## CMT Journal Club on September 10, 2013

Tobias Meng

### Majorana Fermion induced selective equal spin Andreev reflections

James J. He<sup>1</sup>, T. K. Ng<sup>1</sup>, Patrick A. Lee<sup>2</sup> and K. T. Law<sup>1\*</sup> <sup>1</sup> *Department of Physics, Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong, China* <sup>2</sup> *Department of Physics, Massachusetts Institute of Technology, Cambridge MA 02139, USA*

arXiv:1309.1528 (out Monday, September 9)  $\mathcal{L}(\mathcal{S})$  in  $\mathcal{A}$  000  $\mathcal{A}$  FOO (cert Mexceletro Ocertain polari $\mathcal{O}$ ) arxiv. Tous. To so four information of the spin polarization of the spin polarization of the electrons of the

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### Majorana Fermion induced selective equal spin Andreev reflections

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a this work we find that Majorana fermions induce selective equal spin. Andrew reflection spin polarization and the majorant reflection are always reflected as reflected as  $\text{Cov}(t)$ , in which incoming circulous when  $\text{Cov}(t)$  polarization in the idad are interesting rando france functions of the spin-polarization directions of the creations of the spin-polarization directions of the creations of the creations of the contractions of the creations of the contractions of the contractions Andreev reflected channel is selected by the Majorana fermions. Moreover, electrons with opposite spin polarization are always reflected as electrons with unchanged spin. As a result, the charge leads. We point out that SESARs can also be used (ta) detect Majorana fermions) in topological superconductors. In this work, we find that Majorana fermions induce selective equal spin Andreev reflections (SESARs), in which incoming electrons with certain spin polarization in the lead are reflected as counter propagating holes with the same spin. The spin polarization direction of the electrons of this current in the lead is spin-polarized. Therefore, a topological superconductor which supports Majorana fermions can be used as a novel device to create fully spin-polarized currents in paramagnetic



property, a MF has only half the degrees of freedom of

a Dirac fermion and two spatially separated MFs can

form a non-local Dirac fermion. This special property

 $I = \{1, 2, \ldots, N-1\}$  and  $N-1$  is  $N-1$  is  $N-1$  is  $N-1$  is  $N-1$ 

an anti-particle of itself. Due to this self-Hermitian self-Hermitian self-Hermitian self-Hermitian self-Hermi<br>Due to this self-Hermitian self-Hermitian self-Hermitian self-Hermitian self-Hermitian self-Hermitian self-Her

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flections [7, 8], electron teleportations [9, 10], enhanced

### System and idea in pictures In this work, we find that Majorana fermions induce selective equal spin Andreev reflections



(b) Electron of other spin species: ME always reflected as an electron of this spin. a Dirac fermion and two spatially separated MFs can form a non-local Dirac fermion. This special property



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 $\mathbb{R}^n$  is a Majorana fermion (MF)  $\mathbb{R}^n$ 

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$$
H_T=H_L+H_c,
$$

$$
H_L = -iv_F \sum_{\alpha \in \uparrow / \downarrow} \int_{-\infty}^{+\infty} \psi_{\alpha}^{\dagger}(x) \partial_x \psi_{\alpha}(x) dx,
$$

$$
H_c = \tilde{t}\gamma[a\psi_\uparrow(0) + b\psi_\downarrow(0) - a^*\psi_\uparrow^\dagger(0) - b^*\psi_\downarrow^\dagger(0)].
$$

mined by the self-Hermitian property of the MF = *†*.

$$
\gamma = \gamma^\dagger \text{ and } \quad \widetilde{t} \in \mathbb{R} \quad \text{and} \quad |a|^2 + |b|^2 = 1.
$$
 September 13

"(0) *<sup>b</sup>*⇤ *†*

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Moreover, MFs in condensed matter systems obey non-

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$$

**coupling to MF**  $\gamma = \gamma^{\dagger}$  and  $\widetilde{t} \in \mathbb{R}$  and  $|a|^2 + |b|^2 = 1$ . the lead is described by  $\frac{1}{\sqrt{2}}$  is describ  $\overline{\phantom{a}}$ **pling to MF**  $\gamma = \gamma^{\dagger}$  and  $\tilde{t} \in \mathbb{R}$  and  $|a|^2 + |b|^2 = 1$ .  $\frac{1}{13}$ **COUDLING TO ME**  $\alpha = \alpha^{\dagger}$  and  $\widetilde{f} \subset \mathbb{D}$  and  $|a|^2 + |b|^2 = 1$  $\frac{1}{2}$  /  $\frac{1}{2}$  /  $\frac{1}{2}$  and  $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$  .  $\widetilde{\mathcal{H}} = \mathcal{H}^{\dagger}$  and  $\widetilde{t} \in \mathbb{R}$  and  $|a|^2 + |b|^2 = 1.$ 

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Moreover, MFs in condensed matter systems obey non-

 $\overline{1}$ 

# Reminder: Andreev reflection

• Normal metal / superconductor interface: Andreev reflection possible



incoming electron of spin  $\sigma \rightarrow$  hole of spin  $\bar{\sigma}$ 

• Important for instance for Andreev bound states in dots, crossed Andreev reflections (entanglement), ...

### Spin selectivity of the Majorana bound state  $\overline{\phantom{a}}$ an isel<br>Duni se

• Majorana only couples to half of the modes in the normal lead: ten as: property, a MF has only half the degrees of freedom of a Diagorana<br>Magazarta

 $\mapsto$ 

1 *†*

MF

↵2"*/*#  $H_c = \tilde{t} \gamma [a \psi_{\uparrow} (0) + b \psi_{\downarrow} (0) - a^* \psi_{\uparrow}^{\dagger} (0) - b^* \psi_{\downarrow}^{\dagger} (0)].$ fractional  $H$  $\begin{array}{|c|c|c|c|c|}\hline \multicolumn{1}{|c|}{\quad \ \ \, \text{c}}\ \hline \multicolumn{$ 

• Define **e** Define

form a non-local Dirac fermion. This special property

of MFs leads to several interesting phenomena such as

**Define** K.T. Law, P. A. Lee, T. K. Ng, PRL **103**, 237001 (2009) K.T. Law, P. A. Lee, T. K. Ng, PRL **103**, 237001 (2009)

