N

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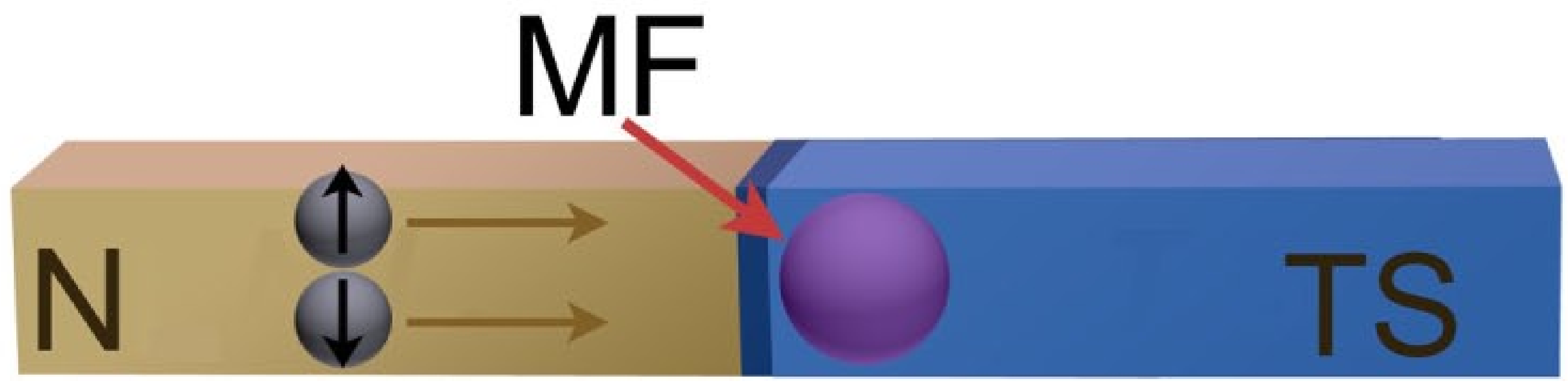
Effective Hamiltonian for N/TS junction



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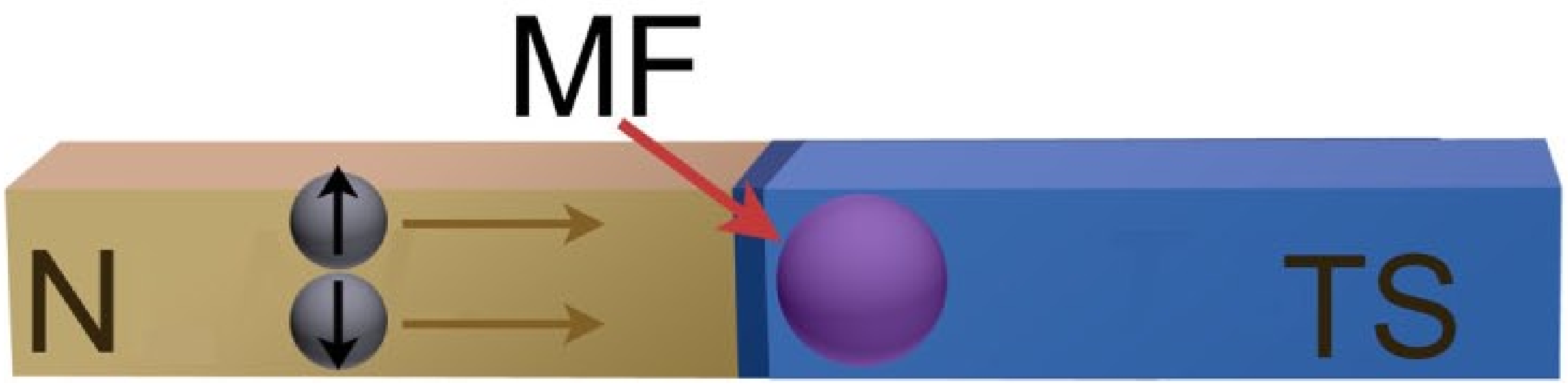
Effective Hamiltonian for N/TS junction



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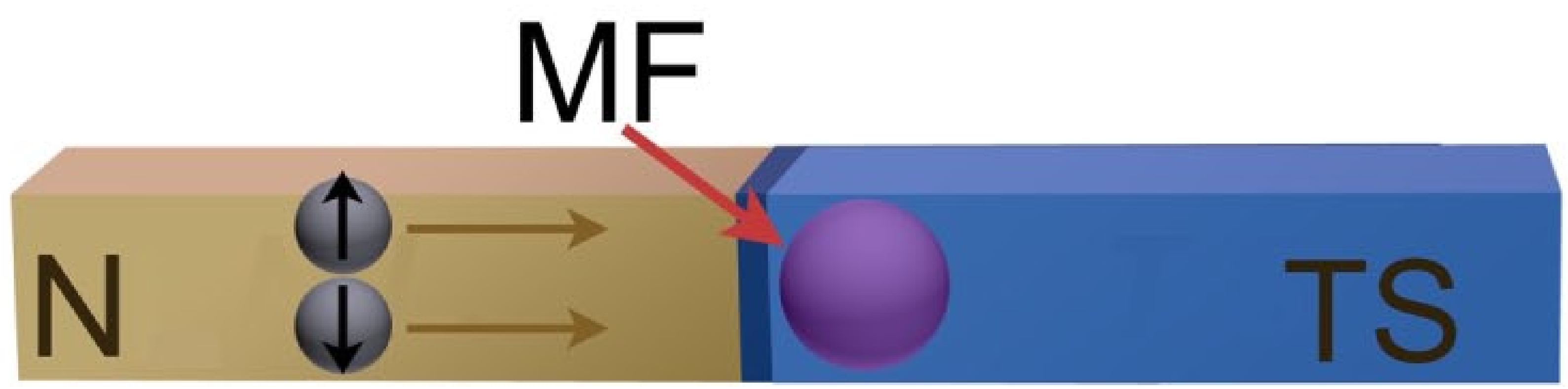
coupling to MF $\gamma = \gamma^\dagger$ and $\tilde{t} \in \mathbb{R}$ and $|a|^2 + |b|^2 = 1.$

