Time-reversal invariant topological superconductivity induced by repulsive interactions in quantum wires

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arxiv: 1310.4525

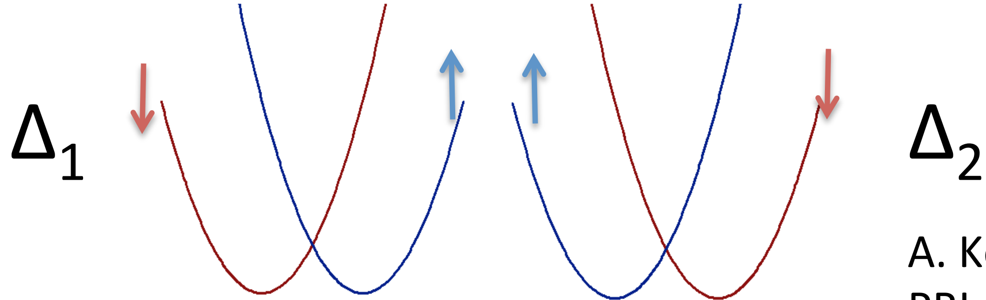
Majorana edge states might be realized in one dimensional semiconductors with s-wave superconducting pairing and spin-orbit interaction.
One needs to break the time reversal symmetry.
Single Majorana edge state.

$$\Delta = \mathrm{const}(\mathrm{k})$$

Majorana edge states without TRS breaking.
Kramers pair at the edge.
Two copies of spinless p-wave superconductors

1. Superconductors with d, s± symmetries
2. π junction + one 2-band semiconducting wire
   (two 1-band wires)

$$\Delta \propto \cos k_x + \cos k_y$$



A. Keselman, L. Fu, A. Stern, and E. Berg
PRL 111, 116402 (2013)

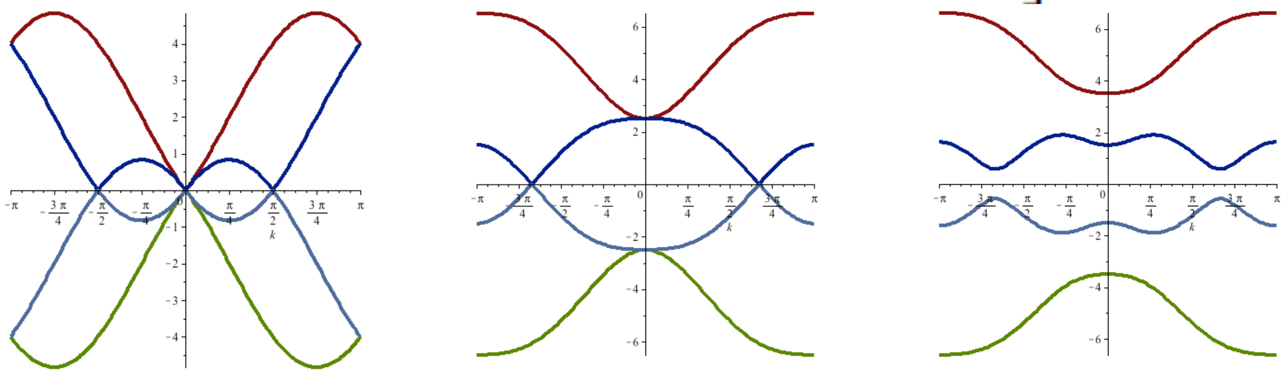
$$H_{0k} = \xi_k + \lambda_k s_z \sigma_z - t_\perp \sigma_x, \qquad E(k) = \pm\Big[\xi_k^2 + \lambda_k^2 + t_\perp^2 + \Delta^2$$