↵(*x*)@*<sup>x</sup>* ↵(*x*)*dx,*

$$
|\Psi_1\rangle=a|\psi_{\uparrow}\rangle+b|\psi_{\downarrow}\rangle\equiv|\uparrow,\vec{n}\rangle
$$

general form of coupling between the MF end states and the MF end states and the MF end states and the MF end<br>The MF end state in the MF end state in the MF end states and the MF end states and the MF end states and the  $\ket{\Psi_2} = -b^* \ket{\psi_\uparrow} + a^* \ket{\psi_\downarrow} \equiv \ket{\downarrow, \vec{n}}$ in fault  $|\Psi_2\rangle$  $h^*|_2$ ,  $\lambda + a^*|_2$ ,  $\lambda = | + \vec{a} \rangle$  $\mathcal{O}$  |  $\varphi$   $\uparrow$  /  $\alpha$  |  $\varphi$   $\downarrow$  /  $\cdots$  |  $\downarrow$  ,  $\mathcal{O}$  /

$$
\vec{n} = \begin{pmatrix} \sin(\theta)\cos(\varphi) \\ \sin(\theta)\sin(\varphi) \\ \cos(\theta) \end{pmatrix} \qquad \theta = 2\arctan\left(\frac{|b|}{|a|}\right) \qquad \varphi = \arg\left(\frac{b}{a}\right)
$$

as electrons with unchanged spin. The control of t<br>The control of the c

#### Spin selectivity of the Majorana bound state enlootivity of the Majorana hound state the lead is described by *Hc*, where *t*  $\overline{\phantom{a}}$ general form of coupling between the MF end state and Spin selectivity of the Majorana bound state  $\overline{\phantom{a}}$ an isel<br>Duni se

• Majorana only couples to half of the modes in the normal lead: *a* and *b* are complex numbers. The form of *H<sup>c</sup>* is determined by the self-Hermitian property of the MF = *†*. property, a MF has only half the degrees of freedom of a Diagorana<br>Magazarta



Here, *H<sup>L</sup>* describes the normal lead with spin up and spin

down electrons "*/*#(*x*) and Fermi velocity *v<sup>F</sup>* . The most

topological superconductor (TS) with MF end states. The

be a spin singlet if spin is not conserved due to spin-orbit

<sup>2</sup> = 1. It

<sup>2</sup> )*<sup>T</sup>* are

"(0) *<sup>b</sup>*⇤ *†*

$$
H_T = H_L + H_c,
$$

$$
H_L = -iv_F \sum_{\alpha \in 1/2} \int_{-\infty}^{+\infty} \Psi^{\dagger}_{\alpha}(x) \partial_x \Psi_{\alpha}(x) dx,
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$$

as electrons with unchanged spin. The control of t<br>The control of the c

 $\bar{\Psi}_{\Omega}$  decouples iunction — like hard wall normal reflection only  $\frac{3}{13}$   $\frac{4}{2}$  decouples. Junction – like riard wall, normal relievable only<br> $\frac{1}{2}$ <sup>2</sup> )*<sup>T</sup>* , while the  $\Rightarrow \Psi_2$  decouples: junction = like hard wall, normal reflection only *<sup>s</sup>*<sup>2</sup> = ( sin ✓ <sup>2</sup> *, e<sup>i</sup>* cos ✓ the lead is described by *Hc*, where *t* ny<br>———————  $\Rightarrow$   $\Psi_2$  decouples: junction = like hard wall, normal reflection only topological superconductors which host MFs has become one of the most important subjects in condensed matter

form a non-local Dirac fermion. This special property

of MFs leads to several interesting phenomena such as

fractional Josephson e↵ects [2–6], resonant Andreev re-

flections [7, 8], electron teleportations [9, 10], enhanced

in fault-tolerant quantum computations [18, 19]. Due

to these remarkable properties of MFs, the search for

### Scattering of electrons at Majorana  $\overline{2}$ *Hectrons at Majo*  $\triangle$ ctrone an anti-particle of itself. Due to this self-Hermitian | Scatter

becomes

 $\mathbb{R}^n$  is a Majorana fermion (MF)  $\mathbb{R}^n$ 

a Dirac fermion and two spatially separated MFs can

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Moreover, MFs in condensed matter systems obey non-

one of the most important subjects in condensed matter



Without loss of generality, one can set *|a|*

is important to note that using a unitary transformation

<sup>1</sup> = *a* "+*b* # and <sup>2</sup> = *b*⇤ "+*a*⇤ #, the Hamiltonian

*†*

• Scattering matrix for incoming electron of energy E in state and outgoing electrons (holes) with energy *E* relative the Fermion incoming electron of energy  $\epsilon$  in state  $|\Psi_1\rangle = |+\rangle$ *<sup>s</sup>*<sup>1</sup> <sup>=</sup> *<sup>|</sup>a|*(1*, b/a*)*<sup>T</sup>* = (cos ✓ <sup>2</sup> *, e<sup>i</sup>* sin ✓ <sup>2</sup> )*<sup>T</sup>* , while the  $\alpha$  ncoming electron of energy E in state  $|\Psi_1\rangle\equiv |\uparrow,\vec{n}\rangle$  $\begin{array}{c} \bullet \text{ Scatterin} \end{array}$ [11] and even resonant [12] crossed Andreev reflections.  $\mathbb{F}$  for incorplination algebra of opensy  $\mathbb{F}$  in otota  $\left| \mathcal{N} \right|$  is  $\left| \rightarrow \infty \right\rangle$ to no modifing ciccuotion of charge  $\mathsf{L} \cap \mathsf{S}$  with  $\mathsf{R} \cap \mathsf{C}$  with  $\mathsf{R} \cap \mathsf{C}$  with  $\mathsf{R} \cap \mathsf{C}$ 

 $10P(1)$  incoming electro totally decoupled from the MF. Denoting the incoming incoming electron

↵(*x*)@*<sup>x</sup>* ↵(*x*)*dx,*

<sup>2</sup> <sup>+</sup> *<sup>|</sup>b<sup>|</sup>*

tial are denoted by *m* and *µ* respectively. The Zeeman

coupling strength. The Pauli matrices *<sup>i</sup>* and ⌧*<sup>i</sup>* act on

cupies the semi-infinite space with *x* 0 and a magnetic

field with magnitude *V<sup>z</sup>* is applied along the *z*-direction,

there exists a MF end state localized near *x* = 0 in the

state satisfies the condition *H*1*D*(*k* ! *i*@*x*) = 0 with

*†* = . In general, the Majorana mode can be written

where *<sup>i</sup>* are the four solutions of the following two quar-

Suppose the one dimensional superconducting wire oc-

the spin and particle-hole space respectively.

field is denoted by *V*

2 = 1. It is a 1. It is

topological regime when *V<sup>z</sup>*

tic equations with positive real parts

(*x*) = X

*,* (3)

<sup>2</sup> )*<sup>T</sup>* are

n = h~

that the 1 electrons are reflected as 1 electrons are reflected as 1 holes with the 1 electrons are reflected a<br>The 1 holes with the 1 ho