Effective Hamiltonian for N/TS junction



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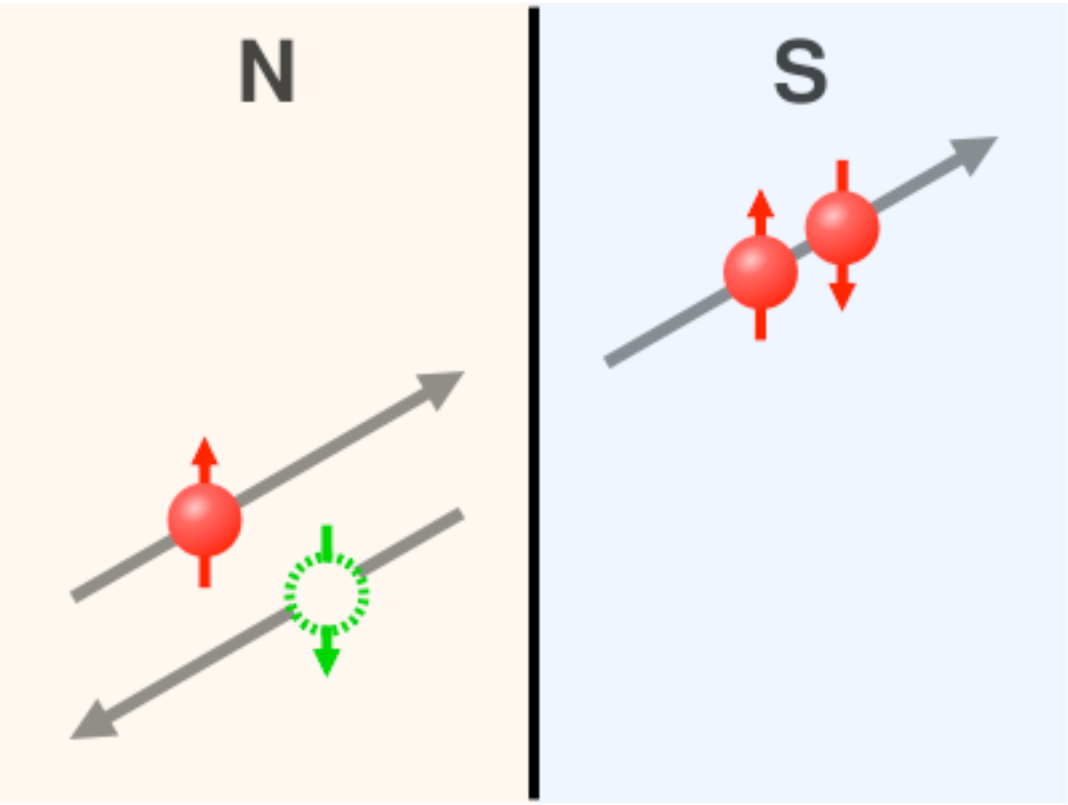
$\gamma = \gamma^\dagger$ and $\tilde{t} \in \mathbb{R}$ and $|a|^2 + |b|^2 = 1.$

Reminder: Andreev reflection

- Normal metal / superconductor interface: Andreev reflection possible

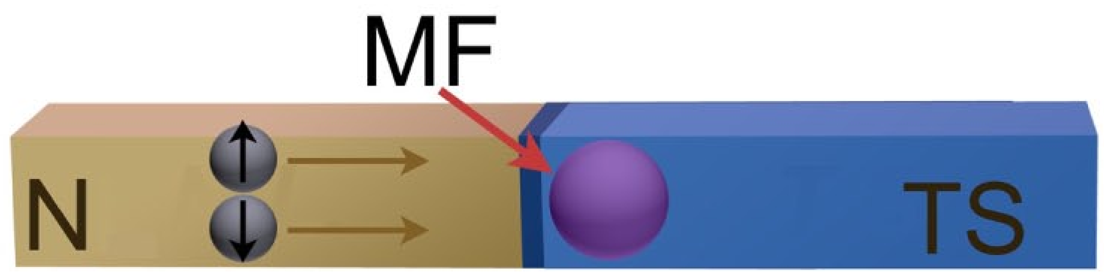


incoming electron of spin $\sigma \longrightarrow$ hole of spin $\bar{\sigma}$

- Important for instance for Andreev bound states in dots, crossed Andreev reflections (entanglement), ...

Spin selectivity of the Majorana bound state

- Majorana only couples to half of the modes in the normal lead:



$$H_c = \tilde{t}\gamma[a\psi_\uparrow(0) + b\psi_\downarrow(0) - a^*\psi_\uparrow^\dagger(0) - b^*\psi_\downarrow^\dagger(0)].$$

- Define

K.T. Law, P. A. Lee, T. K. Ng, PRL **103**, 237001 (2009)

$$|\Psi_1\rangle = a|\psi_\uparrow\rangle + b|\psi_\downarrow\rangle \equiv |\uparrow, \vec{n}\rangle$$

$$|\Psi_2\rangle = -b^*|\psi_\uparrow\rangle + a^*|\psi_\downarrow\rangle \equiv |\downarrow, \vec{n}\rangle$$

$$\vec{n} = \begin{pmatrix} \sin(\theta)\cos(\varphi) \\ \sin(\theta)\sin(\varphi) \\ \cos(\theta) \end{pmatrix} \qquad \theta = 2\arctan\left(\frac{|b|}{|a|}\right) \qquad \varphi = \arg\left(\frac{b}{a}\right)$$

Spin selectivity of the Majorana bound state

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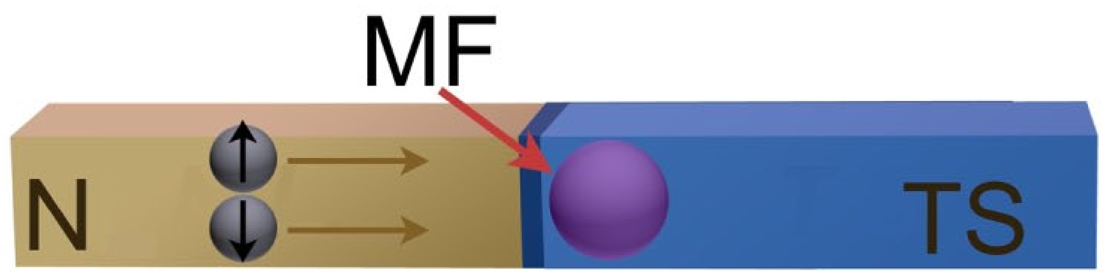


