Measuring topological invariants in photonic systems

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Motivated by the recent theoretical and experimental progress in implementing topological orders with photons, we analyze photonic systems with different topologies and present a scheme to probe their topological features. Specifically, we propose a scheme to modify the boundary phases to manipulate edge state dynamics. Such a scheme allows one to measure the winding number of the edge states. Furthermore, we discuss the effect of loss and disorder on the validity of our approach.

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Introduction

Topology plays a fundamental role in many physical phenomena
- e.g. quantum Hall effect(s) in electronic systems

Topology in non-electronic systems
- ultra cold atomic systems

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- optical/photonic systems

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Introduction

Focus on photonic systems

Implementing gauge fields in experiments? ✓

[12] M. Hafezi, E. A. Demler, M. D. Lukin, and J. M. Taylor, Nat. Phys. **7**, 907 (2011).
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Detection of topological order? ✗
(no Hall conductance measurements)

How to measure integer topological invariants?
- winding numbers of edge states
- Chern number

How do integer values manifest themselves in an optical version of the quantum Hall Hamiltonian?

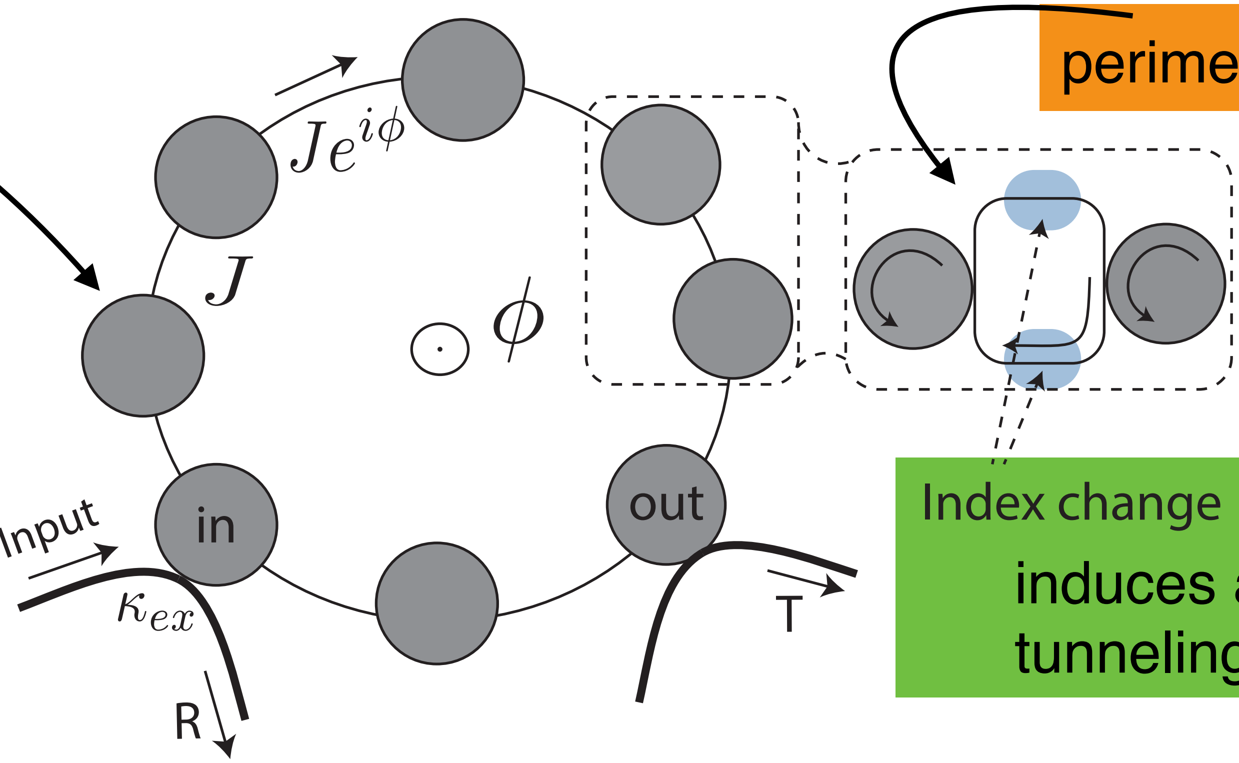
Resonators on a ring

Synthetic magnetic flux threading the ring

perimeter: $m\lambda$

integer — resonant wavelength

perimeter: $m\lambda + 3\lambda/2$



Index change induces a phase in tunneling term

tunneling between the resonators induced by connecting loops

$$H_{ring} = -J \sum_{i=1}^{N-1} \hat{a}_i^\dagger \hat{a}_{i+1} + h.c.$$

coupling between first and last site:

$$-J\hat{a}_1^\dagger \hat{a}_N e^{i\phi} + h.c.$$

charged particles:

phase by introducing a magnetic flux in the middle of the ring

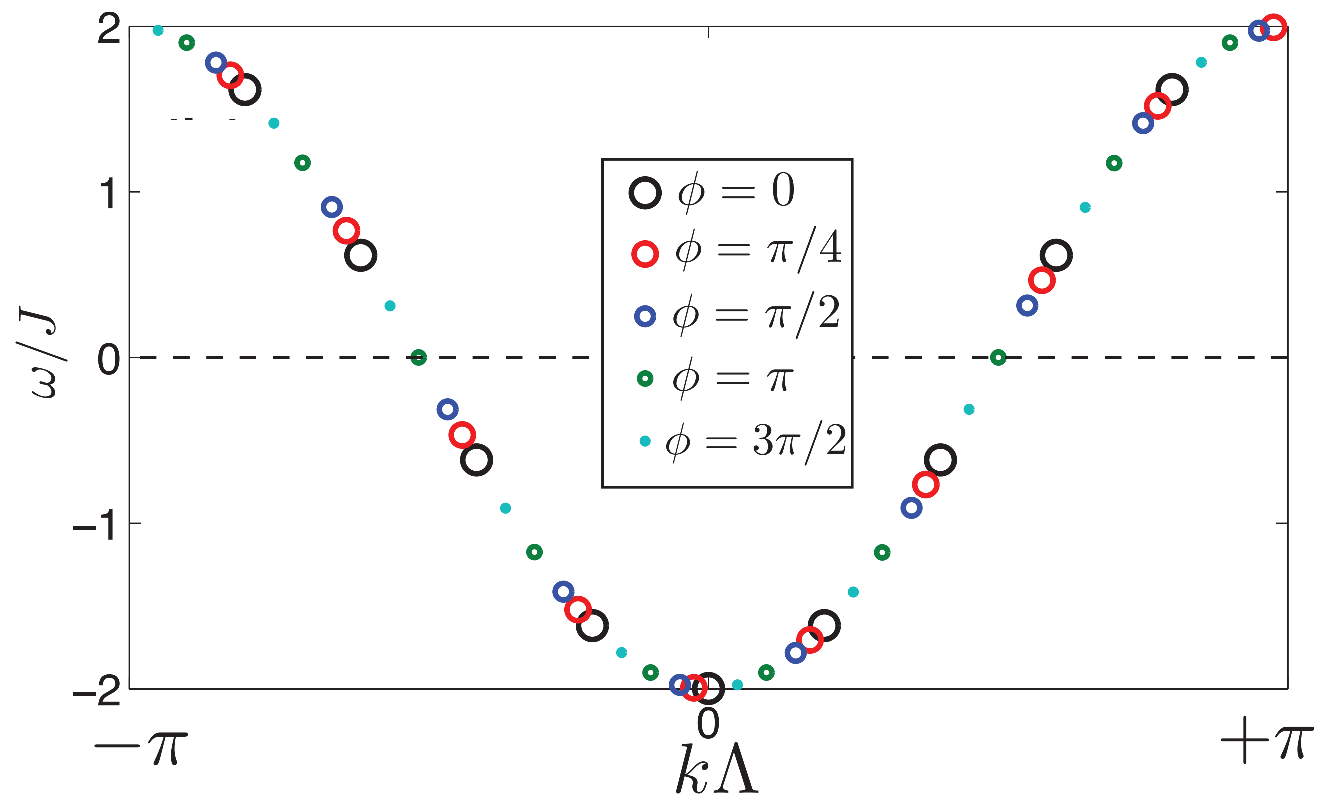
here:

this phase has to be artificially engineered

Resonators on a ring

dispersion relation of the ring

$$\omega = -2J\cos(k\Lambda + \phi/N)$$



Changing ϕ shifts energy spectrum along the dispersion curve

Inserting one flux $\phi = 0 \rightarrow 2\pi$ the energy spectrum returns shifted one state in the Brillouin zone