*s*1*|*~*|*~

*s*1i =

$$
\left(\begin{matrix}\Psi_{1E}(+)\\ \Psi_{1E}^{\dagger}(+) \end{matrix}\right)=\frac{1}{\Gamma+iE}\left(\begin{array}{cc} iE & \Gamma \\ \Gamma & iE \end{array}\right)\left(\begin{matrix}\Psi_{1E}(-)\\ \Psi_{1E}^{\dagger}(-) \end{matrix}\right)
$$
outgoing hole

$$
\Gamma=2\tilde{t}^2/v_F
$$

have spins parallel to the direction  $\mathcal{L}_{\mathcal{A}}$  to the direction  $\mathcal{L}_{\mathcal{A}}$  to the direction  $\mathcal{L}_{\mathcal{A}}$ 

### Scattering of electrons at Majorana an anti-particle of itself. Due to this self-Hermitian  $\overline{2}$ *Hectrons at Majo*

becomes



energy mediate incoming electrontal line inside the scattering matrix of the scattering matrix of the N/TS and N<br>1991 respectively, the scattering matrix of the S/TS and N/TS and N/TS and N/TS and N/TS and N/TS and N/TS an

tial are denoted by *m* and *µ* respectively. The Zeeman

coupling strength. The Pauli matrices *<sup>i</sup>* and ⌧*<sup>i</sup>* act on

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the spin and particle-hole space respectively.

field is denoted by *V*

topological regime when *V<sup>z</sup>*

tic equations with positive real parts

(*x*) = X

FIG. 1: A paramagnetic normal lead (N) is coupled to a

topological superconductor (TS) with MF end states. The

coupling. (b) Electrons with opposite spin are totally reflected

n = h~

*s*1*|*~*|*~

*s*1i =

$$
\left(\begin{array}{c}\Psi_{1E}(+)\\ \Psi_{1E}^{\dagger}(+) \end{array}\right)=\frac{1}{\Gamma+iE}\left(\begin{array}{cc} iE & \Gamma\\ \Gamma & iE \end{array}\right)\left(\begin{array}{c}\Psi_{1E}(-)\\ \Psi_{1E}^{\dagger}(-) \end{array}\right)
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have spins parallel to the direction  $\mathcal{L}_{\mathcal{A}}$  to the direction  $\mathcal{L}_{\mathcal{A}}$  to the direction  $\mathcal{L}_{\mathcal{A}}$ 

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### Scattering of electrons at Majorana to the Fermi energy as 1*E*() ( *†*  $\sim$  Coottoring of plantrons of Majorana Fermion induced selective equal spin Andreev reflections J. Hen, T. Hen, T. Hen, T. K. Ngang J. K. Ngang J. K. Ngang Lee2 and K. Lee2 and K. Lee2 and K. Lee2 and K. Le

 $s_p$  is  $s_p$  in  $s_p$  in  $s_p$ 

topological superconductor (TS) with MF end states. The

current in the lead is spin-polarized. Therefore, a topological superconductor which supports Majo-

which an electron is reflected as a hole with the same spin. The same spin. The same spin. The same spin. The s



*<sup>s</sup>*<sup>1</sup> <sup>=</sup> *<sup>|</sup>a|*(1*, b/a*)*<sup>T</sup>* = (cos ✓

 $\overline{\phantom{a}}$  $t_{\rm b}$ , we can this priendinenon wiajorana induced selective equal s  $p(\lambda)$ From the spinors ~ *s*<sup>1</sup> and ~ *s*2, we note that <sup>1</sup> electrons and induced colocity oqual opins final corrention  $(SFSAR)$ <sup>"</sup> (sin ✓ cos *,*sin ✓ sin *,* cos ✓), and <sup>2</sup> electrons have opcurrent in the lead is spin-polarized. Therefore, a topological superconductor which supports Majo- $\vert \quad$  "We call this phenomenon Majorana-induc leads. We point out that SESARs can also be used to detect Majorana fermions in topological  $\Box$  We call this phenomenon Maiorana-induced selective equal spin-Andree  $\blacksquare$ "We call this phenomenon Majorana-induced selective equal spin Andreev reflection (SESAR)"





 $S_{\rm S}$  and  $S_{\rm S}$  and  $S_{\rm S}$  orbit coupled superconducting superconducting superconducting superconducting superconducting  $S_{\rm S}$ 

Fig. 1: A paramagnetic normal lead (N) is considered to a paramagnetic normal lead (N) is coupled to a paramagnetic normal lead (N) is coupled to a paramagnetic normal lead (N) is coupled to a paramagnetic normal lead (N)

<sup>2</sup> *, e<sup>i</sup>* sin ✓

<sup>2</sup> )*<sup>T</sup>* , while the

spin polarization are always reflected as electrons with unchanged spin. As a result, the charge

### Towards more realistic setups current in the lead is spin-polarized. Therefore, a topological superconductor which supports Majorand fermions can be used as a novel device to consider the core to consider the current spin-polarized current leads. We point out that SESARS can also be used to detect Majorana fermions in topological majorana fermions i<br>That SESARS can also be used to detect Majorana fermions in topological majorana fermions in the second topolo

counter propagating holes with the same spin. The spin polarization direction of the electrons of this

Andreev reflected channel is selected by the Majorana fermions. Moreover, electrons with opposite

spin polarization are always reflected as electrons with unchanged spin. As a result, the charge

↵21*/*2

1<br>1910 - John Barnett, amerikansk matematik<br>1911 - Johann Barnett, amerikansk matematik

*H<sup>T</sup>* = *H<sup>L</sup>* + *Hc,*

So far, couping between Majorana  $\gamma$  and wire modes energy independent: So far, couping between I  $\epsilon$ en Majorana  $\gamma$  and wire 9*)*<br>"  $\alpha$  and  $\alpha$  n+ ical superconductor can be realized experimentally [33–

$$
H_c = \tilde{t}\gamma \left[a\psi_\uparrow(0) + b\psi_\downarrow(0) - a^*\psi_\uparrow^\dagger(0) - b^*\psi_\downarrow^\dagger(0)\right] \quad \text{with} \quad \frac{a}{b} = \text{const.}
$$



What about more realistic setup, e.g. Rashba wire + superconductor? *<sup>H</sup>*1*D*(*k*) = [( *<sup>k</sup>*<sup>2</sup> what about more realistic setup in a Rashba wire + superconductor?  $\mathcal{M}$  is consequented matter systems obey nonistic setup, e.g. Rasnba wire + superconductor?