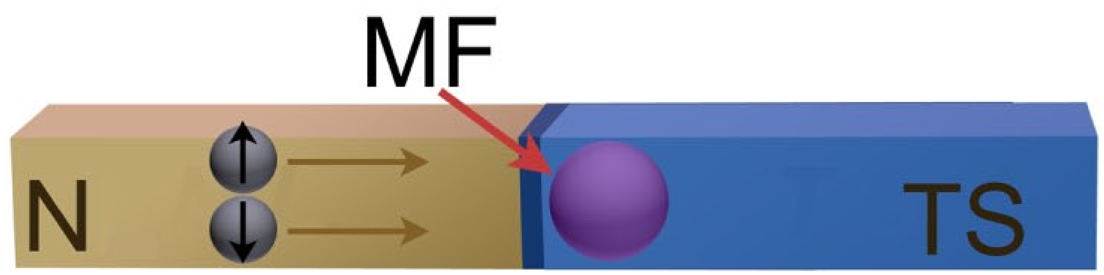
$$H_T = H_L + H_c,$$

$$H_L = -iv_F \sum_{\alpha \in 1/2} \int_{-\infty}^{+\infty} \Psi_\alpha^\dagger(x) \partial_x \Psi_\alpha(x) dx,$$

$$H_c = \tilde{t}\gamma[\Psi_1(0) - \Psi_1^\dagger(0)].$$

$\Rightarrow \quad \Psi_2$ decouples: junction = like hard wall, normal reflection only

Scattering of electrons at Majorana



- Scattering matrix for incoming electron of energy E in state $|\Psi_1\rangle \equiv |\uparrow, \vec{n}\rangle$

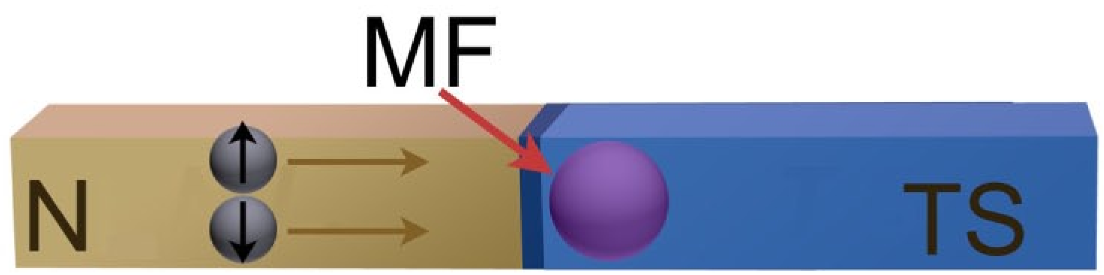
incoming electron

$$\begin{pmatrix} \Psi_{1E}(+) \\ \Psi^\dagger_{1E}(+) \end{pmatrix} = \frac{1}{\Gamma + iE} \begin{pmatrix} iE & \Gamma \\ \Gamma & iE \end{pmatrix} \begin{pmatrix} \Psi_{1E}(-) \\ \Psi^\dagger_{1E}(-) \end{pmatrix}$$

outgoing hole

$$\Gamma = 2\tilde{t}^2 / v_F$$

Scattering of electrons at Majorana



incoming electron

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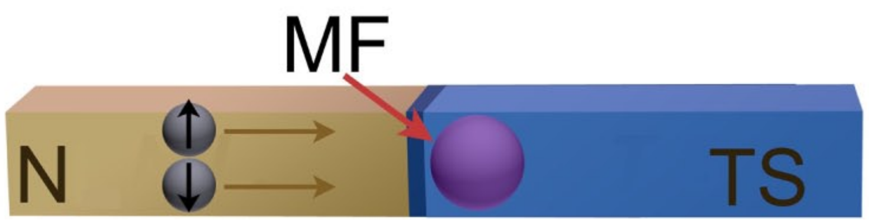
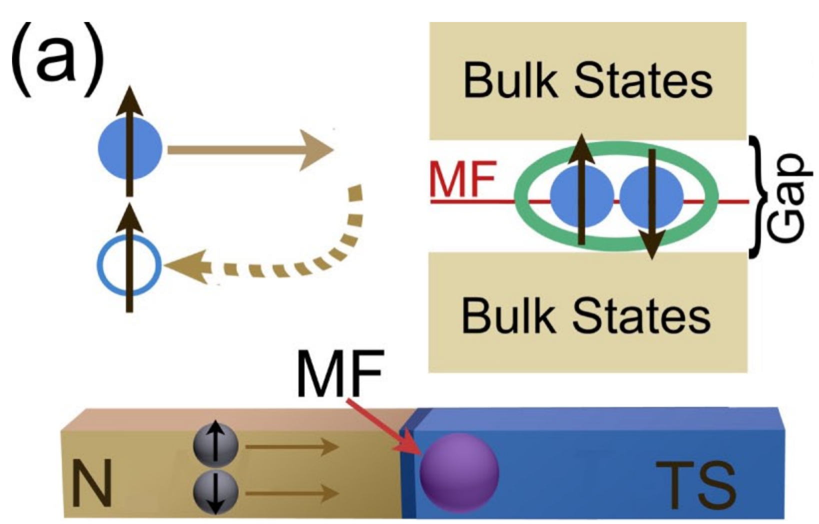