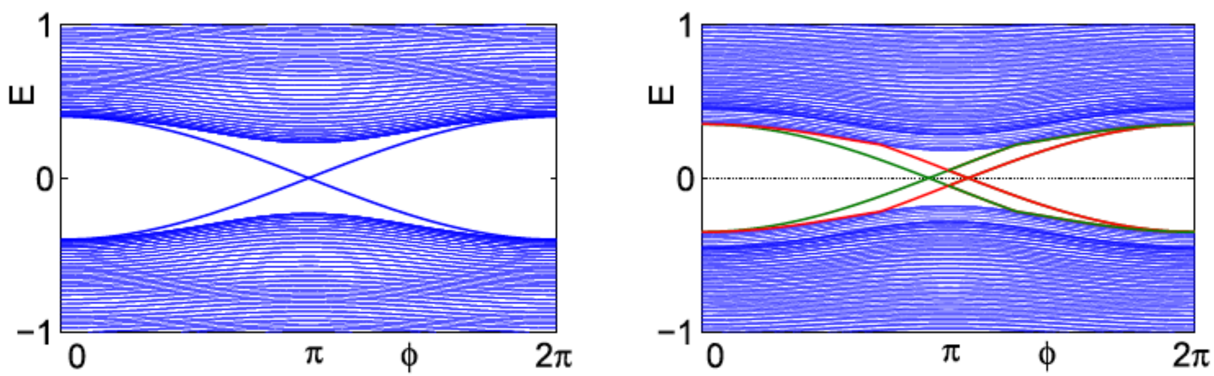
$$H_\Delta = \Delta \sigma_z, \qquad \pm 2\sqrt{\xi_k^2 t_\perp^2 + \xi_k^2 \lambda_k^2 + t_\perp^2 \Delta^2}\Big]^{1/2}.$$

$t_\perp = 0, \Delta = 0$

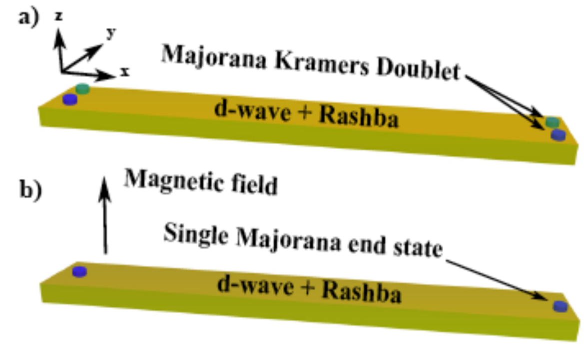
$t_\perp = 2.5, \Delta = 0$

$t_\perp = 2.5, \Delta = 1$

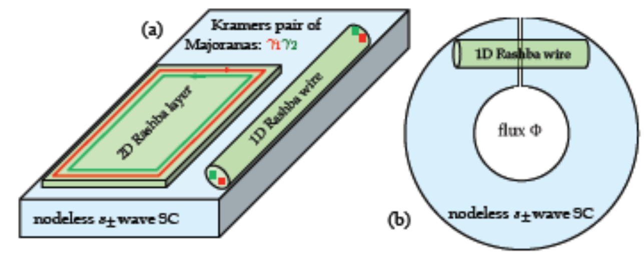




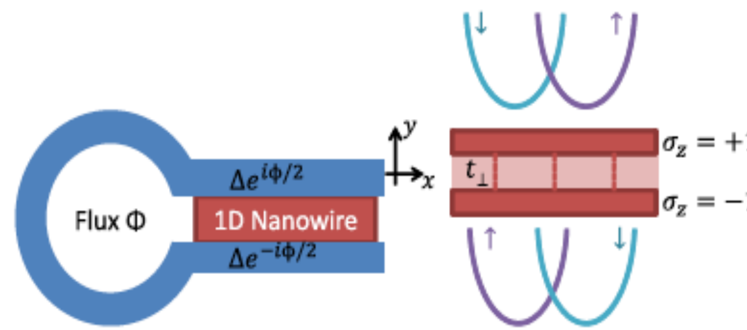
Majorana Kramers Doublets in **d**-wave Superconductors with Rashba Spin-Orbit Coupling. C. L. M. Wong, K. T. Law
Phys. Rev. B **86**, 184516 (2012)



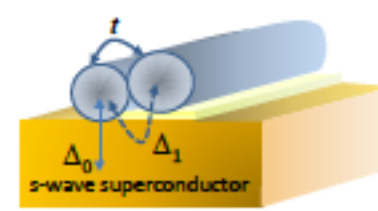
Time-Reversal-Invariant Topological Superconductivity and Majorana Kramers Pairs
F. Zhang, C. L. Kane, and E. J. Mele
Phys. Rev. Lett. 111, 056402 (2013)

