Nonequilibrium Model of Photon Condensation

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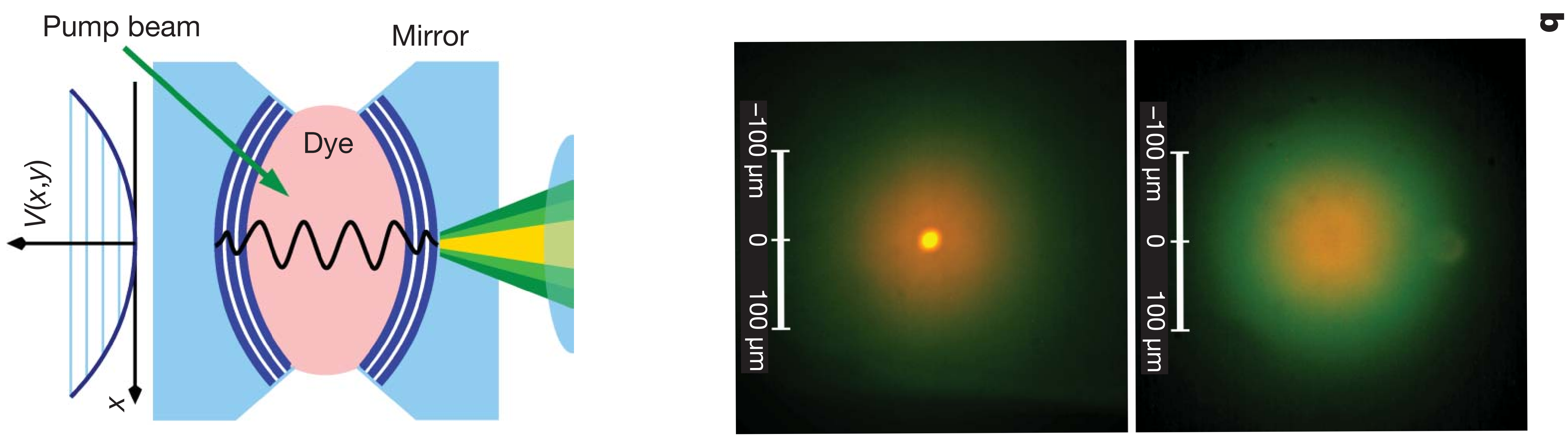
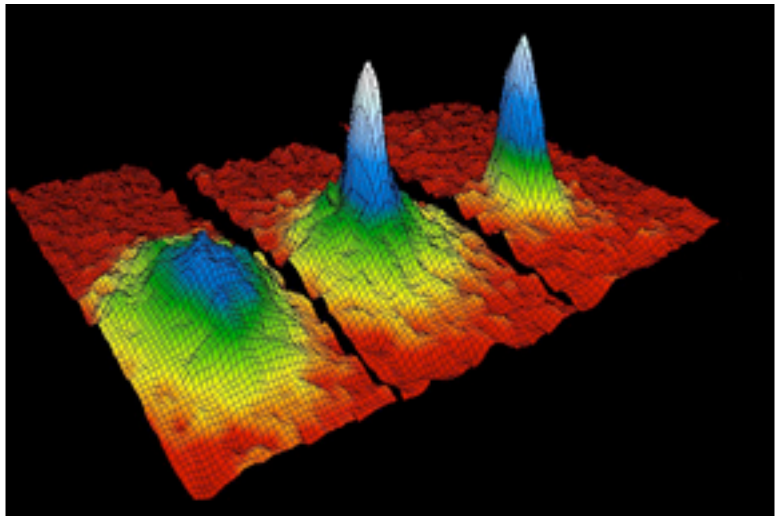
We develop a nonequilibrium model of condensation and lasing of photons in a dye filled microcavity. We examine in detail the nature of the thermalization process induced by absorption and emission of photons by the dye molecules, and investigate when the photons are able to reach a thermal equilibrium Bose-Einstein distribution. At low temperatures, or large cavity losses, the absorption and emission rates are too small to allow the photons to reach thermal equilibrium and the behavior becomes more like that of a conventional laser.



Bose condensation



ultracold atomic gases

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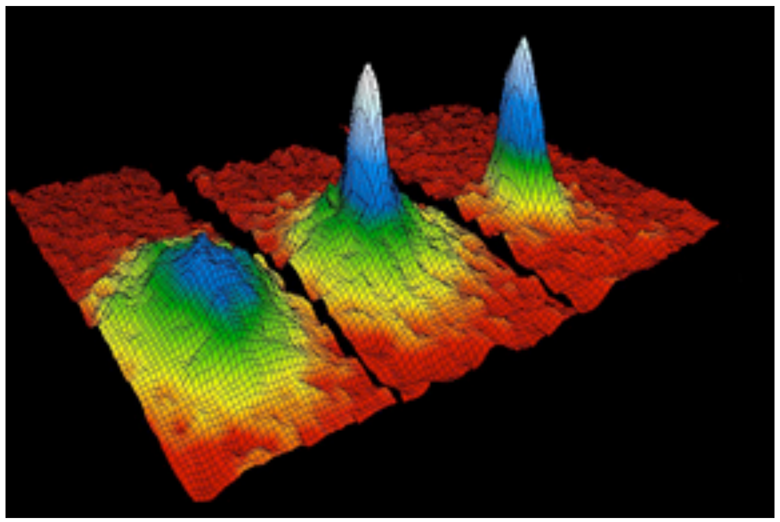

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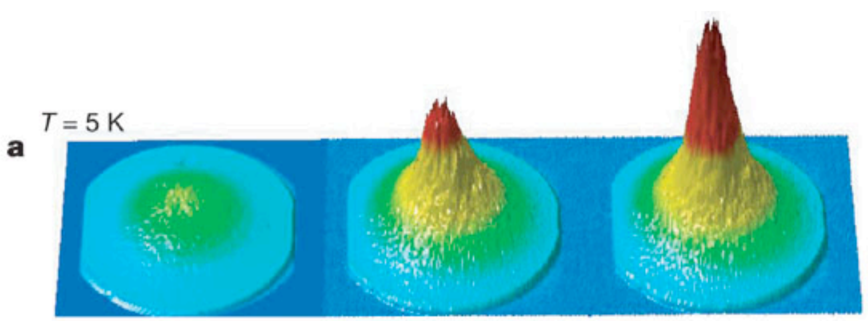

(exciton-)polariton

[3] J. Kasprzak, M. Richard, S. Kundermann, A. Baas, P. Jeambrun, J. M. J. Keeling, F. M. Marchetti, M. H. Szymaska, R. André, J. L. Staehli, V. Savona, P. B. Littlewood, B. Deveaud, and L. S. Dang, Nature (London) **443**, 409 (2006).

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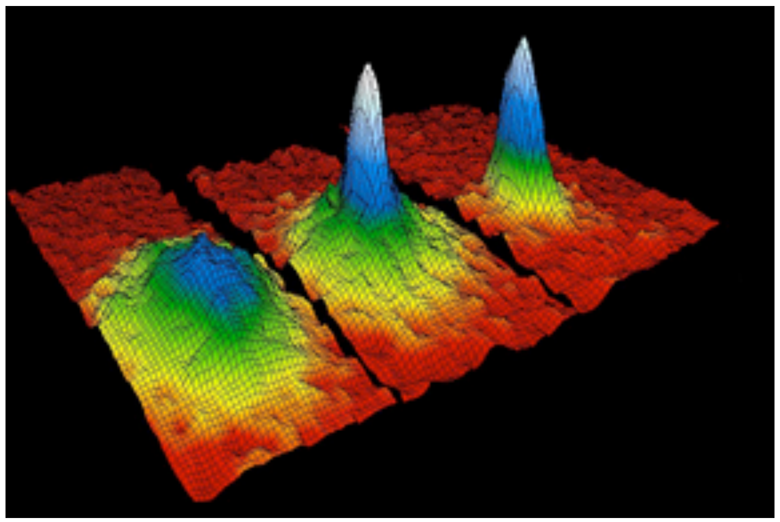