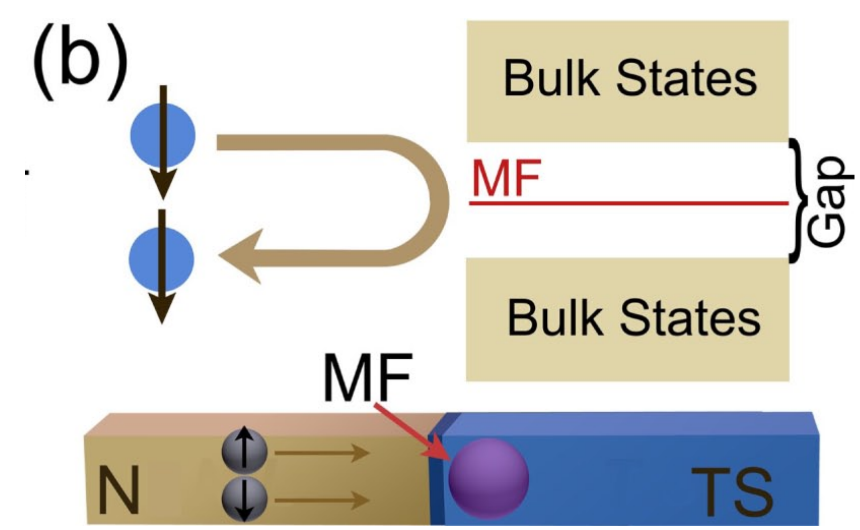
incoming electron

$$\begin{pmatrix} \Psi_{1E}(+) \\ \Psi^{\dagger}_{1E}(+) \end{pmatrix} = \frac{1}{\Gamma + iE} \begin{pmatrix} iE & \Gamma \\ \Gamma & iE \end{pmatrix} \begin{pmatrix} \Psi_{1E}(-) \\ \Psi^{\dagger}_{1E}(-) \end{pmatrix}$$

outgoing hole

„We call this phenomenon Majorana-induced selective equal spin Andreev reflection (SESAR)"

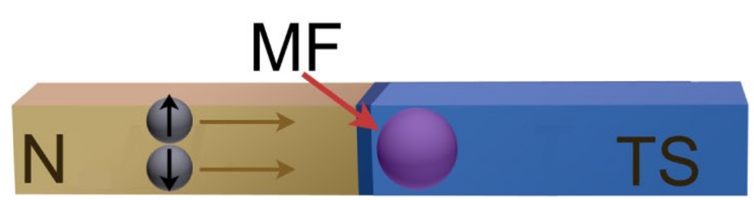




Towards more realistic setups

So far, couping between Majorana γ and wire modes energy independent:

$$H_c = \tilde{t}\gamma[a\psi_\uparrow(0) + b\psi_\downarrow(0) - a^*\psi_\uparrow^\dagger(0) - b^*\psi_\downarrow^\dagger(0)] \quad \text{with} \quad \frac{a}{b} = \text{const.}$$



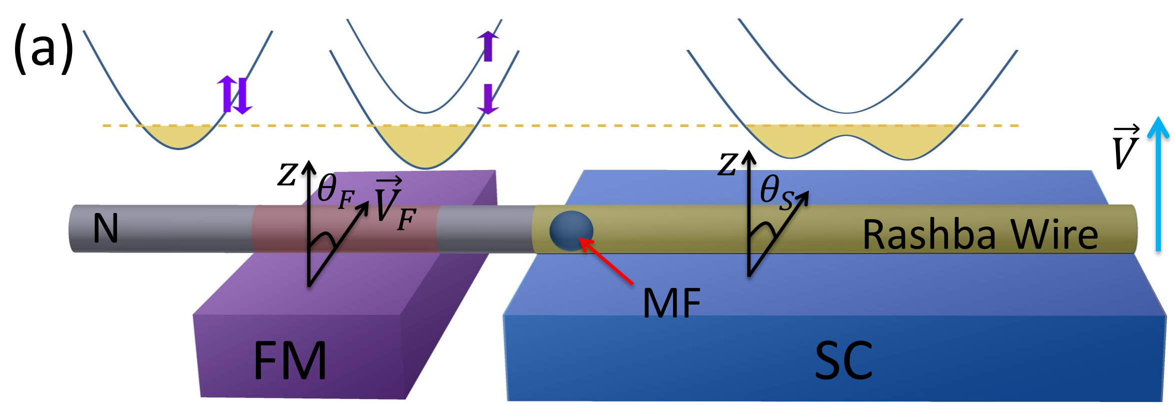
What about more realistic setup, e.g. Rashba wire + superconductor?

Does Majorana couple to electron spin $||$ same $\vec{n}$ for any incoming energy?

Rashba nanowire + superconductor

- Setup: Rashba nanowire + proximity coupling to s-wave superconductor

$$H_{1D}(k) = [(\frac{k^2}{2m} - \mu)\sigma_0 + \vec{V}\cdot\vec{\sigma} + \alpha_R k \sigma_y)]\tau_z - \Delta\sigma_y\tau_y.$$



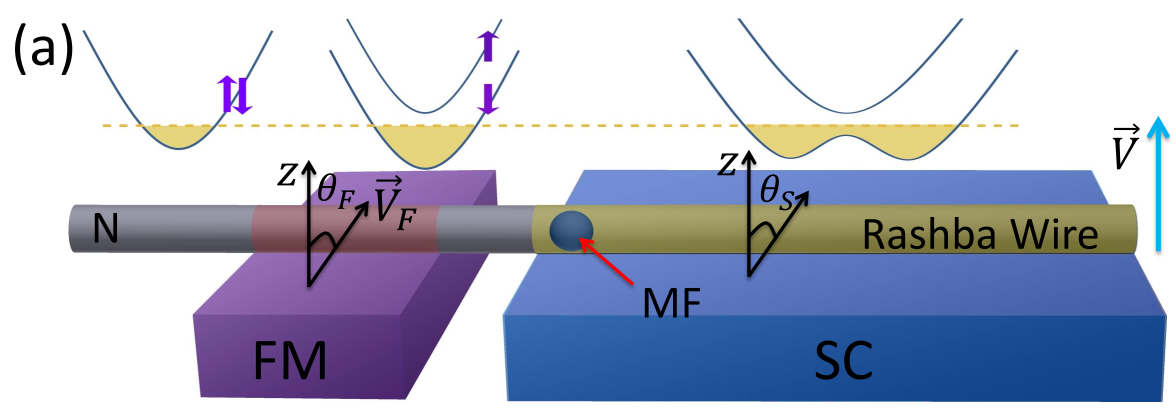
- System: semi-infinite TSC wire + normal metal / ferromagnetic metal lead