Detect state transfer with transmission spectroscopy

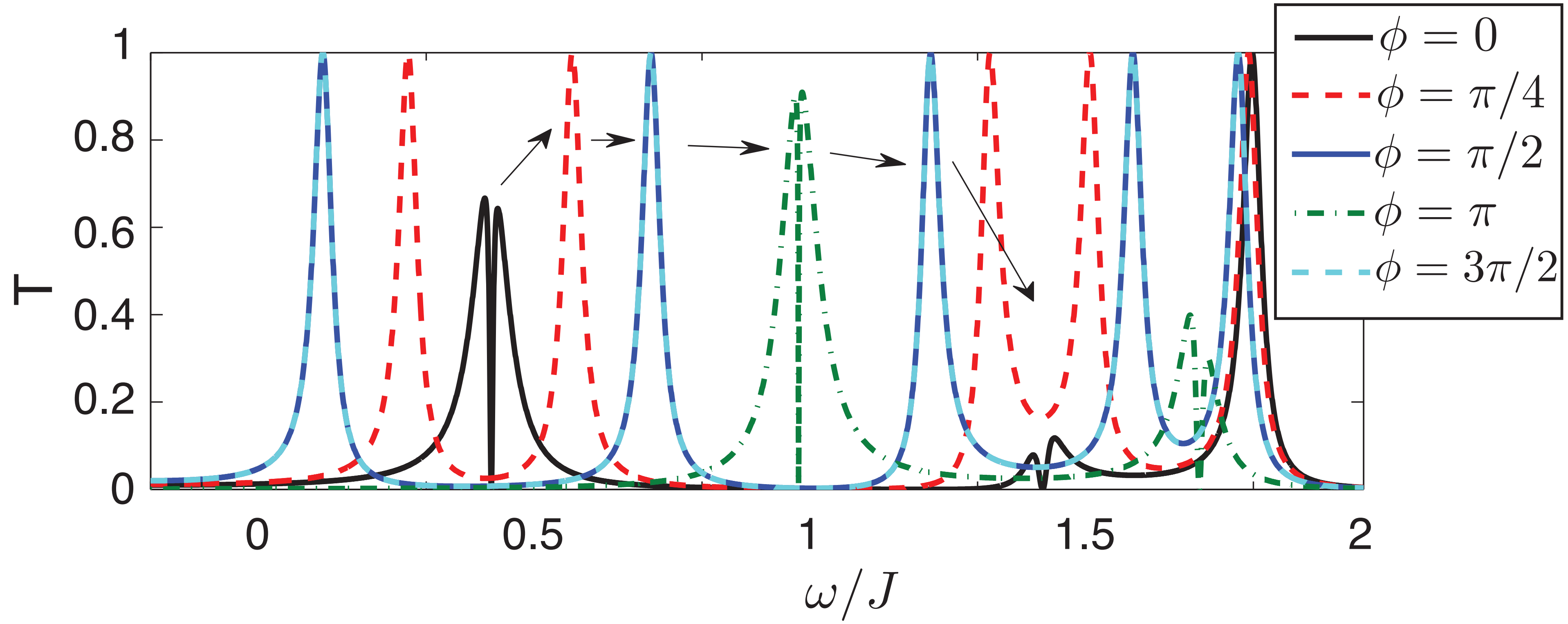
Resonators on a ring

input-output formalism

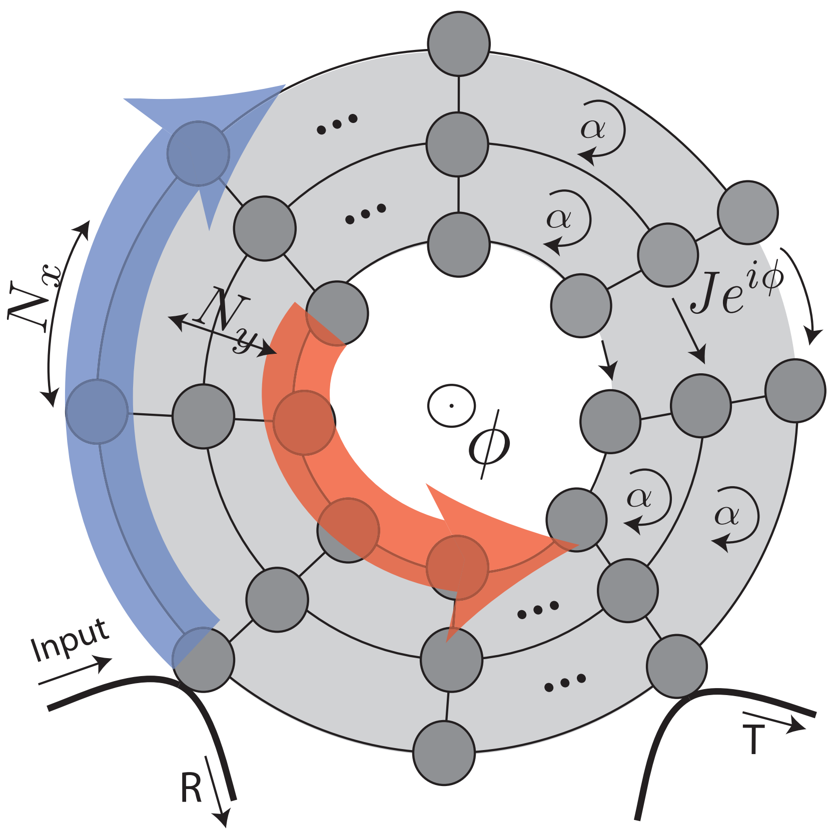
$$\dot{\hat{a}}_j = i[H, \hat{a}_j] - (\delta_{j,\text{in}} + \delta_{j,\text{out}})\kappa_{ex}\hat{a}_j - \delta_{j,\text{in}}\sqrt{2\kappa_{ex}}\mathcal{E}_{in}e^{-i\omega t}$$