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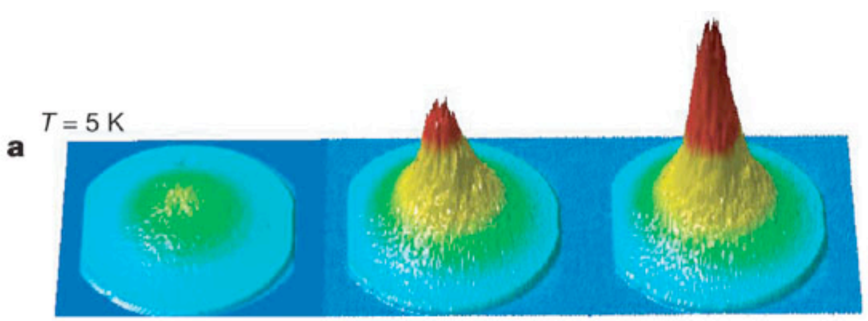

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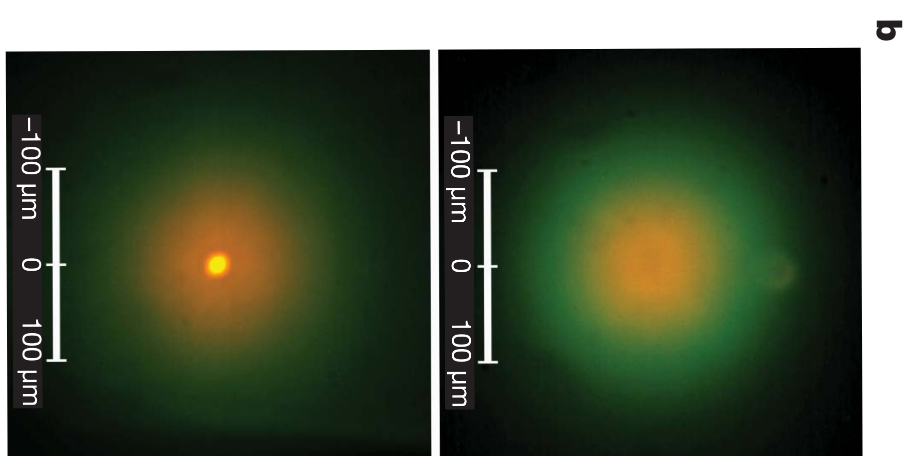
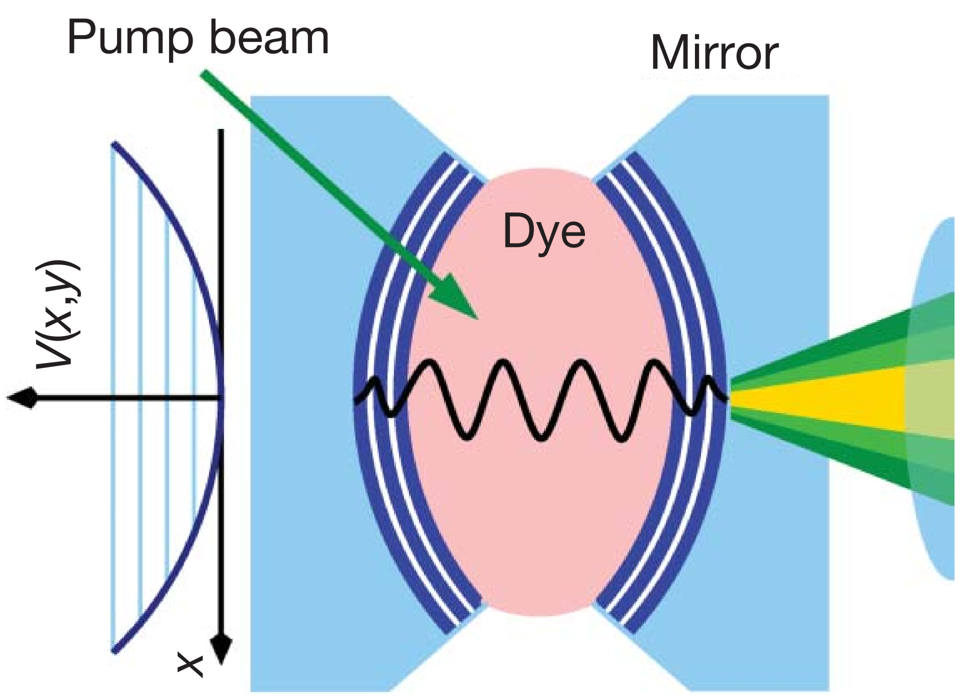


gas of photons

[10] J. Klaers, J. Schmitt, F. Vewinger, and M. Weitz, Nature (London) **468**, 545 (2010).