Rashba nanowire + superconductor

- Setup: Rashba nanowire + proximity coupling to s-wave superconductor

$$H_{1D}(k) = [(\frac{k^2}{2m} - \mu)\sigma_0 + \vec{V}\cdot\vec{\sigma} + \alpha_R k \sigma_y)]\tau_z - \Delta\sigma_y\tau_y .$$

magnetic field　　Rashba SOI　　proximity induced SC



- System: semi-infinite TSC wire + normal metal / ferromagnetic metal lead

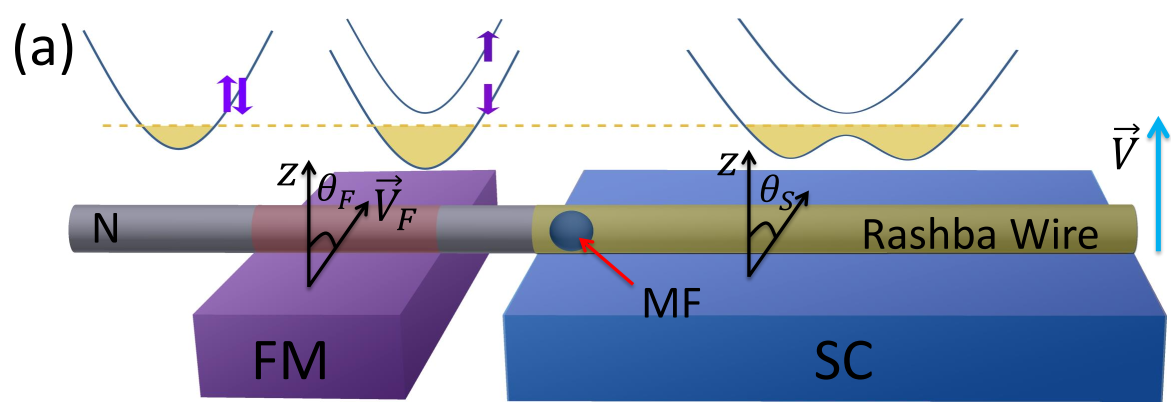
Rashba nanowire + superconductor

- Setup: Rashba nanowire + proximity coupling to s-wave superconductor

spin space — particle-hole space

$$H_{1D}(k) = [(\frac{k^2}{2m} - \mu)\sigma_0 + \vec{V}\cdot\vec{\sigma} + \alpha_R k \sigma_y)]\tau_z - \Delta\sigma_y\tau_y.$$

magnetic field — Rashba SOI — proximity induced SC



- System: semi-infinite TSC wire + normal metal / ferromagnetic metal lead

1) Analytics @ $\mathcal{O}(\alpha_R^0)$

Majorana mode

- Majorana mode present if „magnetic field > superconductivity“

$$H_{1D}(k) = [(\frac{k^2}{2m} - \mu)\sigma_0 + \vec{V}\cdot\vec{\sigma} + \alpha_R k \sigma_y)]\tau_z - \Delta\sigma_y\tau_y.$$

with $V_z{}^2 > \mu^2 + \Delta^2$

- Majorana mode ansatz

$$\gamma(x) = \sum_{i=1}^{3} \beta_i \begin{pmatrix} \vec{\phi}_i \\ \vec{\phi}_i \end{pmatrix} e^{-\lambda_i x} + \beta_4 \begin{pmatrix} \vec{\phi}_4 \\ -\vec{\phi}_4 \end{pmatrix} e^{-\lambda_4 x}$$

- Parameters defined by using that the Majorana has zero energy: λ_i solve

$$\left(\frac{\lambda^2}{2m} + \mu\right)^2 + (\alpha_R\lambda \pm \Delta)^2 - V_z^2 = 0.$$

Majorana mode (2)

$$\gamma(x) = \sum_{i=1}^{3} \beta_i \begin{pmatrix} \vec{\phi}_i \\ \vec{\phi}_i \end{pmatrix} e^{-\lambda_i x} + \beta_4 \begin{pmatrix} \vec{\phi}_4 \\ -\vec{\phi}_4 \end{pmatrix} e^{-\lambda_4 x}$$

$$\lambda_1 = \lambda_2^* = i\lambda_0 + \delta$$

$$\lambda_3 = \lambda_0 - \delta$$

$$\lambda_4 = \lambda_0 + \delta$$

$$\lambda_0 = \sqrt{2m}\,(V_z^2 - \Delta^2)^{1/4}$$

$$\vec{\phi}_i = (\lambda_i^2/(2m) + V_z, -\Delta - \alpha_R \lambda_i)^T \quad \text{for} \quad i = 1, 2, 3$$

$$\vec{\phi}_4 = (\lambda_4^2/(2m) + V_z, \Delta - \alpha_R \lambda_4)^T$$

Lead modes

- Lead Hamiltonian: $H_L = (k^2/2m_L - \mu)\sigma_0\tau_z$
- Decompose lead modes

$$\Psi_L = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} e^{ik_F x} + \begin{pmatrix} d_{e\uparrow} \\ d_{e\downarrow} \\ d_{h\uparrow} \\ d_{h\downarrow} \end{pmatrix} e^{-ik_F x}$$

incoming spin up electron

outgoing mode: electron & hole, spin up & down

Boundary conditions

- At boundary: wave function continuous (Ψ_S = general SC wave function)

$$\Psi_L(x)|_{x=0} = \Psi_S(x)|_{x=0}$$

- Current continuous: $J_x \Psi_L(x)|_{x=0} \; = \; J_x \Psi_S(x)|_{x=0} \, ,$

$$J_x = \frac{\partial H_{1D}(k)}{\partial k}|_{k \to -i\partial_x} = \begin{pmatrix} -i\partial_x/m & -i\alpha_R \\ i\alpha_R & -i\partial_x/m \end{pmatrix} \tau_z .$$

Andreev and normal reflection

- Solve boundary condition equations, find scattering matrix to order $\mathcal{O}(\alpha_R^0)$
- Scattering matrix for an electron $\begin{pmatrix}\Psi_\uparrow(k)\\ \Psi_\downarrow(k)\end{pmatrix}$ to be reflected into a hole $\begin{pmatrix}\Psi_\uparrow(-k)^\dagger\\ \Psi_\downarrow(-k)^\dagger\end{pmatrix}$