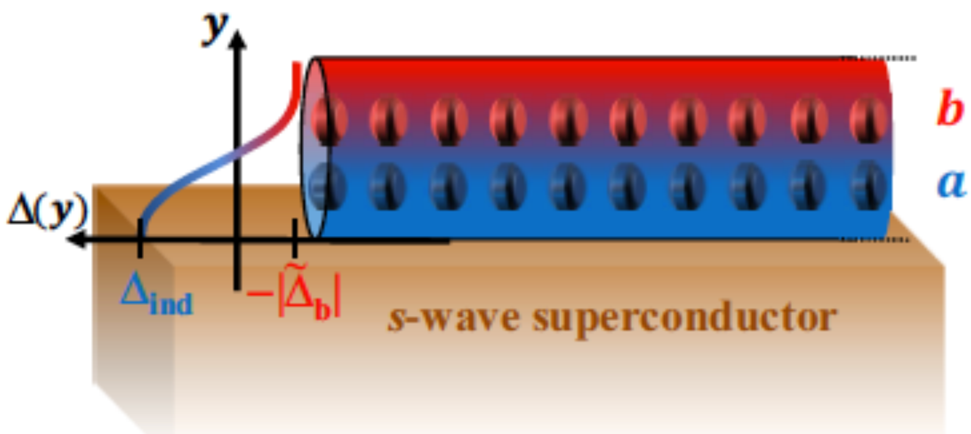


Inducing Time-Reversal-Invariant Topological Superconductivity
and Fermion Parity Pumping in Quantum Wires
A. Keselman, L. Fu, A. Stern, and E. Berg
PRL 111, 116402 (2013)



Majorana bound states in two-channel time-reversal-symmetric nanowire systems
E. Gaidamauskas, J. Paaske, and K. Flensberg,
arxiv1309.2808





$$H = \frac{1}{2}\sum_k \Psi_k^\dagger \mathcal{H}_{0,k}\Psi_k + \sum_{i,\sigma} U_\sigma \hat{n}_{i\sigma\uparrow}\hat{n}_{i\sigma\downarrow}$$

$$\mathcal{H}_{0,k} = \left[\bar{\xi}_k + \delta\xi_k\sigma^z - (\bar{\alpha} + \delta\alpha\sigma^z)\sin k\ s^z + t_{ab}\sigma^x\right]\tau^z$$
$$+\Delta_{\rm ind}/2\ \cdot (1+\sigma^z)\,\tau^x,$$

where $\Psi_k^\dagger = (\psi_k^\dagger,\ -is^y\psi_{-k})$. We model the two sub-bands of the wire using two chains labeled a and b, such that $\psi_k^\dagger = (c_{a,k\uparrow}^\dagger\ \ c_{b,k\uparrow}^\dagger\ \ c_{a,k\downarrow}^\dagger\ \ c_{b,k\downarrow}^\dagger)$. $\vec{\tau}, \vec{\sigma}$, and $\vec{s}$ are Pauli matrices operating on particle-hole (PH), chain and spin degrees of freedom, respectively. Here, $\bar{\xi}_k, \delta\xi_k, \bar{\alpha}$ and $\delta\alpha$ are defined as $(\xi_{k,a} \pm \xi_{k,b})/2$ and $(\alpha_a \pm \alpha_b)/2$ respectively, and $\xi_{k,\sigma} = 2t_\sigma\,(1-\cos k) - \mu_\sigma$, $\sigma = a, b$. The

Repulsive interactions on each of the chains

Where is the inter-wire interaction?

Hartree-Fock Analysis.—We consider a set of trial wave-functions which are ground states of the following quadratic Hamiltonian:

$$H_{\rm HF} = \frac{1}{2}\sum_k \Psi_k^\dagger \mathcal{H}_k^{\rm HF} \Psi_k,$$

$$\mathcal{H}_k^{\rm HF} = \tilde{\mathcal{H}}_{0,k} + \tilde{\Delta}_b/2 \ \cdot (1-\sigma^z)\,\tau^x,$$

Due to repulsive interactions
Effective pairing potential
on chain b has an opposite sign
with respect to chain a.

We choose the four effective parameters such that they minimize the expectation value of the full Hamiltonian,

$$\langle H\rangle_{\rm HF} = E_0 + \frac{1}{L}\sum_\sigma U_\sigma \left(N_{\sigma,\uparrow}N_{\sigma,\downarrow} + |P_\sigma|^2\right)$$

$$N_{\sigma,s} = \textstyle\sum_k \langle c_{\sigma,k,s}^\dagger c_{\sigma,k,s}\rangle_{\rm HF},\ \ P_\sigma = \sum_k \langle c_{\sigma,k,\uparrow}^\dagger c_{\sigma,-k\downarrow}^\dagger\rangle_{\rm HF}$$

$$E_0 = \frac{1}{2}\sum_{k,m,n} \mathcal{H}_{0,k,mn}\langle \Psi_{k,m}^\dagger \Psi_{k,n}\rangle_{\rm HF}$$