Model



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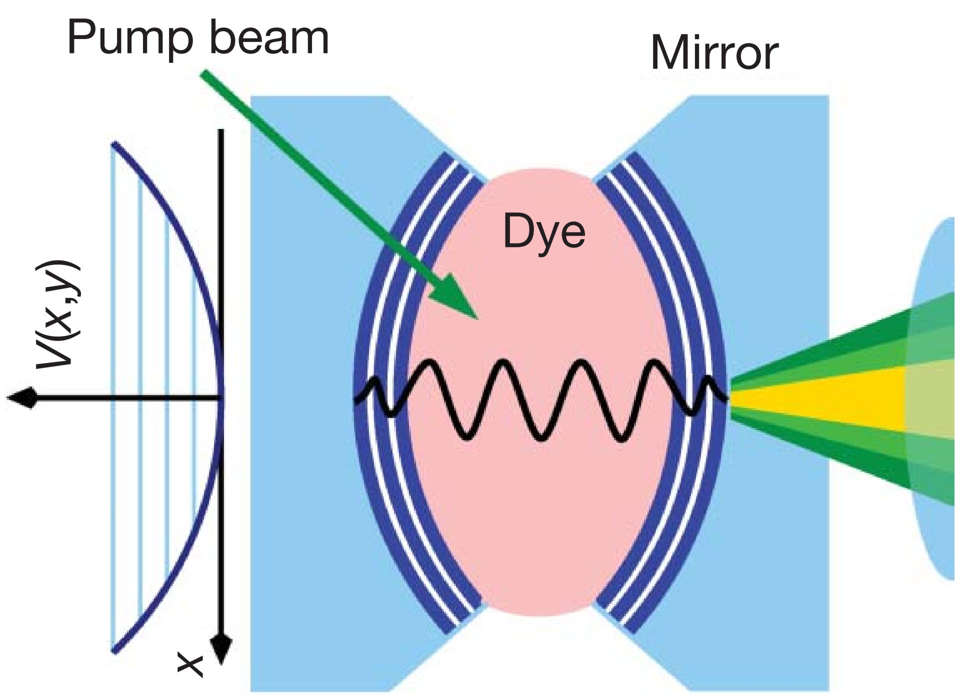
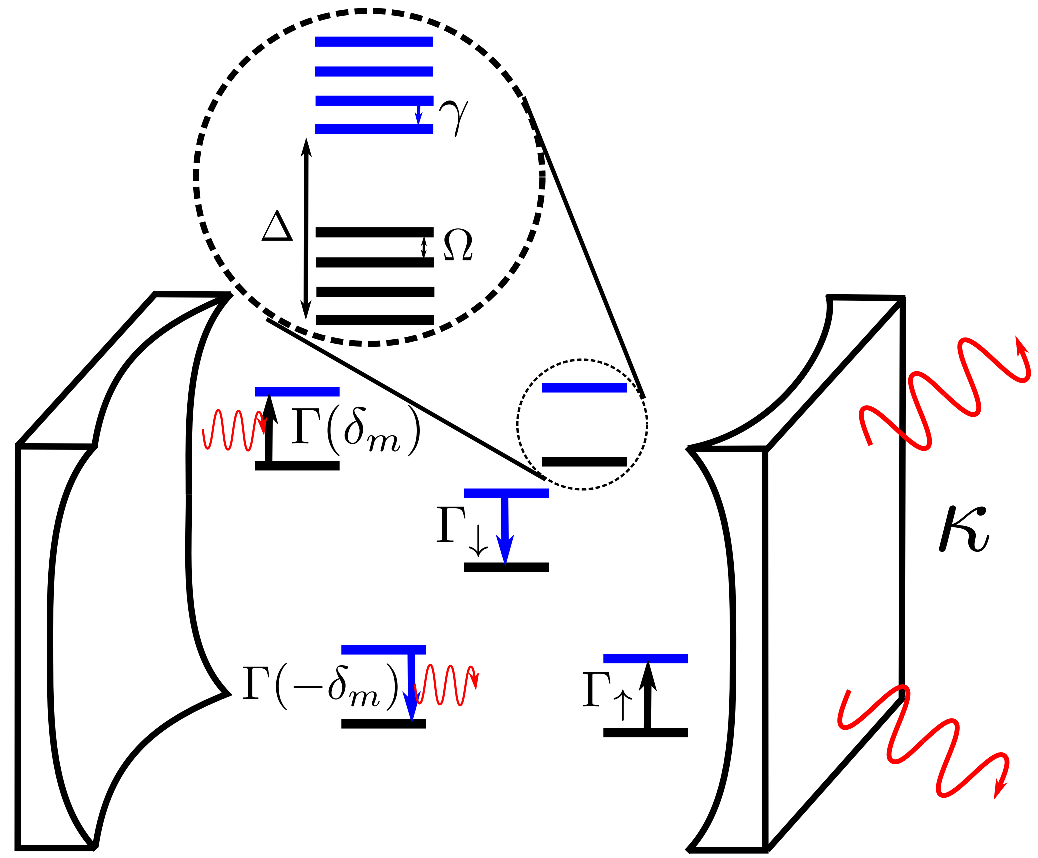
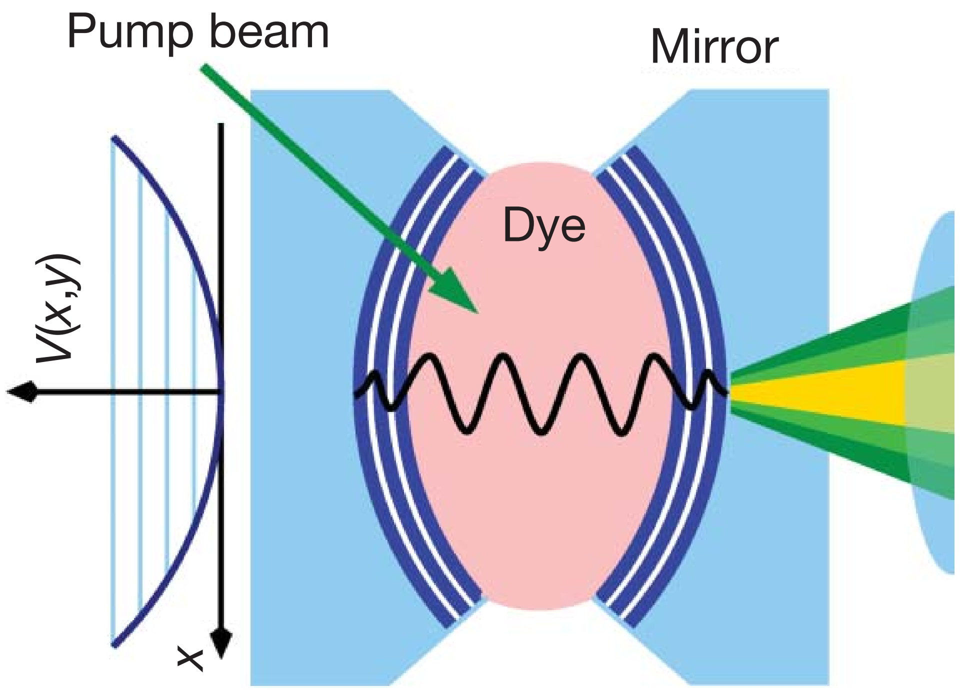
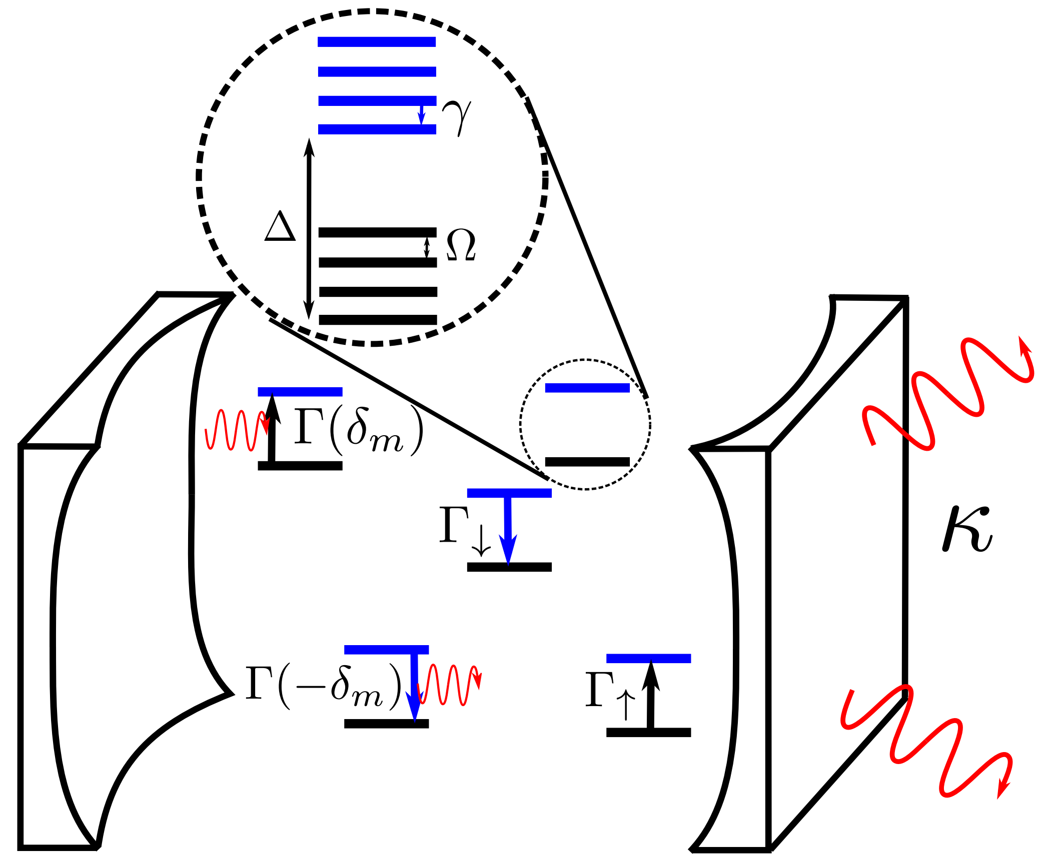

Pump beam
Mirror
Dye
V(x,y)
x


γ
Δ
Ω
$\Gamma(\delta_m)$
$\Gamma_\downarrow$
$\Gamma(-\delta_m)$
$\Gamma_\uparrow$
κ

Model





$$H = \sum_m \omega_m a_m^\dagger a_m + \sum_i \frac{\Delta}{2}\sigma_i^z + \Omega\left(b_i^\dagger b_i + \sqrt{S}\sigma_i^z(b_i + b_i^\dagger)\right)$$

$$+ g\sum_{m,i}(a_m\sigma_i^+ + a_m^\dagger\sigma_i^-), \qquad (1)$$

$$\omega_m = \omega_0 + m\epsilon$$
$$g_m = m + 1$$

Polaron transformation

$$H = \sum_m \omega_m a_m^\dagger a_m + \sum_i \frac{\Delta}{2}\sigma_i^z + \Omega\Big(b_i^\dagger b_i + \sqrt{S}\sigma_i^z(b_i + b_i^\dagger)\Big)$$

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$$H_0 = \sum_{m,i}\delta_m a_m^\dagger a_m + 0\,\sigma_i^z \qquad \delta_m = \omega_m - \Delta$$

$$H_g = g\sum_{m,i} a_m\sigma_i^+ e^{i\sqrt{S}\int_{-\infty}^t [b_i(t') + b_i^\dagger(t')]dt'} + H.c.$$

Fermi's Golden Rule

$$H_g = g \sum_{m,i} a_m \sigma_i^+ e^{i\sqrt{S}\int_{-\infty}^{t}[b_i(t')+b_i^\dagger(t')]dt'} + H.c.$$

$\langle n, \uparrow | H_g | n+1, \downarrow \rangle$ leads to photon-TLS coupling rates $\Gamma(\pm\delta_m)$

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$\langle n, \uparrow | H_g | n+1, \downarrow \rangle$ leads to photon-TLS coupling rates $\Gamma(\pm\delta_m)$

$$\Gamma(\omega) = 2\,\mathrm{Re}[K(\omega)]$$

$$K(\omega) = g^2 \int_0^\infty dt f(t) \mathrm{e}^{-(\Gamma_\uparrow+\Gamma_\downarrow)|t|/2} \mathrm{e}^{-i\omega t}, \qquad (3)$$

$$f(t) = \exp\left[-\frac{2S\gamma}{\pi}\int_{-\infty}^{\infty} d\nu \frac{2\sin^2\frac{\nu t}{2}\coth\frac{\beta\nu}{2} + i\sin\nu t}{(\Omega-\nu)^2+\frac{\gamma^2}{4}}\right]. \qquad (4)$$