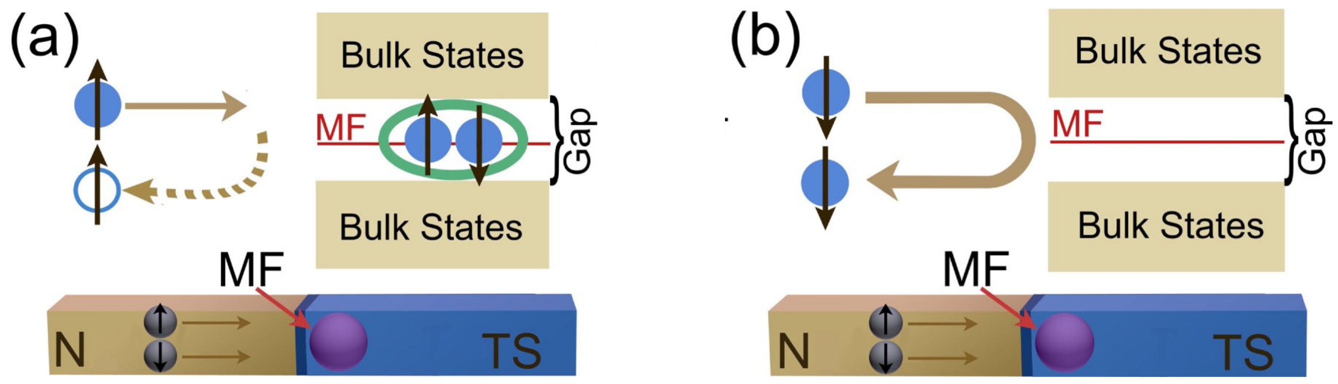
$$r_{he}(V_z) = \begin{pmatrix} \frac{V_z-\sqrt{V_z^2-\Delta^2}}{2V_z} & -\frac{\Delta}{2V_z} \\ -\frac{\Delta}{2V_z} & \frac{V_z+\sqrt{V_z^2-\Delta^2}}{2V_z} \end{pmatrix}$$

- Scattering matrix for an electron $\begin{pmatrix}\Psi_\uparrow(k)\\ \Psi_\downarrow(k)\end{pmatrix}$ to be reflected into an electron

$$r_{ee}(V_z) \;=\; r_{he}(-V_z)e^{i\chi(k)}, \;\text{where}\; e^{i\chi(k)} \;=\; \frac{k/m_L - i\lambda_0/m}{k/m_L + i\lambda_0/m}$$

Andreev and normal reflection at $\mathcal{O}(\alpha_R^0)$

- Result with
$$|s_0\rangle = |\uparrow, \vec{n}\rangle$$
$$|u_0\rangle = |\downarrow, \vec{n}\rangle$$



$$\vec{n} = \begin{pmatrix} \sin(\theta)\cos(\varphi) \\ \sin(\theta)\sin(\varphi) \\ \cos(\theta) \end{pmatrix} \quad , \quad \cos(\theta/2) = -\frac{\Delta}{\mathcal{N}} \quad , \quad \sin(\theta/2)\, e^{i\varphi} = \frac{V_z + \sqrt{V_z^2 - \Delta^2}}{\mathcal{N}}$$

- $r_{he}\vec{s}_0 = \vec{s}_0^{\,*}$ and $r_{ee}\vec{s}_0 = 0.$ $\quad$ $\vec{s}_0$-mode reflected as hole

  $r_{ee}\vec{u}_0 = e^{i\chi}\vec{u}_0.$ $\quad$ $\vec{u}_0$-mode reflected as electron

2) Numerics

Numerical model

- Tight-binding model

$$H_S = \sum_{i>0,\sigma} -t\psi^\dagger_{Si\sigma}\psi_{Si+1,\sigma} + (V_z\sigma - \mu + 2t)\psi^\dagger_{Si\sigma}\psi_{Si\sigma}$$
$$-\frac{1}{2}\alpha_R\sigma\psi^\dagger_{Si\sigma}\psi_{Si+1,-\sigma} + \Delta\psi^\dagger_{Si\sigma}\psi^\dagger_{Si-\sigma} + h.c.$$

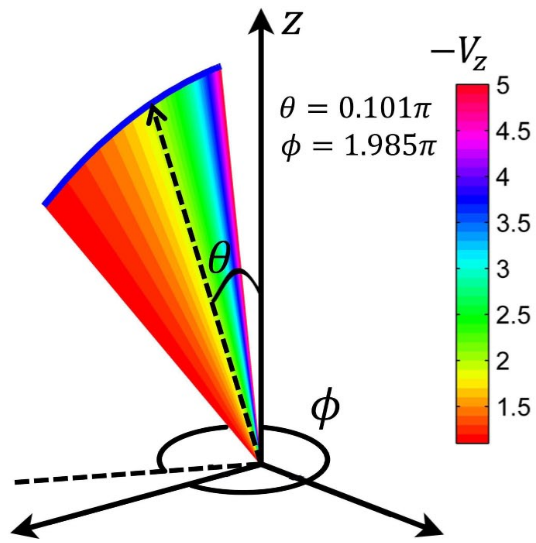
$$H_L = \sum_{i<0,\sigma} -t'\psi^\dagger_{Li\sigma}\psi_{Li+1,\sigma} - \mu\psi^\dagger_{Li\sigma}\psi_{Li\sigma} + h.c.$$

$$H_t = \sum_\sigma t_c\psi^+_{L0\sigma}\psi_{S1\sigma} + h.c..$$

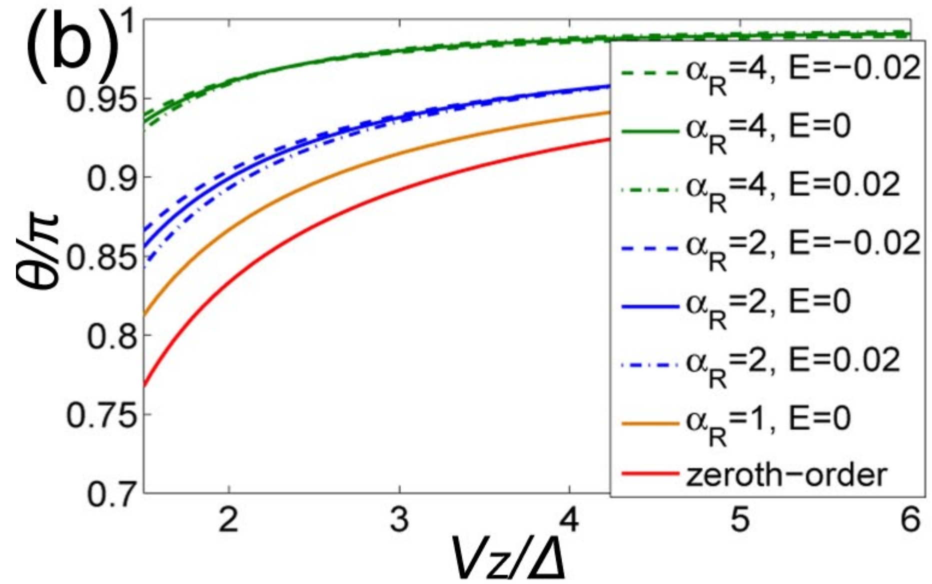
Andreev and normal reflection in topological regime