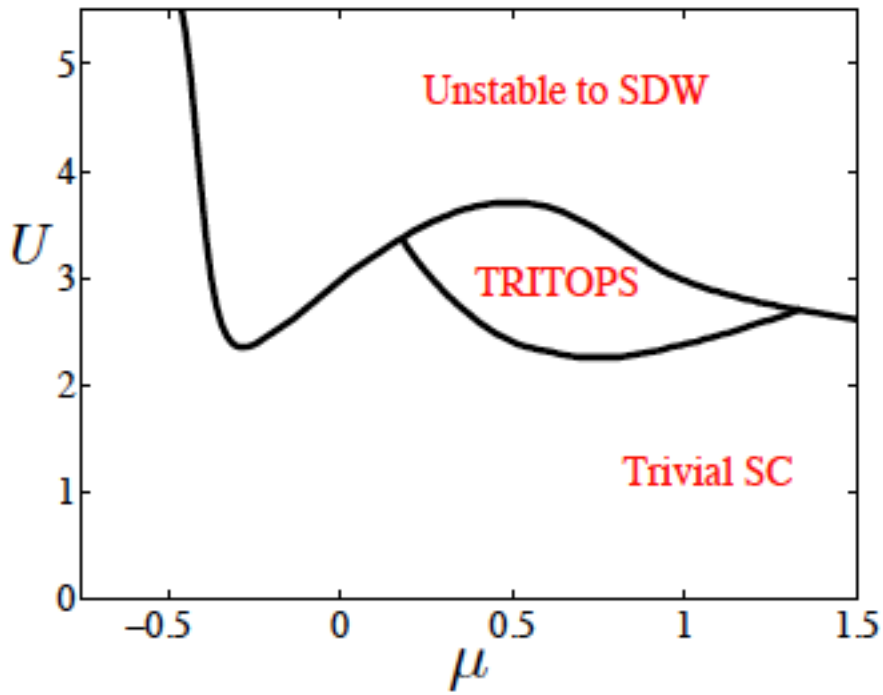
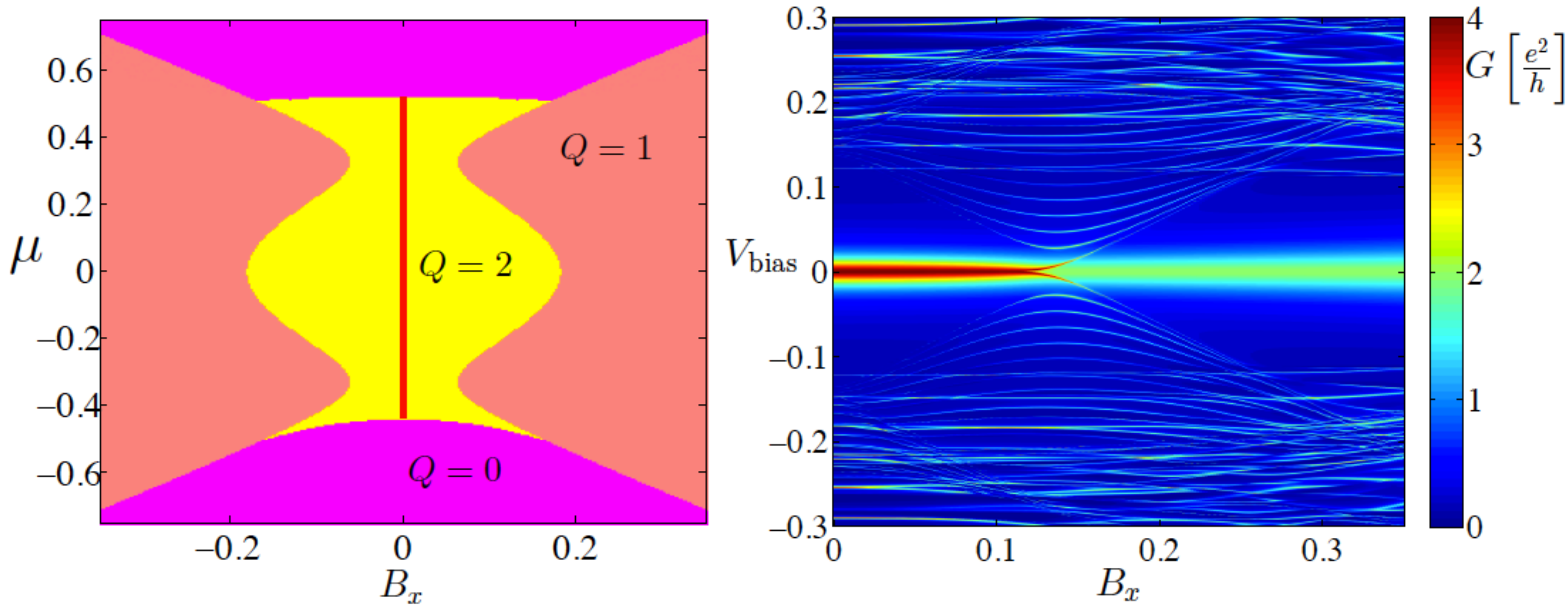