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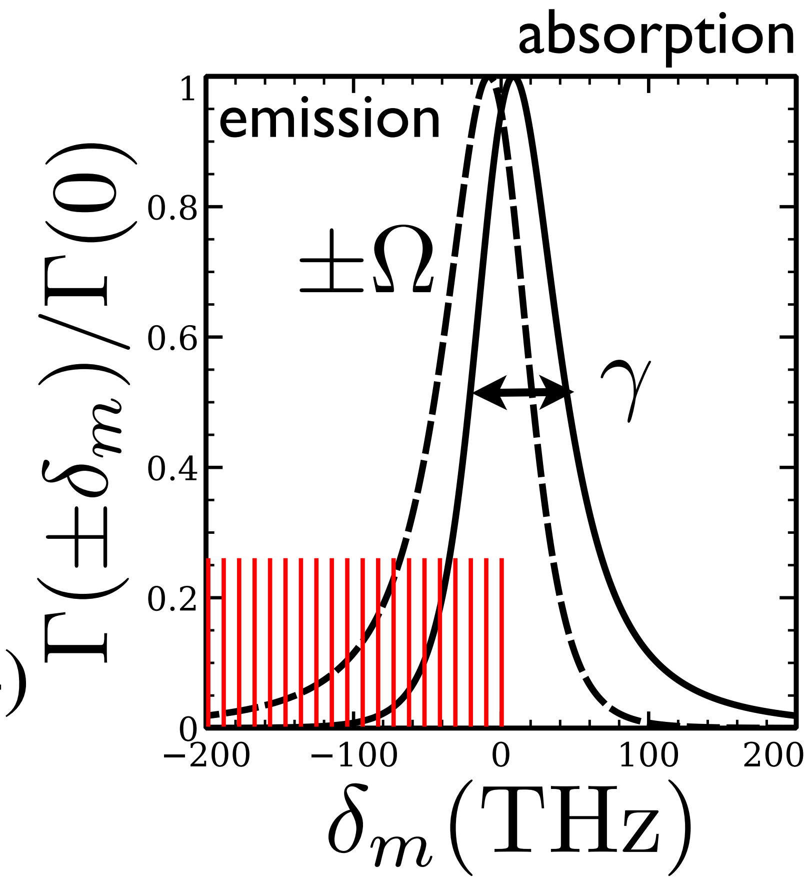

Effective master equation

$$\dot{\rho} = -i[H_0, \rho] - \sum_{i,m} \Big\{ \frac{\kappa}{2} \mathcal{L}[a_m] + \frac{\Gamma_\uparrow}{2} \mathcal{L}[\sigma_i^+] + \frac{\Gamma_\downarrow}{2} \mathcal{L}[\sigma_i^-]$$

$$+ \frac{\Gamma(-\delta_m)}{2} \mathcal{L}[a_m^\dagger \sigma_i^-] + \frac{\Gamma(\delta_m)}{2} \mathcal{L}[a_m \sigma_i^+] \Big\} \rho. \qquad (2)$$

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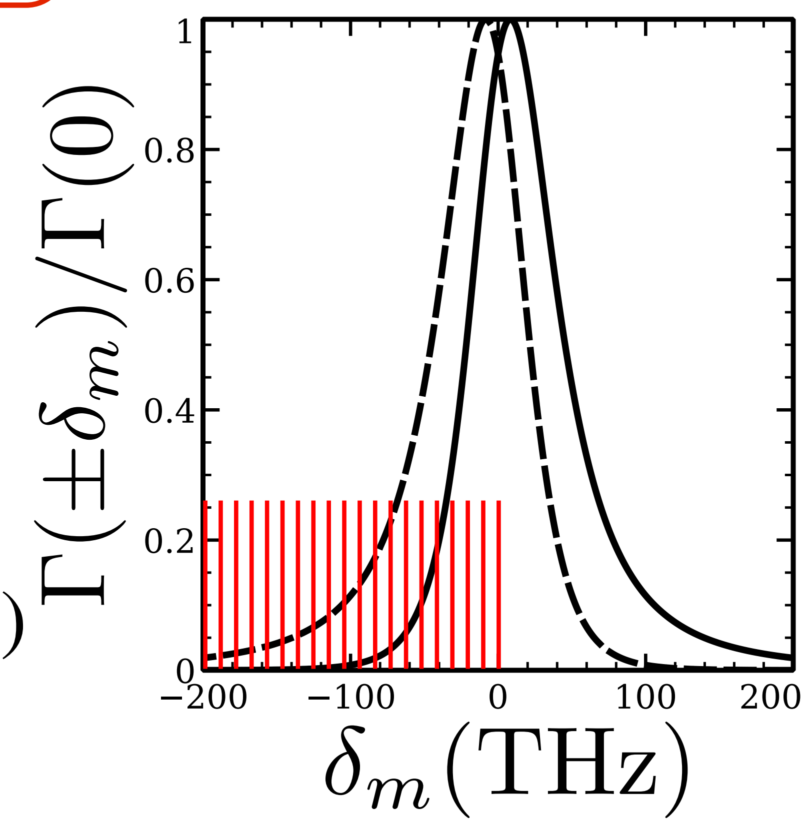


$$\mathcal{L}[X]\rho = \{X^\dagger X, \rho\} - 2X\rho X^\dagger$$

[29] M. Marthaler, Y. Utsumi, D. S. Golubev, A. Shnirman, and G. Schön, Phys. Rev. Lett. **107**, 093901 (2011).

Rate equation

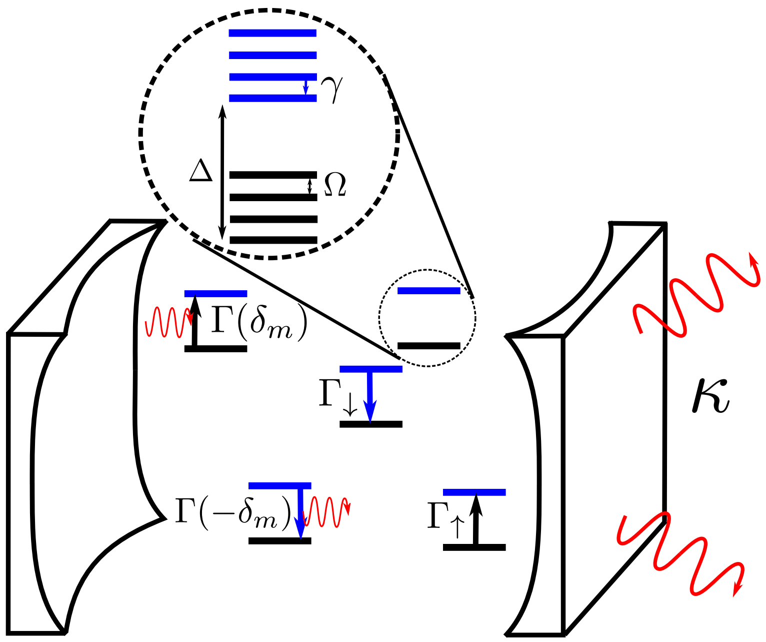
Adiabatically eliminating the TLS (dye molecules) [validity?]

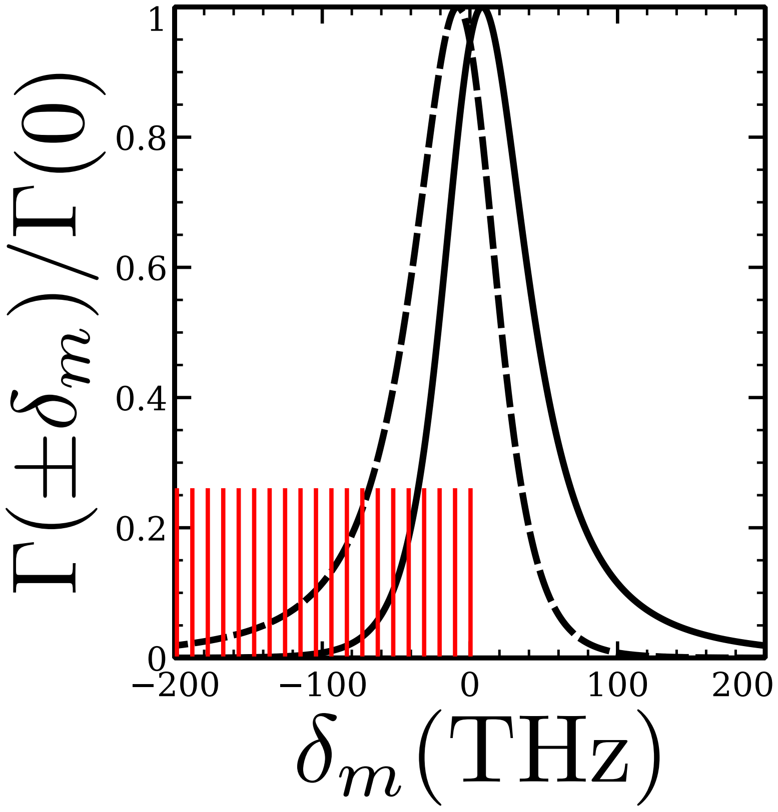
$$\frac{\partial n_m}{\partial t} = -\kappa n_m + N \frac{\Gamma(-\delta_m)(n_m+1)\tilde{\Gamma}_\uparrow - \Gamma(\delta_m) n_m \tilde{\Gamma}_\downarrow}{\tilde{\Gamma}_\uparrow + \tilde{\Gamma}_\downarrow}, \qquad (5)$$