- Numerics vs. $\mathcal{O}(\alpha_R^0)$:

$$|s_n\rangle = |\uparrow, \vec{n}\rangle$$
$$|u_n\rangle = |\downarrow, \vec{n}\rangle$$



$\vec{n}$ for $\alpha_R = 2$



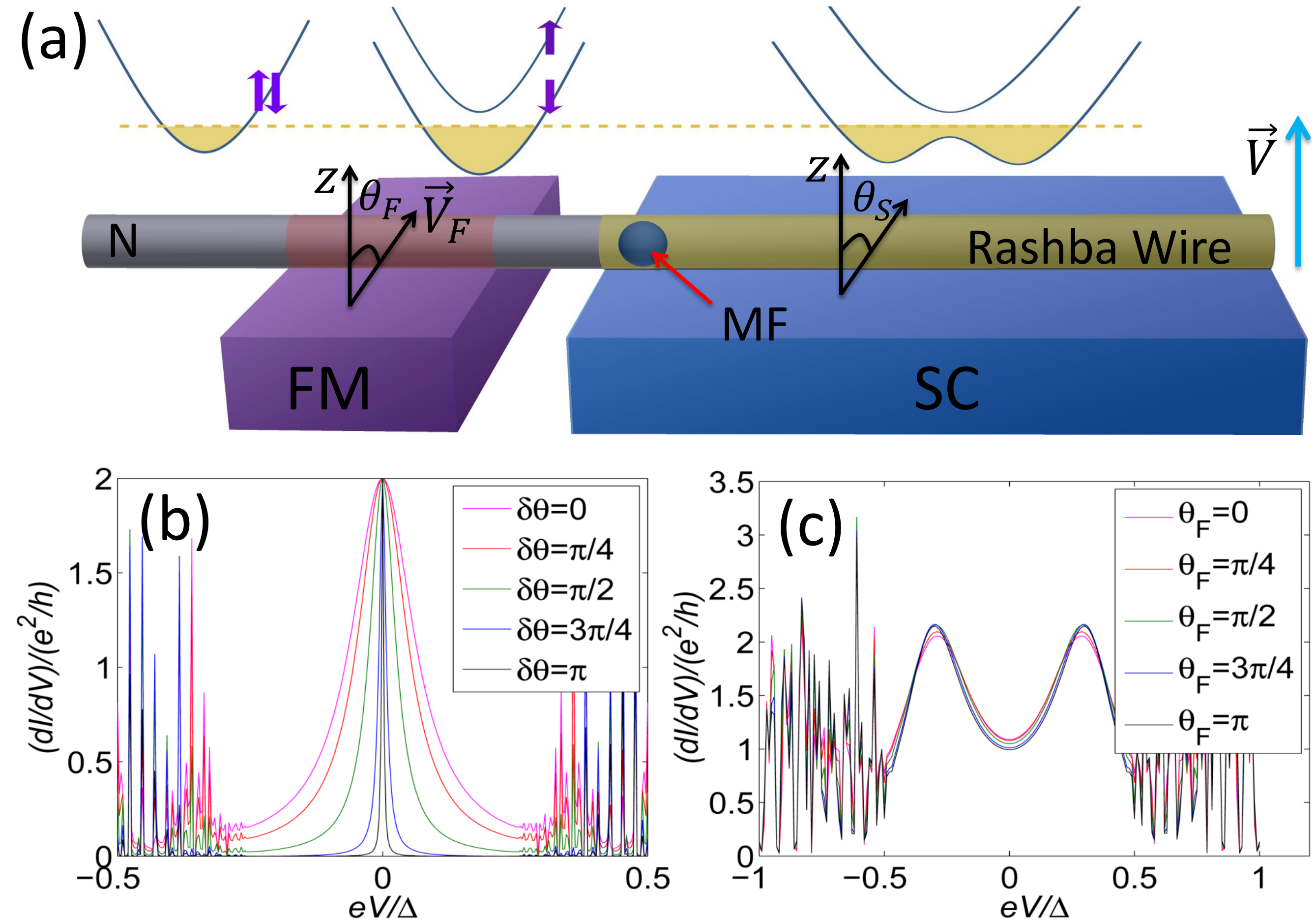
- $\vec{s}_n$ partially reflected as hole of same spin, partially as electron of same spin

$$\widetilde{r}_{ee}\vec{s}_n = m_1\,\vec{s}_n \quad \text{and} \quad \widetilde{r}_{he}\vec{s}_n = m_1'\,\vec{s}_n$$
$$\widetilde{r}_{ee}\vec{u}_n = m_2\,\vec{u}_n \quad \text{and} \quad \widetilde{r}_{he}\vec{u}_n = 0$$
$$|m_1| < 1 \quad , \quad |m_1'| \leq 1 \quad , \quad |m_2| = 1$$