Does Majorana couple to electron spin || same  $\vec{n}$  for any incoming energy? Without loss of generality, one can set *|a|* <sup>2</sup> <sup>+</sup> *<sup>|</sup>b<sup>|</sup>* Does Majorana couple to electron spin  $\mid$  same  $\vec{n}$  for any incom energy?  $A$  and  $A$  and  $A$  and  $A$  and  $A$  and  $A$  applications  $\mathbb{R}$ polarization can undergo equal spin  $\mathbb{R}^2$  $\mathsf b$  to electron spin $\parallel$ same  $\vec n$  for any incoming energy?

(2)

in realistic topological superconductors. Moreover, the

tonian for the semi-conductor wire in proximity to a sum-conductor  $\mathcal{L}$ 

momentum *k*, the e↵ective mass and the chemical poten-

tial are denoted by *m* and *µ* respectively. The Zeeman

coupling strength. The Pauli matrices *<sup>i</sup>* and ⌧*<sup>i</sup>* act on

e↵ective Hamiltonian cannot determine ~

superconductor as depicted in Fig.3a.

In the Nambu basis ( *<sup>k</sup>*"*, <sup>k</sup>*#*, †*

field is denoted by *V*

perconductor can be written as [29–32]:

property, a MF has only half the degrees of freedom of

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to these remarkable properties of MFs, the search for

topological superconductors which host MFs has become

one of the most important subjects in condensed matter

### Rashba nanowire + superconductor

In the Nambu basis ( *<sup>k</sup>*"*, <sup>k</sup>*#*, †*

• Setup: Rashba nanowire + proximity coupling to s-wave superconductor *k*"*, † k*#), the Hamil-

$$
H_{1D}(k) = \left[ \left( \frac{k^2}{2m} - \mu \right) \sigma_0 + \vec{V} \cdot \vec{\sigma} + \alpha_R k \sigma_y \right] \tau_z - \Delta \sigma_y \tau_y.
$$



● System: semi-infinite TSC wire + normal metal / ferromagnetic metal lead *eV* and di↵erent ↵*R*. The results are shown in Fig.2b.

<sup>1</sup>. This is consistent with the

The di↵erential conductance *dI/dV* , as a function of

voltage bias *V* between the lead and the superconductor,

is shown in Fig.2a. As expected, the zero bias conduc-

tance is quantized to 2*e*<sup>2</sup>*/h* as the MF couples to only a

To study the spin polarization vector ~n = h~*sn|*~*|*~*sn*i, we

plot the angle ✓ calculated from the tight-binding model

The zeroth order analytic result at zero bias, which is

single conducting channel of the lead.

### Rashba nanowire + superconductor

In the Nambu basis ( *<sup>k</sup>*"*, <sup>k</sup>*#*, †*

• Setup: Rashba nanowire + proximity coupling to s-wave superconductor *k*"*, † k*#), the Hamil-

$$
H_{1D}(k) = \left[ (\frac{k^2}{2m} - \mu)\sigma_0 + \vec{V} \cdot \vec{\sigma} + \alpha_R k \sigma_y \right] \tau_z - \Delta \sigma_y \tau_y.
$$
  
\n
$$
H_{\text{magnetic field}}
$$
  
\n
$$
H_{\text{Rashba SOI}}
$$
  
\n
$$
H_{\text{Rashba SOI}}
$$
  
\n
$$
H_{\text{Rashba Wire}}
$$
  
\n
$$
H_{\text{Rashba Wire}}
$$
  
\n
$$
H_{\text{MIF}}
$$
  
\n
$$
H_{\text{MIF}}
$$
  
\n
$$
H_{\text{MIF}}
$$
  
\n
$$
H_{\text{MIF}}
$$

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The zeroth order analytic result at zero bias, which is

# 1) Analytics  $\mathcal{O}(\alpha_R^0)$

#### Majorana mode Suppose the one dimensional superconducting wire oc- $\mathbf{S}$  is the one dimensional superconduction  $\mathbf{S}$ Here, *H<sup>L</sup>* describes the normal lead with spin up and spin down electrons "*/*#(*x*) and Fermi velocity *v<sup>F</sup>* . The most  $\blacksquare$  a mode to the semi-conductor wire in proximity to a su-conductor wire in proximity to a su-conductor  $\mathcal{L}$ coupling strength. The Pauli matrices *<sup>i</sup>* and ⌧*<sup>i</sup>* act on rtychand particle-hole space respectively.

↵(*x*)@*<sup>x</sup>* ↵(*x*)*dx,*

<sup>2</sup> *, ei* cos ✓

#(0)]*.*

<sup>1</sup>*E*() ) and <sup>1</sup>*E*(+)

◆

<sup>2</sup> = 1. It

*s*1*|*~*|*~

<sup>2</sup> )*<sup>T</sup>* , while the

<sup>2</sup> *, e<sup>i</sup>* cos ✓

*s*2, we note that <sup>1</sup> electrons

<sup>2</sup> *, e<sup>i</sup>* cos ✓

*s*2, we note that <sup>1</sup> electrons

Evidently, the MF only couples to the <sup>1</sup> electrons with

Evidently, the MF only couples to the <sup>1</sup> electrons with

general form of coupling between the MF end state and

mined by the self-Hermitian property of the MF = *†*.

is important to note that using a unitary transformation

totally decoupled from the MF. Denoting the incoming

totally decoupled from the MF. Denoting the incoming

and outgoing electrons (holes) with energy *E* relative

and outgoing electrons (holes) with energy *E* relative

that the <sup>1</sup> electrons are reflected as <sup>1</sup> holes with the

same spin with Andreev reflection amplitude */*(+*iE*).

that the <sup>1</sup> electrons are reflected as <sup>1</sup> holes with the

same spin with Andreev reflection amplitude */*(+*iE*).

(sin ✓ cos *,*sin ✓ sin *,* cos ✓), and <sup>2</sup> electrons have op-

totally decoupled from the MF. Denoting the incoming

and outgoing electrons (holes) with energy *E* relative

same spin with Andreev reflection amplitude */*(+*iE*).