$$\tilde{\Gamma}_\uparrow = \Gamma_\uparrow + \sum_m g_m \Gamma(\delta_m) n_m$$

$$\tilde{\Gamma}_\downarrow = \Gamma_\downarrow + \sum_m g_m \Gamma(-\delta_m)(n_m+1)$$

$$\Gamma(\omega) = \Gamma(\omega, g, S, \beta, \Omega, \gamma)$$





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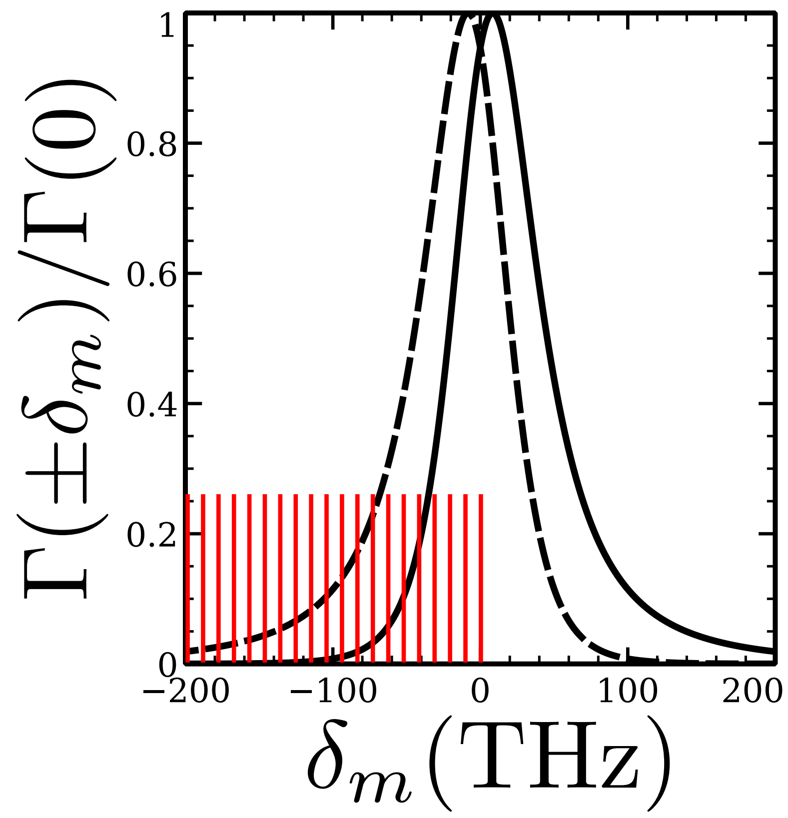
Equilibrium limit $\kappa, \Gamma_\uparrow, \Gamma_\downarrow \to 0$

$$\Gamma(\delta) = \mathrm{e}^{\beta\delta}\Gamma(-\delta)$$

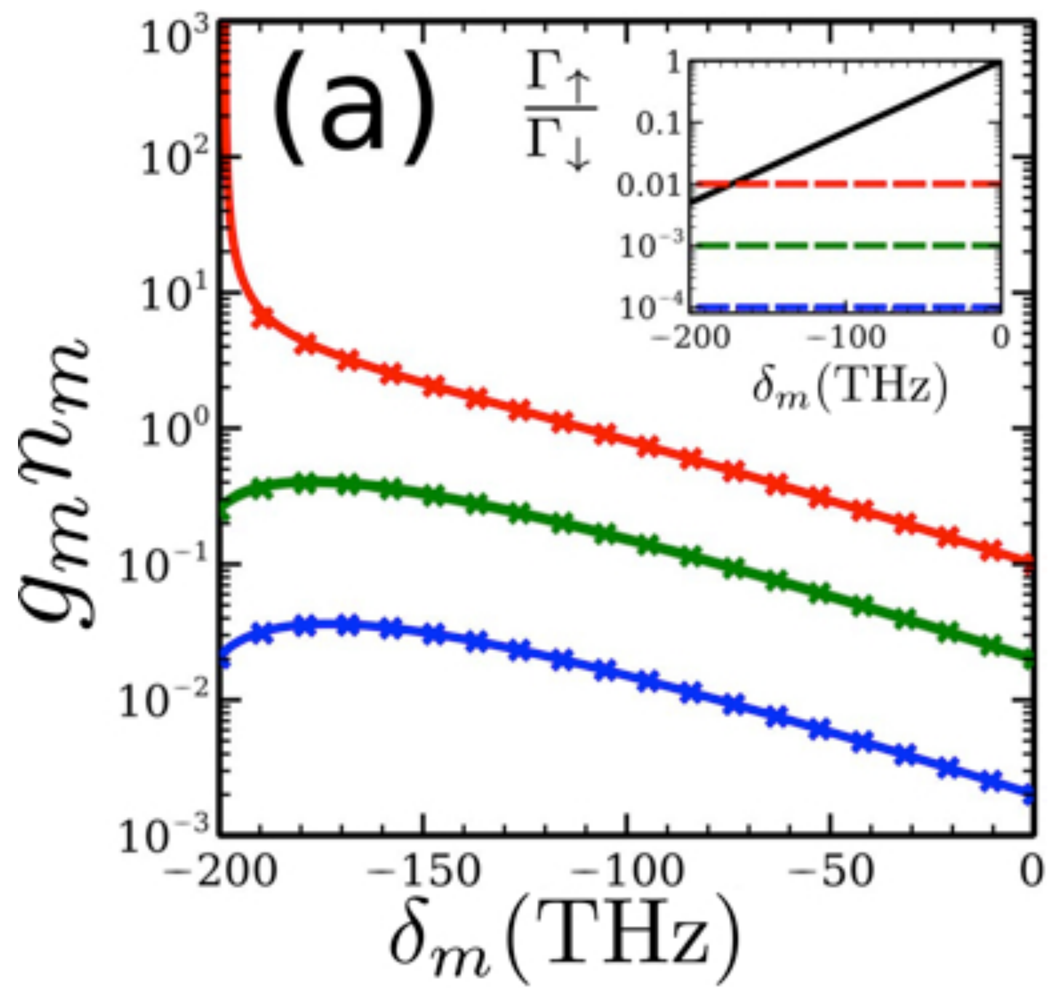
$$(n_m + 1)/\, n_m = e^{\beta\delta_m}\tilde{\Gamma}_\downarrow/\tilde{\Gamma}_\uparrow$$

In this limit we recover textbook BEC.