FIG. 2: Hartree-Fock phase diagram as a function of chemical potential $\mu_a = \mu_b = \mu$, and interaction strength U, for $t_a = t_b = 1, t_{ab} = 0.4, \alpha_a = 0, \alpha_b = 1$, and $\Delta_{\text{ind}} = 1$. The diagram includes a TR-invariant topological superconductor phase (TRITOPS), a trivial superconductor phase, and a region in which the Hartree-Fock solution is locally unstable to the formation of spin-density waves [36].

[36] This ordered phase is not permissible in the continuum limit in 1D, as it requires the breaking of a continuous symmetry. We therefore expect that long-range fluctuations (which are not accounted for in the HF treatment) would result in a Luttinger liquid phase.



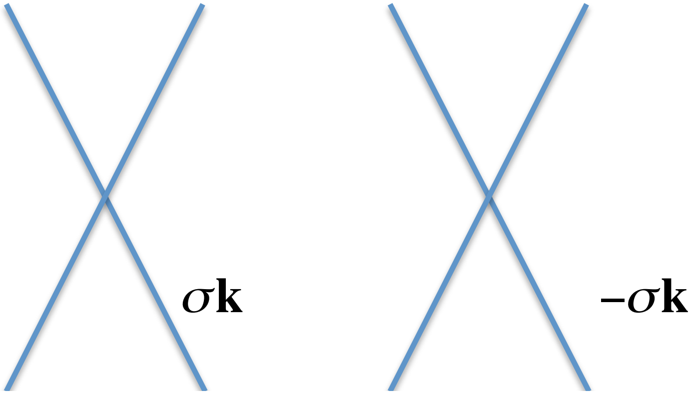
We next use the scattering matrix formalism to calculate the differential conductance through a single lead coupled to the system [30, 41, 42], described by the Hamiltonian of Eq. (2) with an additional Zeeman field along x. As is evident in Fig. 4b, there indeed exists a ZBP quantized to $4e^2/h$ which does not split at low

To estimate the strength of the repulsive interaction U in a realistic setup, we take the wire diameter d, the spin-orbit length and the Fermi wavelength to be of order $100nm$. This gives $U \sim e^2/\varepsilon d \sim 1-10K$, for a reasonable dielectric constant of $\varepsilon \sim 10-100$. Hence, U can become comparable with other energy scales in the system such as the Fermi energy and $\Delta_{\rm ind}$, in agreement with our

Resume:
In the superconductor and normal metal
the coupling constant in the Cooper channel has opposite signs.

1. Superconductors with d, s± symmetries
2. π junction + 2 band semiconductor

3. Phase slips.
4. LOFF, S-F contacts
5. Multiband superconductors with number of bands >2,
   frustration



$$H = \sum_{\mathbf{k}} v_F [\psi^\dagger_{R\mathbf{k}} \sigma \cdot \mathbf{k} \psi_{R\mathbf{k}} - \psi^\dagger_{L\mathbf{k}} \sigma \cdot \mathbf{k} \psi_{L\mathbf{k}}]$$

$$+ \frac{1}{2} \sum_{\mathbf{k}} [|\Delta_R| e^{i\theta_R} \psi^\dagger_{R-\mathbf{k}} i\sigma_y \psi^\dagger_{R\mathbf{k}}$$

$$+ |\Delta_L| e^{i\theta_L} \psi^\dagger_{L-\mathbf{k}} i\sigma_y \psi^\dagger_{L\mathbf{k}} + \text{H.c.}].$$

$$S_{\text{eff}} = \int d^4x \Big[\frac{\theta_L - \theta_R}{64\pi^2} \epsilon^{\mu\nu\sigma\tau} F_{\mu\nu} F_{\sigma\tau} - \frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu}$$

$$+ \frac{1}{2} \rho_L (\partial_\mu \theta_L - 2A_\mu)^2 + \frac{1}{2} \rho_R (\partial_\mu \theta_R - 2A_\mu)^2$$

$$+ J \cos(\theta_L - \theta_R) \Big].$$