• Majorana mode present if "magnetic field > superconductivity" cupies the semi-infinite space with *x* 0 and a magnetic • Majorana mode present if "magnetic field > superconductivity"  $\begin{array}{ccc} \bullet & \Lambda A \end{array}$ *a* and *b* are complex numbers. The form of *H<sup>c</sup>* is deterperconductor can be written as [29–32]: Majorana mode present if "magnetic field > superconductivity"<br><sup>7</sup> <sup>2</sup>

$$
H_{1D}(k) = \left[ \left( \frac{k^2}{2m} - \mu \right) \sigma_0 + \vec{V} \cdot \vec{\sigma} + \alpha_R k \sigma_y \right] \Big] \tau_z - \Delta \sigma_y \tau_y.
$$
  
with  $V_z^2 > \mu^2 + \Delta^2$ 

cupies the semi-infinite space with *x* 0 and a magnetic

tial are denoted by *m* and *µ* respectively. The Zeeman

 $35$  by applying a magnetic field to a magnetic field to a spin-orbit coupled to a spin-orbit coupled

tial are denoted by *m* and *µ* respectively. The Zeeman

tial are denoted by *m* and *µ* respectively. The Zeeman

· Majorana mode ansatz <sup>1</sup>*E*() ) and <sup>1</sup>*E*(+) ie IVIaj<br>I  $\overline{\phantom{a}}$   $\bullet$  Ma  $Maiorana mode ansatz$ │ ● Ma *f*  $\alpha$ <sub>1</sub> = . In general, the Majorana mode can be written with the Majorana mode can be written with the majorana mode can be written with the majorana model with the majorana model with the majorana model with the majo

<u>,</u><br>⊥ 2022 — 2022 — 2022 — 2022 — 2022 — 2022 — 2022 — 2022 — 2022 — 2022 — 2022 — 2022

+ *µ*

$$
\gamma(x) = \sum_{i=1}^{3} \beta_i \begin{pmatrix} \vec{\phi}_i \\ \vec{\phi}_i \end{pmatrix} e^{-\lambda_i x} + \beta_4 \begin{pmatrix} \vec{\phi}_4 \\ -\vec{\phi}_4 \end{pmatrix} e^{-\lambda_4 x}
$$

• Parameters defined by using that the Majorana has zero energy:  $\,\lambda_i\,$  solve ˜<sup>2</sup>*/v<sup>F</sup>* . From the scattering matrix, we note tic equations with positive real parts with positive real parts with positive real parts with positive real pa<br>The contract of the contract o ˜<sup>2</sup>*/v<sup>F</sup>* . From the scattering matrix, we note where *<sup>i</sup>* are the four solutions of the following two quar- $\epsilon$ ters defined by using that the Majorana has zero energy:  $\,\lambda_i$  sol $\,$ Evidently, the MF only couples to the <sup>1</sup> electrons with *i*=1  $\overline{\phantom{a}}$ ~ *i* a ha  $\overline{z}$ d

$$
\left(\frac{\lambda^2}{2m} + \mu\right)^2 + (\alpha_R \lambda \pm \Delta)^2 - V_z^2 = 0.
$$

For realistic semi-conductor wires with <sup>2</sup>*m*↵<sup>2</sup>

+ (↵*R ±* )

state satisfies the condition *H*1*D*(*k* ! *i*@*x*) = 0 with

*<sup>z</sup>* <sup>2</sup> ⌧ <sup>1</sup>

*z* 2 *i* 2 <sup>1</sup>

<u>p</u>

<sup>2</sup> *<sup>V</sup>* <sup>2</sup>

*V* <sup>2</sup>

*<sup>z</sup>* = 0*.* (6)

<sup>2</sup> )*<sup>T</sup>* are

### Majorana mode (2) total<br>India and outgoing electrons (holes) with energy *E* relative state satisfies the condition *H*1*D*(*k* ! *i*@*x*) = 0 with

$$
\gamma(x) = \sum_{i=1}^{3} \beta_i \begin{pmatrix} \vec{\phi}_i \\ \vec{\phi}_i \end{pmatrix} e^{-\lambda_i x} + \beta_4 \begin{pmatrix} \vec{\phi}_4 \\ -\vec{\phi}_4 \end{pmatrix} e^{-\lambda_4 x}
$$

cupies the semi-infinite space with *x* 0 and a magnetic

$$
\lambda_1 = \lambda_2^* = i\lambda_0 + \delta
$$
  
\n
$$
\lambda_3 = \lambda_0 - \delta
$$
  
\n
$$
\lambda_4 = \lambda_0 + \delta
$$
  
\n
$$
\lambda_0 = \sqrt{2m} (V_z^2 - \Delta^2)^{1/4}
$$
  
\n
$$
\vec{\phi}_i = (\lambda_i^2 / (2m) + V_z, -\Delta - \alpha_R \lambda_i)^T \text{ for } i = 1, 2, 3
$$
  
\n
$$
\vec{\phi}_4 = (\lambda_4^2 / (2m) + V_z, \Delta - \alpha_R \lambda_4)^T
$$

<sup>4</sup> = [<sup>2</sup>

<sup>4</sup>*/*(2*m*) + *Vz,* ↵*R*4]

*T* .

for

~

<sup>1</sup>, ~

2 and 2

~

3, and 3, an

~

<sup>2</sup> )*<sup>T</sup>* , while the

<sup>2</sup> *, ei* cos ✓

<sup>1</sup>*E*() ) and <sup>1</sup>*E*(+)

 $\mathbb{E}_{\mathbf{z}}\left(\mathbf{z}\right)$  , the MF only couples to the 1 electrons with  $\mathbf{z}$ 

<sup>1</sup>*E*(+)) respectively, the scattering matrix of the N/TS

that the <sup>1</sup> electrons are reflected as <sup>1</sup> holes with the

same spin with Andreev reflection amplitude */*(+*iE*).

(sin ✓ cos *,*sin ✓ sin *,* cos ✓), and <sup>2</sup> electrons have op-

posite spins, where ~ is the Pauli vector. Therefore,

ple to the MF and undergo equal spin Andreev reflec-

tions, whereas electrons with opposite spin are totally

˜<sup>2</sup>*/v<sup>F</sup>* . From the scattering matrix, we note

*s*2, we note that <sup>1</sup> electrons

*s*1*|*~*|*~

n directions can cou-

#### Lead modes —<br>| mo  $\overline{C}$ des

• Lead Hamiltonian: Lead Hamiltonian:  $H_L \; = \; (k^2/2m_L \,- \, \mu) \sigma_0 \tau_z$ 

and 3*/4 = 0*  $\mu$  *, where*  $0$  *= 0*  $\mu$  *, where*  $0$  *= 0*  $\mu$  *, where*  $0$  *= p<sup>2</sup>m<sup>m</sup> , where*  $0$  *= p<sup>2</sup>mm , where*  $0$  *=* 

• Decompose lead modes tion in the lead at the Fermi energy can be written Lecompose lead modes *es*  $\frac{1}{2}$   $\frac{1$ 



note that the wavefunction has to satisfy the continuity

 $A$ ssuming that the lead can be described by th

 *<sup>S</sup>*(*x*) can be written as the linear combination of the incoming spin up electron and the outgoing mode: electron & hole, spin up & down

*V* <sup>2</sup>

*<sup>z</sup>* <sup>2</sup><sup>1</sup>*/*<sup>4</sup> and

*e*2*eik<sup>F</sup> <sup>x</sup>* +

*<sup>i</sup>* in Eq.5. We

*T*

*T* .