From BEC to lasing

$\kappa = 10$ MHz



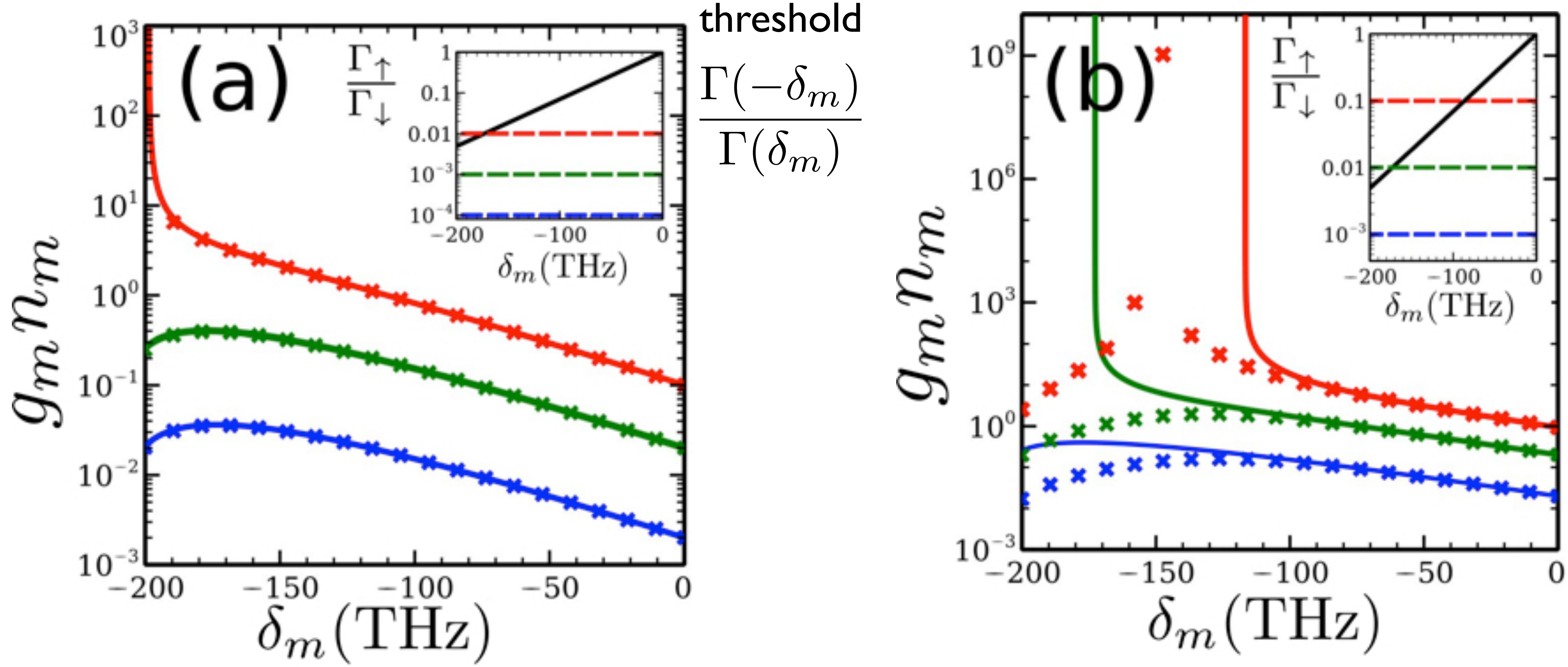
threshold

$$\frac{\Gamma(-\delta_m)}{\Gamma(\delta_m)}$$

$\Gamma_\downarrow = 1$ GHz, $S = 0.5$, $\Omega = 1$ THz, $N = 10^{11}$, $g = 0.1$ GHz, $T =$ 1THz 300K, $\delta_0 = -200$ THz, and the mode spacing $\epsilon = 10$ THz. $\gamma = 100$THz,

From BEC to lasing

$\kappa = 10$ MHz $\qquad\qquad$ $\kappa = 5$ GHz



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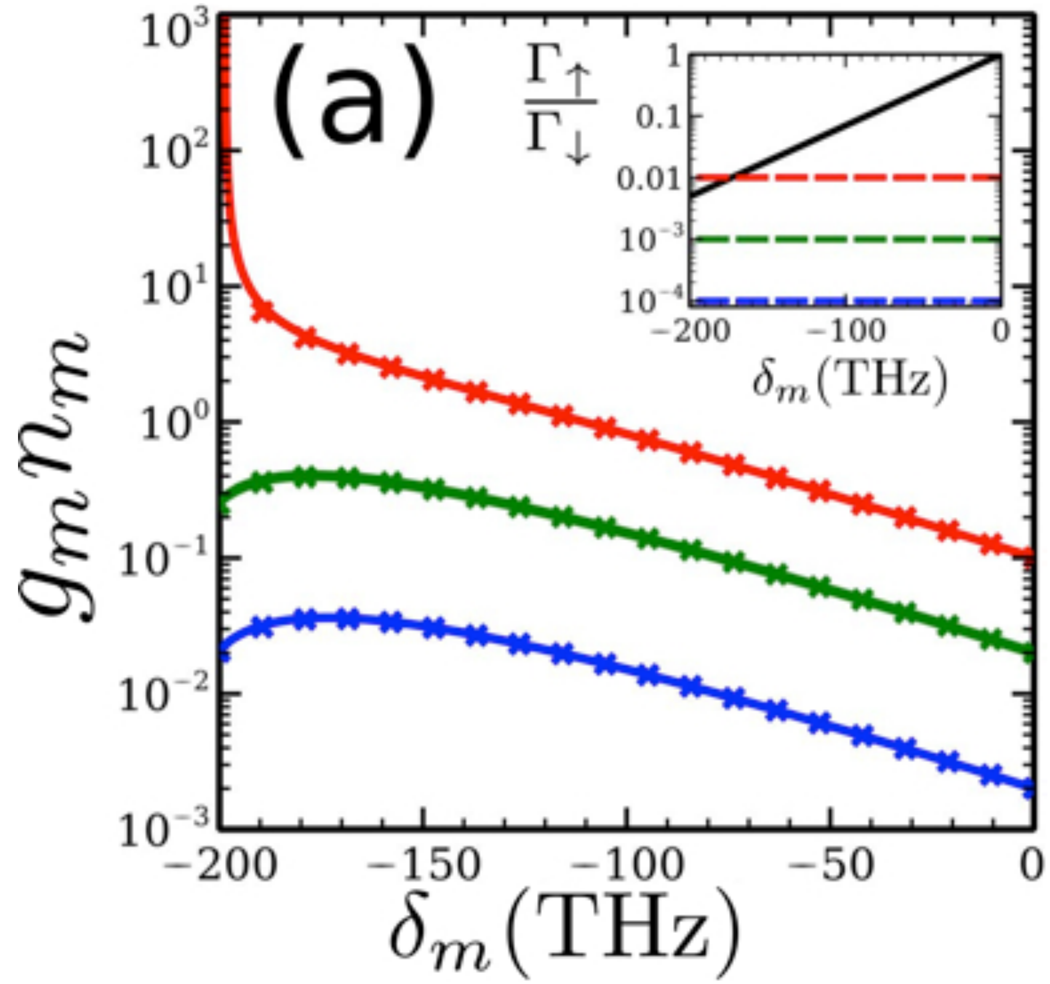

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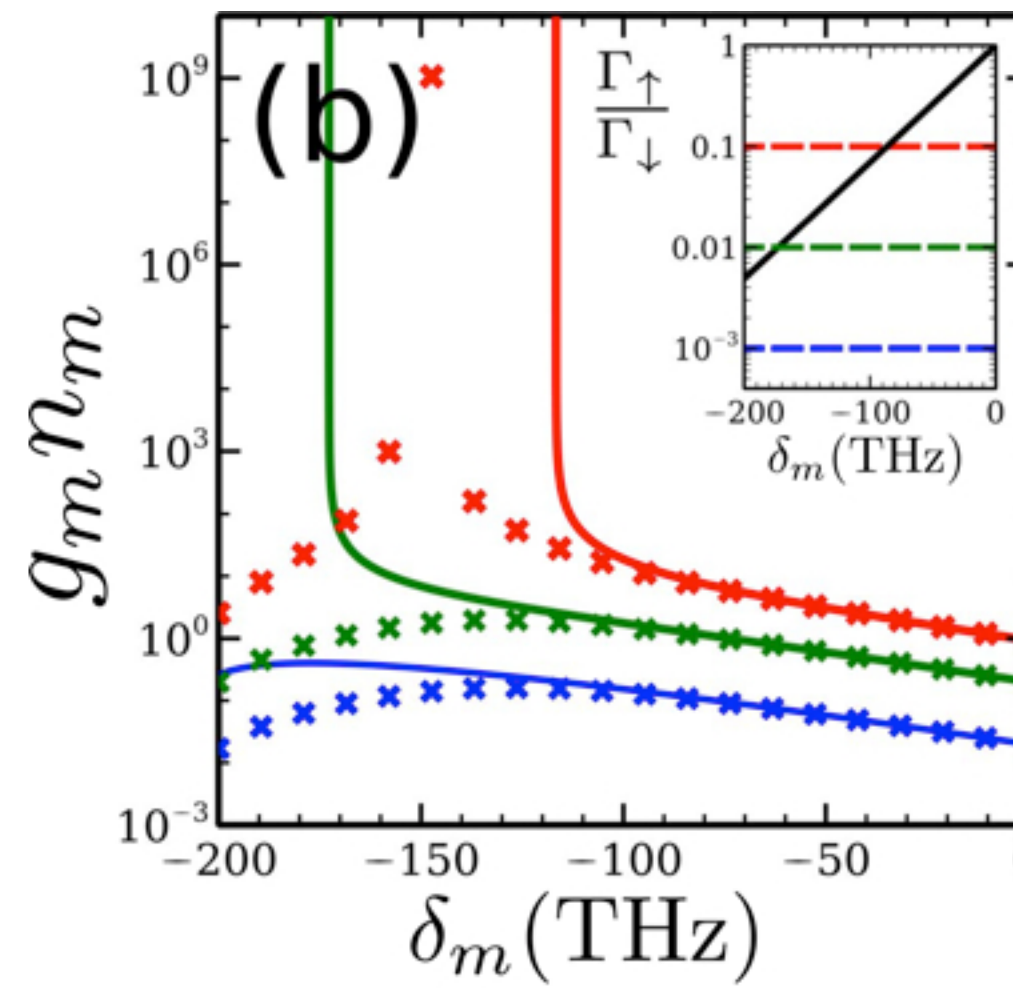