- $\vec{u}_n$ always reflected as electron of same spin

Majorana and spin-polarized leads

- Setup:



topologial trivial

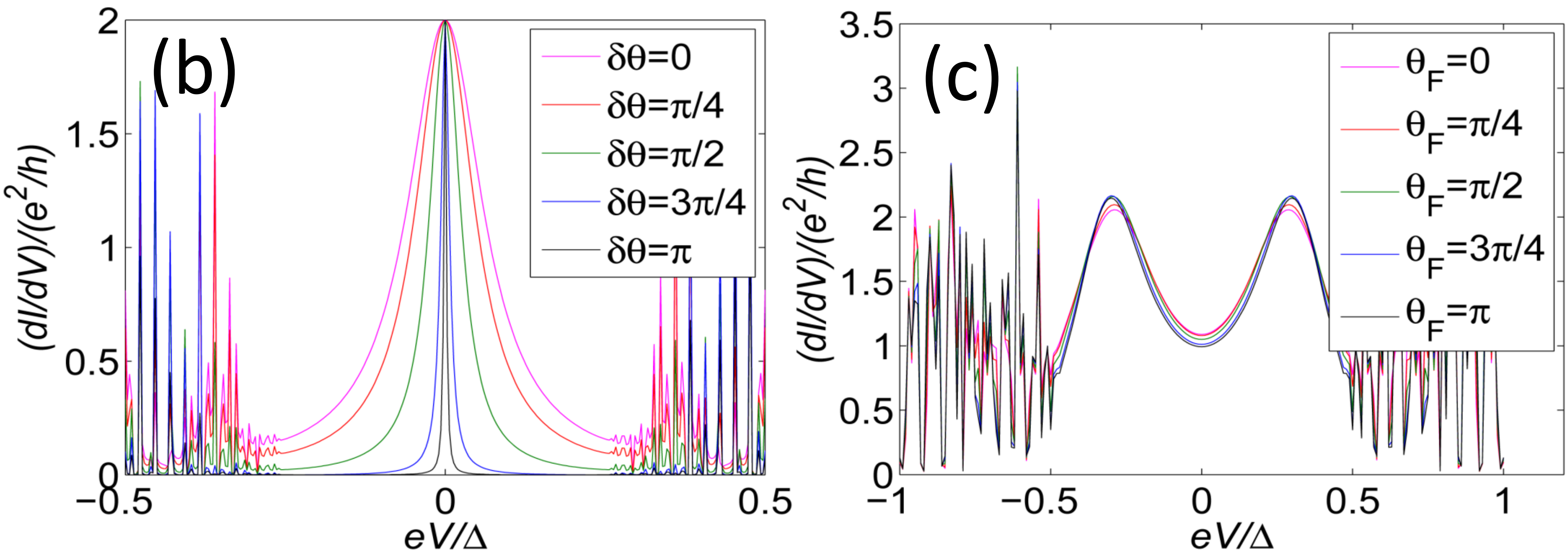
$$\delta\theta = \theta_F - \theta_S$$

- Normal leads spin || to spin mode coupling to Majorana, $\delta\theta \approx 0$:

most incoing electrons undergo equal spin Andreev reflection

width of conductance peak = coupling strength = large

Majorana and spin-polarized leads



topologial

trivial

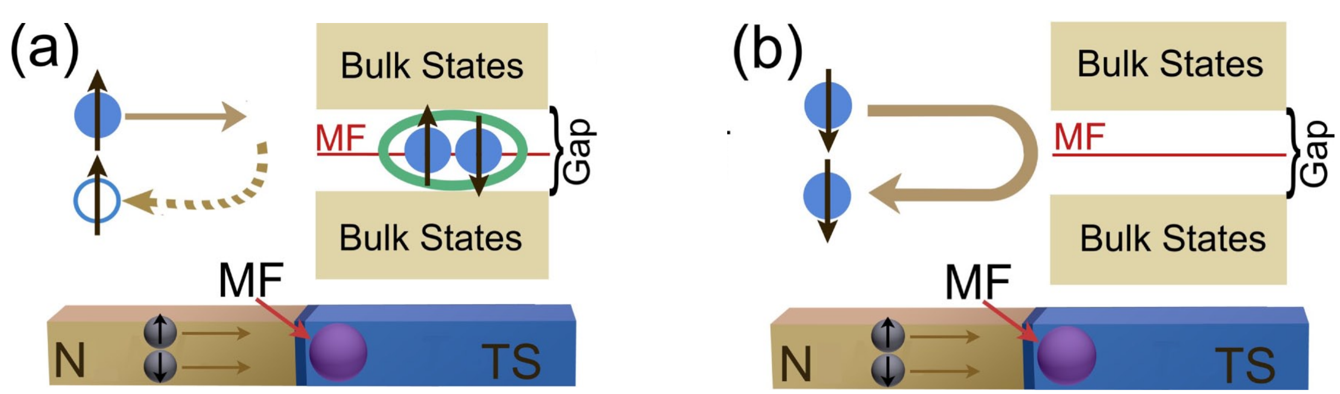
- Normal leads spin $||$ to spin mode coupling to Majorana, $\delta\theta \approx 0$:

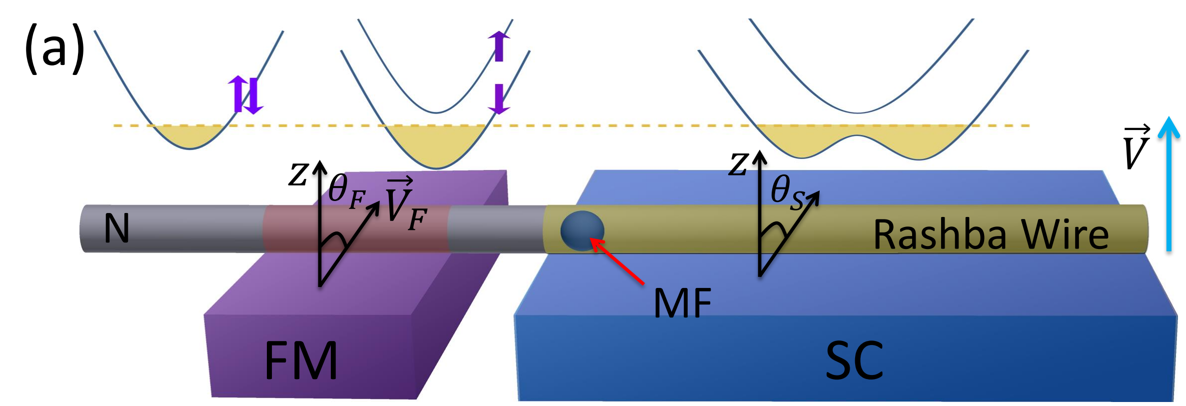
  most incoing electrons undergo equal spin Andreev reflection

  width of conductance peak = coupling strength = large

Conclusions

Conclusion— In short, we show in this work that MFs induce SESARs.* As a result, topological superconductors can be used as novel devices to generate spin-polarized currents in paramagnetic leads. The SESARs can also be used to detect MFs if spin-polarized leads are used.





* **s**elective **e**qual **s**pin **A**ndreev **r**eflection