### Boundary conditions

 $\bullet$  At boundary: wave function continuous ( $\Psi_S$  = general SC wave function)

$$
\Psi_L(x)|_{x=0} = \Psi_S(x)|_{x=0}
$$

• Current continuous:  $\min_{\mathbf{C} \in \mathcal{C}} |I \mathbf{U}_\tau(\alpha)|$ condition *<sup>L</sup>*(*x*)*|x*=0 = *<sup>S</sup>*(*x*)*|x*=0 and current conserva-Current continuous:  $J_x \Psi_L(x)|_{x=0} \ = \ J_x \Psi_S(x)|_{x=0},$ continuous.  $J_x \Psi_L(\psi)/x=0$  =  $J_x \Psi_S(\psi)/x=0$ ,

$$
J_x = \frac{\partial H_{1D}(k)}{\partial k}|_{k \to -i\partial_x} = \begin{pmatrix} -i\partial_x/m & -i\alpha_R \\ i\alpha_R & -i\partial_x/m \end{pmatrix} \tau_z.
$$

trons ( *<sup>k</sup>*"*, <sup>k</sup>*#)*<sup>T</sup>* with the outgoing holes ( *†*

reflection matrix *rhe*, which relates the incoming elec-

 $\sqrt{2}$ 

found. At zeroth order in ↵*<sup>R</sup>* with ↵*<sup>R</sup>* ! 0, the Andreev

reflection matrix *rhe*, which relates the incoming elec-

**(al/dV)/(e<sup>2</sup>/h)**<br>a<br>d/dV)/(e<sup>2</sup>/h)

*k*#)*<sup>T</sup>*

*k*"*, †*

### Andreev and normal reflection (c) trix of the N/TS junction at the N/TS junction at the N/TS junction at the N/TS junction at the Fermi energy c<br>TS junction at the Fermi energy can be the Fermi energy can be the Fermi energy can be the Fermi energy can be

@*k |<sup>k</sup>*!*i*@*<sup>x</sup>*

@*H*1*D*(*k*)

• Solve boundary condition equations, find scattering matrix to order trix of the N/TS junction at the N/TS junction at the N/TS junction at the Fermi energy can be Fermi energy ca<br>The Fermi energy can be the Fermi ener foundary condition equations, inidiscattering matrix to order Solve boundary condition equations, find scattering matrix to order  $\,{\cal O}(\alpha_R^0)$   $\blacktriangleleft$ 

up and spin down incoming electrons, the spin down income  $\alpha$  the scattering mass  $\alpha$ 

 $-8.\overline{3}$ 

• Scattering matrix for an electron  $\left(\begin{array}{c} \frac{1}{2} \int_{\Omega} \sqrt{1-x^2} dx \end{array}\right)$  to be reflected into a hole  $(\Psi_{\ast}(k))$ the reflected into a hole intity for an electron  $\left(\frac{\dot{\Psi}^{+}_k(k)}{\Psi^{+}_k(k)}\right)$  to be reflected into a hole  $\bar{t}$  $\left(\Psi_{\uparrow}(k)\right)$  $\Psi_\downarrow(k)$ to be reflected into a hole  $\begin{pmatrix} \Psi_{\uparrow}(-k)^{\dagger} & \Psi_{\downarrow}(-k)^{\dagger} & \Psi_{\downarrow}(\cdot-k)^{\dagger} & \Psi_{\downarrow}(\cdot-k)^{\$  $\langle \mathbf{r} \rangle$  is the incomentation matrix  $\langle \mathbf{r} \rangle$ Scattering matrix for an electron  $\binom{\Psi \uparrow (k)}{\Psi \downarrow (k)}$  to be reflected into a hole  $\binom{\Psi \uparrow (-k)^{\dagger}}{\Psi \downarrow (-k)^{\dagger}}$ 

$$
r_{he}(V_z) = \begin{pmatrix} \frac{V_z - \sqrt{V_z^2 - \Delta^2}}{2V_z} & -\frac{\Delta}{2V_z} \\ -\frac{\Delta}{2V_z} & \frac{V_z + \sqrt{V_z^2 - \Delta^2}}{2V_z} \end{pmatrix} \tag{e}
$$

✓ *i*@*x/m i*↵*<sup>R</sup>*

*<sup>i</sup>*↵*<sup>R</sup> i*@*x/m* ◆

*<sup>i</sup>*↵*<sup>R</sup> i*@*x/m* ◆

• Scattering matrix for an electron  $\begin{pmatrix} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \end{pmatrix}$  to be reflected into an electron  $(\Psi_{\uparrow}(k))$  $\left(\frac{1}{\Psi} \sqrt{2\pi k} \right)$  to be reflected into an electron  $\left(\frac{1}{\Psi} \sqrt{k}\right)$ attering matrix for an electron  $\begin{pmatrix} \Psi_{\uparrow}(k) \ \mathbf{x} \end{pmatrix}$  to be reflected into an electron  $x$  $\left(\begin{array}{c}\n x \downarrow (n)\n \end{array}\right)$  $\left(\Psi_{\uparrow}(k)\right)$  $\Psi_{\downarrow}(k)$ ◆

$$
r_{ee}(V_z) = r_{he}(-V_z)e^{i\chi(k)}, \text{ where } e^{i\chi(k)} = \frac{k/m_L - i\lambda_0/m}{k/m_L + i\lambda_0/m}
$$

is the phase acquired by the reflected electrons at

*s*<sup>0</sup> = (cos ✓<sup>0</sup>

<sup>2</sup> *, e<sup>i</sup>*<sup>0</sup> sin ✓<sup>0</sup>

FIG. 2: = 1, *t* = 25, *t*

 $\Psi_{\downarrow}(-k)^{\dagger}$ 

<sup>2</sup> )*<sup>T</sup>* =

◆

The di↵erential conductance *dI/dV* of the N/TS junction as

Andrew and normal reflection at $\mathcal{O}(\alpha_R^0)$	
Result with $\begin{bmatrix}  s_0\rangle =  \uparrow, \vec{n}\rangle \\  u_0\rangle =  \downarrow, \vec{n}\rangle \end{bmatrix}$	(a) MEE
$\vec{n} = \begin{pmatrix} \sin(\theta)\cos(\varphi) \\ \sin(\theta)\sin(\varphi) \\ \cos(\theta) \end{pmatrix}$ , $\cos(\theta/2) = -\frac{\Delta}{N}$ , $\sin(\theta/2)e^{i\varphi} = \frac{1}{N}$	
$r_{he}\vec{s}_0 = \vec{s}_0^*$ and $r_{ee}\vec{s}_0 = 0$ .	$\vec{s}_0$ -mode reflected as hole
$r_{ee}\vec{u}_0 = e^{i\chi}\vec{u}_0$ .	$\vec{u}_0$ -mode reflected as electron