The origin of the destruction of thermalization is the competition between optical loss and emission rate.

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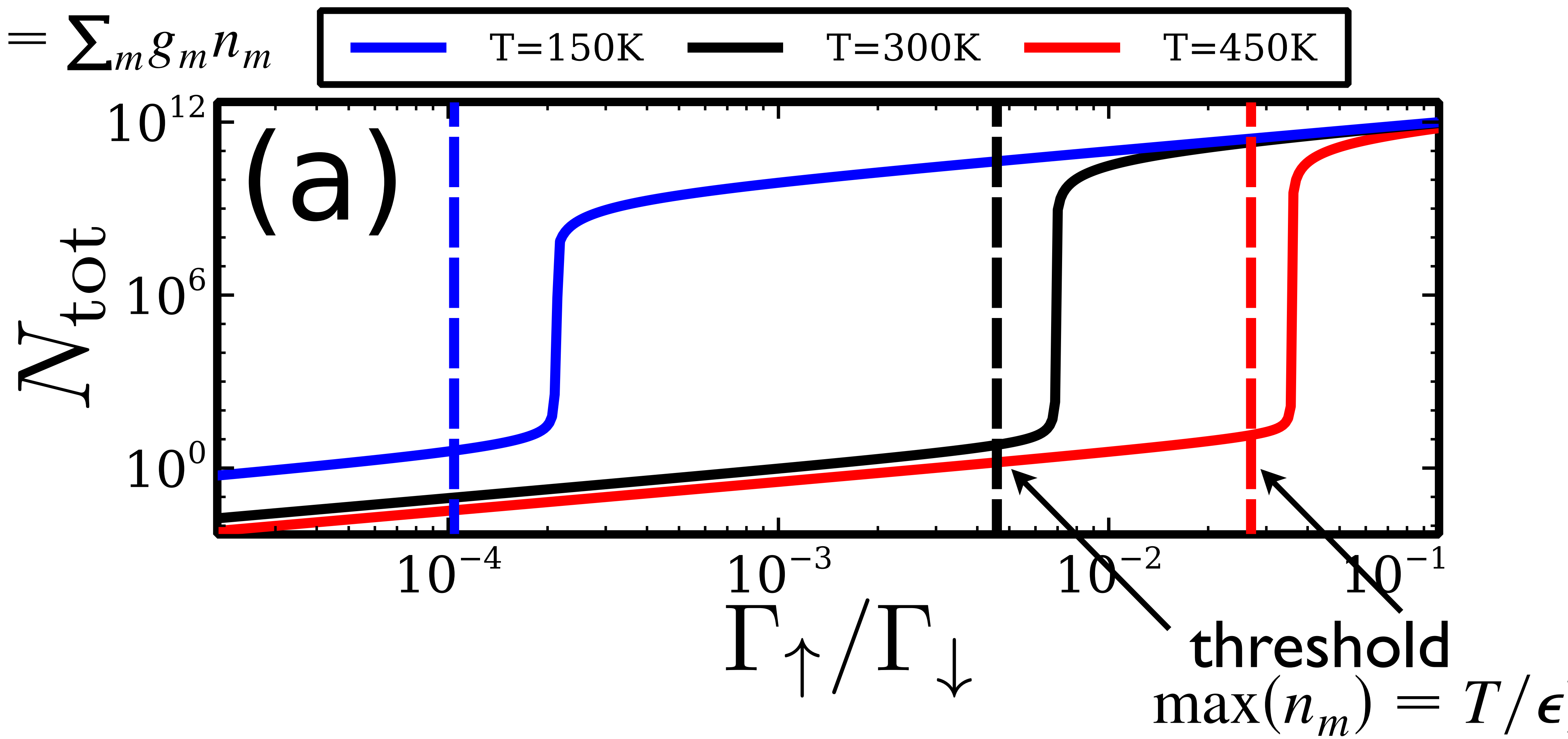
Degree of thermalization

$N_{\rm tot} = \sum_m g_m n_m$



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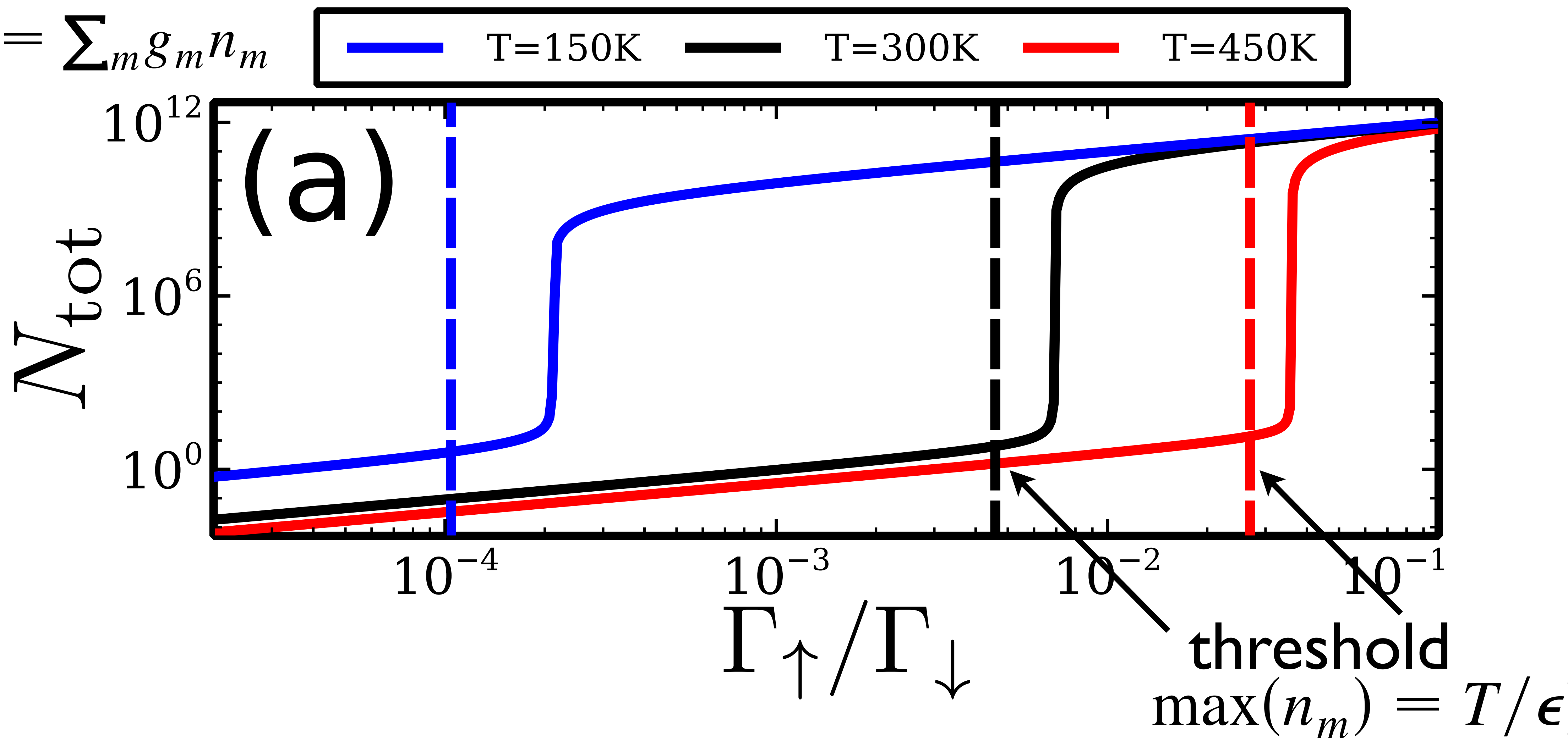

Increasing temperature decreases asymmetry of emission and absorption, threshold power and number increase.

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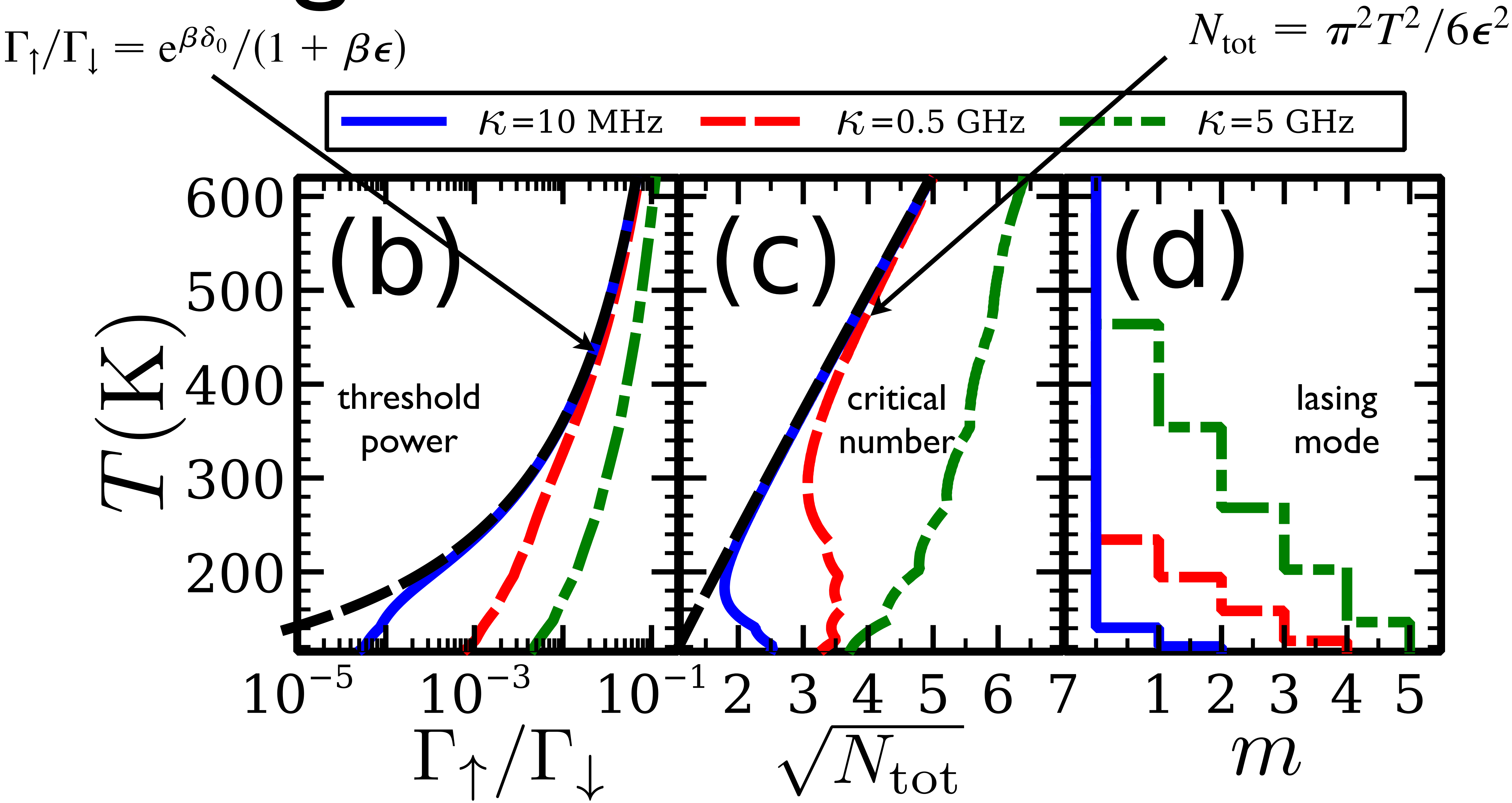

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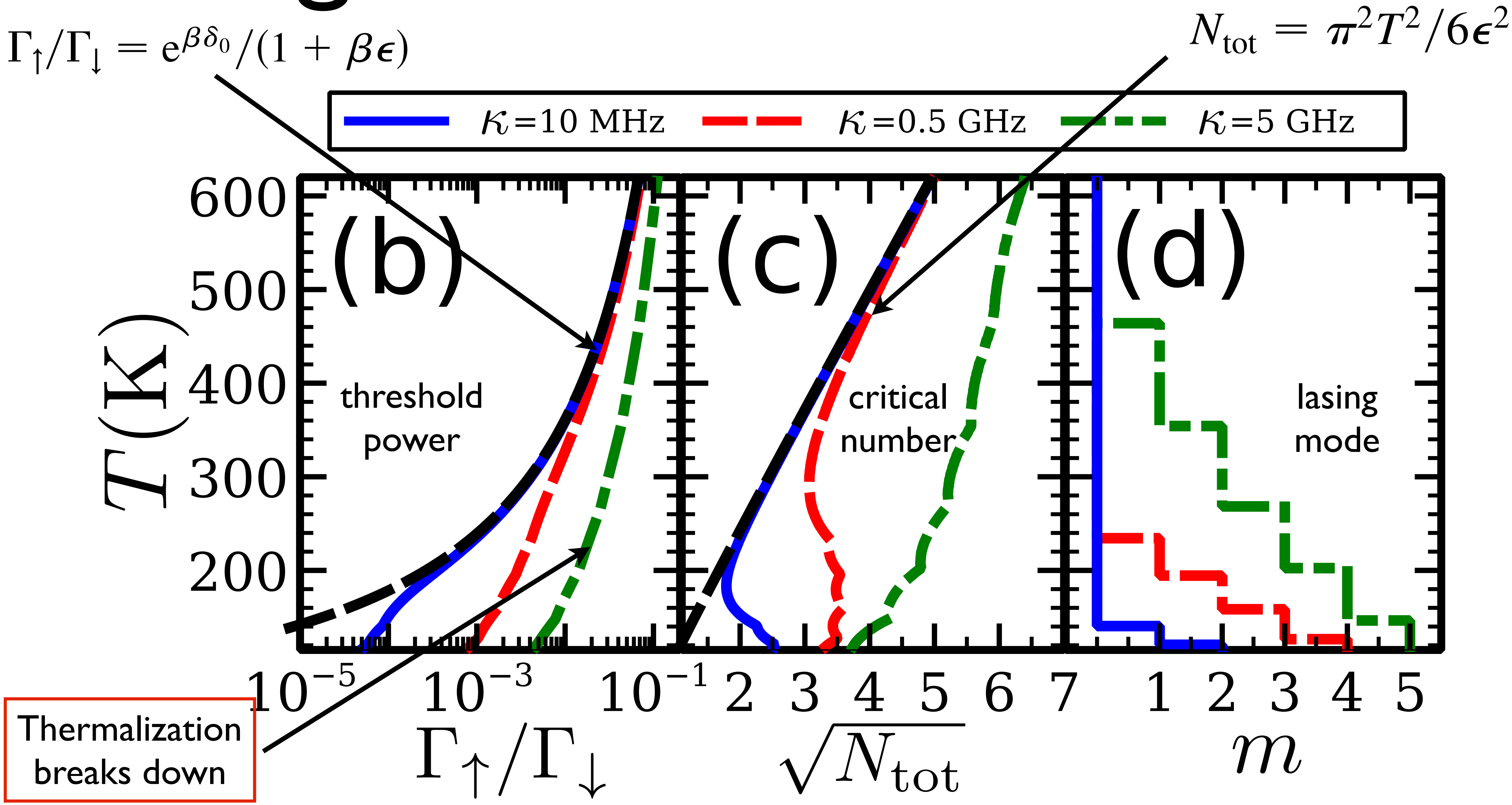

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Conclusions

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- At larger losses, lower temperature, or larger detuning, a crossover to lasing is predicted, thermalization is suppressed.
- As for polariton condensates, there is a crossover from lasing to condensation.