~

In the following sections, we first show, we first show, we first show, using an e $\mu$ 

↵*<sup>R</sup>* = 2 and di↵erent *Vx*. *V<sup>x</sup>* = 2 for the dashed vector (f)

 $\mathcal{F}(\mathcal{F})$  the polarization vector  $\mathcal{F}(\mathcal{F})$  the polarization vector  $\mathcal{F}(\mathcal{F})$ 

for the numerical results for small ↵*R*, is also presented. (c)-

 $\overline{\phantom{a}}$  and  $\overline{\phantom{a}}$  and  $\overline{\phantom{a}}$  and  $\overline{\phantom{a}}$  and  $\overline{\phantom{a}}$  and  $\overline{\phantom{a}}$  and  $\overline{\phantom{a}}$ 

n with *V<sup>x</sup>* = 2 and di↵erent ↵*R*. ↵*<sup>R</sup>* = 2 for the dashed

electrons with spinor ~

anti-parallel to ~

we calculate the scattering matrix of the scattering matrix of the N/TS junction  $\mathcal{M}^{\mathcal{M}}$ 

 $\sim$ n is determined by the properties of the topological sub-topological su-topological su-topological su-topological su-

perconductor. Importantly, electrons with opposite spin

*u*<sup>0</sup> = *e<sup>i</sup>*~

# 2) Numerics

#### Numerical model To further verify the analytic results and generalize the analytic results an urrericar riouer umerical model To further verify the analytic results and generalize the analytic results an anti-parallel to ~<br>anti-parallel to ~ anti-parallel to ~ anti-parallel to ~ anti-parallel to ~ anti-parallel to ~ anti-parallel<br>anti-parallel to ~ anti-parallel to ~ anti-parallel to ~ anti-parallel to ~ anti-parallel to ~ nd are totally reflected as electrons with the control of the control of the control of the control of the con<br>The control of the c

anti-parallel to ~

• Tight-binding model results to a radius to a result of a r<br>Unchanged spin since *r* we calculate the scattering model. The further verify the analytic results and generalize the analytic results and generalize the substantial control of the substantial control of the substantial control of the substantial control of the substantial control

$$
H_S = \sum_{i>0,\sigma} -t\psi^{\dagger}_{Si\sigma}\psi_{Si+1,\sigma} + (V_z\sigma - \mu + 2t)\psi^{\dagger}_{Si\sigma}\psi_{Si\sigma} - \frac{1}{2}\alpha_R\sigma\psi^{\dagger}_{Si\sigma}\psi_{Si+1,-\sigma} + \Delta\psi^{\dagger}_{Si\sigma}\psi^{\dagger}_{Si-\sigma} + h.c.
$$

0 respectively. The coupling between the coupling between the su-spectrum coupling between the su-spectrum coup<br>The su-spectrum coupling between the su-spectrum coupling between the su-spectrum coupling between the su-spec

we calculate the scattering matrix of the scattering matrix of the N/TS junction  $\mathcal{M}^{\mathcal{M}}$ 

*<sup>u</sup>*<sup>0</sup> = ( sin ✓<sup>0</sup>

no are totally reflected as electrons with  $\alpha$  as electrons with  $\alpha$  as electrons with  $\alpha$ 

<sup>2</sup> *, e<sup>i</sup>*<sup>0</sup> cos ✓<sup>0</sup>

n<sup>0</sup> = h~

*s*0*|*~*|*~

*s*0i =

 $f(x) = \frac{1}{2\pi} \int_0^x \frac{dx}{(x-x)^2} dx$ 

zeroth order result from ~

of the dashed vector. (c) ~

 $f(x)$ 

zeroth order result from ~

of the dashed vector. (c) ~

 $f(x)$ 

of the dashed vector. (c) ~

vector.

vector.

Here, ˜*r*

vector.

Here, ˜*r*

Here, ˜*r*

by ↵

and ~

by ↵

by ↵

*m*<sup>1</sup>

*V<sup>z</sup>* = 2 for the dashed vector. (d) ~

*V<sup>z</sup>* = 2 for the dashed vector. (d) ~

*V<sup>z</sup>* = 2 for the dashed vector. (d) ~

 $v_{\rm c}$ 

n as a function of *Vz*, for di↵erent ↵*<sup>R</sup>* and voltage bias. The

↵*<sup>R</sup>* = 2, *V<sup>z</sup>* = 2. (b) The angle ✓ of the polarization vector

for the numerical results for small ↵*R*, is also presented. (c)-

 $v_{\rm c}$ 

for the numerical results for small ↵*R*, is also presented. (c)-

↵*<sup>R</sup>* = 2 and di↵erent *Vx*. *V<sup>x</sup>* = 2 for the dashed vector (f)

 $v_{\rm c}$ 

n with *V<sup>x</sup>* = 2 and di↵erent ↵*R*. ↵*<sup>R</sup>* = 2 for the dashed

↵*<sup>R</sup>* = 2 and di↵erent *Vx*. *V<sup>x</sup>* = 2 for the dashed vector (f)

n with *V<sup>x</sup>* = 2 and di↵erent ↵*R*. ↵*<sup>R</sup>* = 2 for the dashed

↵*<sup>R</sup>* = 2 and di↵erent *Vx*. *V<sup>x</sup>* = 2 for the dashed vector (f)

n with *V<sup>x</sup>* = 2 and di↵erent ↵*R*. ↵*<sup>R</sup>* = 2 for the dashed

 $t$ 

matrix element of the retarded Green's function *G<sup>r</sup>* of

 $t_{\rm eff}$ 

the superconductor. The broadening function is denoted

 $t_{\rm eff}$ 

matrix element of the retarded Green's function *G<sup>r</sup>* of

the superconductor. The broadening function is denoted

 $\frac{d}{dt}$ 

the superconductor. The broadening function is denoted

 $\blacksquare$ 

 $\frac{d}{dt}$ 

matrix element of the retarded Green's function *G<sup>r</sup>* of

<sup>0</sup> where ↵ denotes electron (*e*) or hole (*h*). [*G<sup>r</sup>*]

<sup>0</sup> where ↵ denotes electron (*e*) or hole (*h*). [*G<sup>r</sup>*]

<sup>0</sup> where ↵ denotes electron (*e*) or hole (*h*). [*G<sup>r</sup>*]

di↵erent ↵*R*. ↵*<sup>R</sup>* = 2 for the dashed vector. (e) ~

di↵erent ↵*R*. ↵*<sup>R</sup>* = 2 for the dashed vector. (e) ~

di↵erent ↵*R*. ↵*<sup>R</sup>* = 2 for the dashed vector. (e) ~

$$
H_L = \sum_{i<0,\sigma} -t' \psi_{Li\sigma}^{\dagger} \psi_{Li+1,\sigma} - \mu \psi_{Li\sigma}^{\dagger} \psi_{Li\sigma} + h.c.
$$

$$
H_t = \sum_{\sigma} t_c \psi_{L0\sigma}^+ \psi_{S1\sigma} + h.c.
$$

### tion condition *J<sup>x</sup> <sup>L</sup>*(*x*)*|x*=0 = *J<sup>x</sup> <sup>S</sup>*(*x*)*|x*=0, where the ✓ *i*@*x/m i*↵*<sup>R</sup>* Andreev and normal reflection in topological regime

3



•  $\vec{s}_n$  partially reflected as hole of same spin, partially as electron of same spin •  $\vec{s}_n$  partially reflected  $\overrightarrow{C}$  portially  $U_{\ell}$  particuity

$$
\begin{aligned}\n\widetilde{r}_{ee}\vec{s}_n &= m_1 \,\vec{s}_n \quad \text{and} \quad \widetilde{r}_{he}\vec{s}_n = m'_1 \,\vec{s}_n \\
\widetilde{r}_{ee}\vec{u}_n &= m_2 \,\vec{u}_n \quad \text{and} \quad \widetilde{r}_{he}\vec{u}_n = 0 \\
|m_1| < 1 \quad , \quad |m'_1| \le 1 \quad , \quad |m_2| = 1\n\end{aligned}
$$

 $\overline{\phantom{a}}$  and  $\overline{\phantom{a}}$  a

 $\bullet$   $\vec{u}_n$  always reflected as electron of same spin  $\alpha_n$  anti-parallel to  $n$  $\lambda$  $\vec{u}_n$  always reflected as electron of same spin  $\overrightarrow{l}$ 

⌧*z.* (7)

flected as holes with the same spin. On the contrary,

condition *<sup>L</sup>*(*x*)*|x*=0 = *<sup>S</sup>*(*x*)*|x*=0 and current conserva-

current operator is

*<sup>r</sup>ee*(*Vz*) = *<sup>r</sup>he*(*Vz*)*e<sup>i</sup>*(*k*)

tor, we have *rhe*~*s*<sup>0</sup> = ~*s*⇤

*<sup>N</sup>* (*, V<sup>z</sup>* <sup>+</sup>p*<sup>V</sup>* <sup>2</sup>

trix of the N/TS junction at the Fermi energy can be

found. At zeroth order in ↵*<sup>R</sup>* with ↵*<sup>R</sup>* ! 0, the Andreev

reflection matrix *rhe*, which relates the incoming elec-

trons ( *<sup>k</sup>*"*, <sup>k</sup>*#)*<sup>T</sup>* with the outgoing holes ( *†*

the interface. Denoting ~*s*<sup>0</sup> = (cos ✓<sup>0</sup>

# Majorana and spin-polarized leads



the spin polarization direction direction of the income  $\sigma$  the incomendation of the incomendation of the income

• Normal leads spin  $||$  to spin mode coupling to Majorana,  $\delta \theta \approx 0$  : so that the current at finite bias is also spin-polarized. pected, the projection of ~n on the *z*-axis increases as *V<sup>z</sup>* increase in the other hand,  $\sim$ width of conductance peak = coupling strength = large Rashba strength. The ~n dependent on ↵*<sup>R</sup>* for fixed *V<sup>z</sup>* is superconductor (SC). The wire can support MF end states.  $\bullet$  Normal leads spin  $\parallel$  to spin mode coupling to Majorana,  $\delta \theta \approx 0$  : polarize the electrons of the wire. The schematic band strucmost incoing electrons undergo equal spin Andreev reflection  $\omega$ racture poar – coupling stronger – large width of conductance  $peak = coupling$  strength  $= large$ 

ductance as a function of  $\overline{a}$  in the topological regime with the topological regime with  $\overline{a}$ 

Majorana and spin-polarized leads



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 $\bullet$  Normal leads spin  $\parallel$  to spin mode coupling to Majorana,  $\delta \theta \approx 0$  : most incoing electrons undergo equal spin Andreev reflection width of conductance peak = coupling strength = large

**Conclusion**— In short, we show in this work that MFs induce SESARs. As a result, topological superconductors can be used as novel devices to generate spin-polarized currents in paramagnetic leads. The SESARs can also be used to detect MFs if spin-polarized leads are used. Induce SESARs. As a r James J. He<sup>1</sup>, T. K. Ng<sup>1</sup>, Patrick A. Lee<sup>2</sup> and K. T. Law<sup>1</sup>⇤ <sup>2</sup> *Department of Physics, Massachusetts Institute of Technology, Cambridge MA 02139, USA* can be used as nov  $A = \frac{1}{\sqrt{2}}$  reflected by the Majorana fermions. Moreover, electrons with opposite  $\mathbf{e}$ spin polarization are always reflection are always reflective as  $c$ current in the lead is spin-polarized. Therefore, a topological supports  $\mathbf{I}_{\text{max}}$ leads. We also detect that SESARS we use the SESARS can also be used the James J. He<sup>1</sup>, T. K. Ng<sup>1</sup>, Patrick A. Lee<sup>2</sup> and K. T. Law<sup>1</sup>⇤ <sup>1</sup> *Department of Physics, Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong, China* <sup>2</sup> *Department of Physics, Massachusetts Institute of Technology, Cambridge MA 02139, USA* (SESARs), in which incoming electrons with certain spin polarization in the lead are reflected as can be used as novel of the spin polarization of the spin. The spin polarization of the electrons of th Andreev reflected channel is selected by the Majorana fermions. Moreover, electrons with opposite currents in paramagne  $\epsilon$  current is spin-polarized. The supports  $\epsilon$  $r_{\rm{ho}}$  used to detect  $\rm{MFe}$ leads. We added to detect with  $S$ 

tion processes dominate. As a result, the conductance



 $\bigstar$ Selective equal spin Andreev reflection  $\downarrow$ the bulk gap of the  $\mathcal{L}_\text{max}$  electrons with a specific spin  $\mathcal{L}_\text{max}$  $\star$ the bulk gap of the  $\mathcal{L}_\mathcal{D}$  Electrons with a specific spin a specific spin a specific spin a specific spin a  $\downarrow$ *eV* and di↵erent ↵*R*. The results are shown in Fig.2b.

flexting  $\mathbb{R}$ 

 $\mathbf{1}$ 

Moreover, MFs in condensed matter systems obey non-

 $\mathbf{1}$ 

in fault-tolerant quantum computations [18, 19]. Due

to these remarkable properties of MFs, the search for