Transmission in output channel

$$T = |a_{out}/\mathcal{E}_{in}|^2 \qquad \kappa_{ex} < 4J/N$$



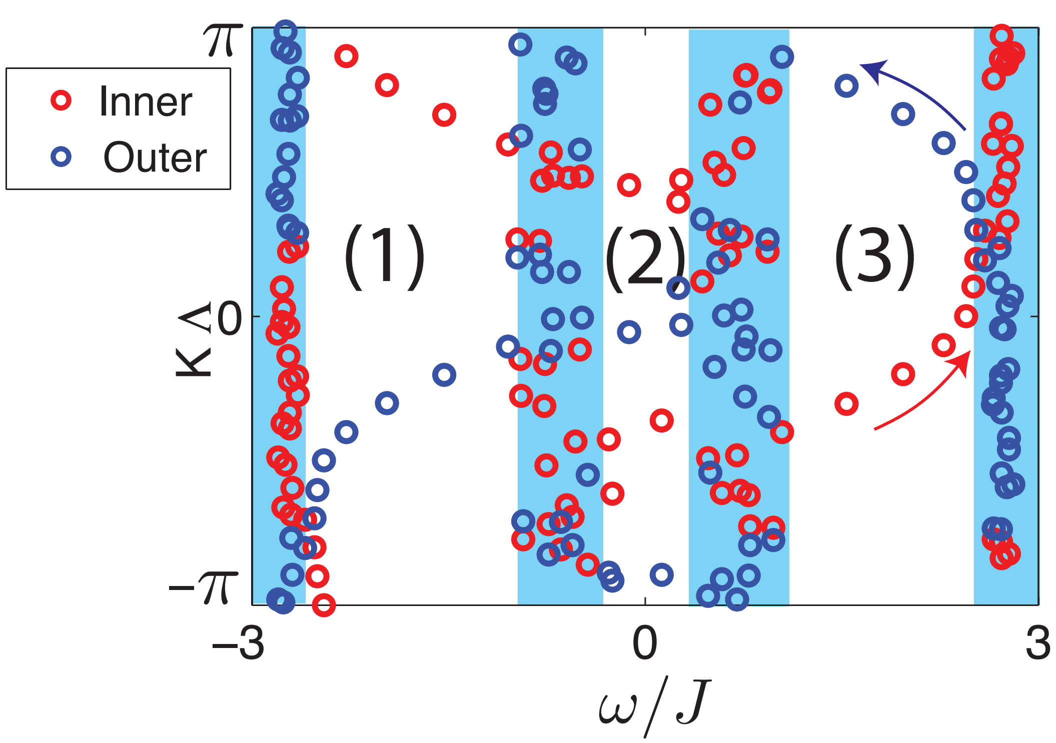
Resonators on an annulus



- 2D lattice, uniform perpendicular magnetic field
- photon hopping (clockwise) around a plaquette acquires $2\pi\alpha$

[12] M. Hafezi, E. A. Demler, M. D. Lukin, and J. M. Taylor, Nat. Phys. **7**, 907 (2011).
[13] M. Hafezi, S. Mittal, J. Fan, A. Migdall, and J. Taylor, AOP Nature Photon. (2013).

$$H_{\text{mag}} = -J \sum_{x,y} \hat{a}^\dagger_{x+1,y}\hat{a}_{x,y}e^{i2\pi\alpha y} + \hat{a}^\dagger_{x,y}\hat{a}_{x+1,y}e^{-i2\pi\alpha y} + \hat{a}^\dagger_{x,y+1}\hat{a}_{x,y} + \hat{a}^\dagger_{x,y}\hat{a}_{x,y+1}$$



infinite system:

Hofstadter butterfly spectrum;

if $\alpha = p/q$, q gapped bands

finite annulus:

edge states (in the spectral gap) spatially confined at the edges

$$n = s_n q + t_n p \quad |t_n| \leq q/2$$

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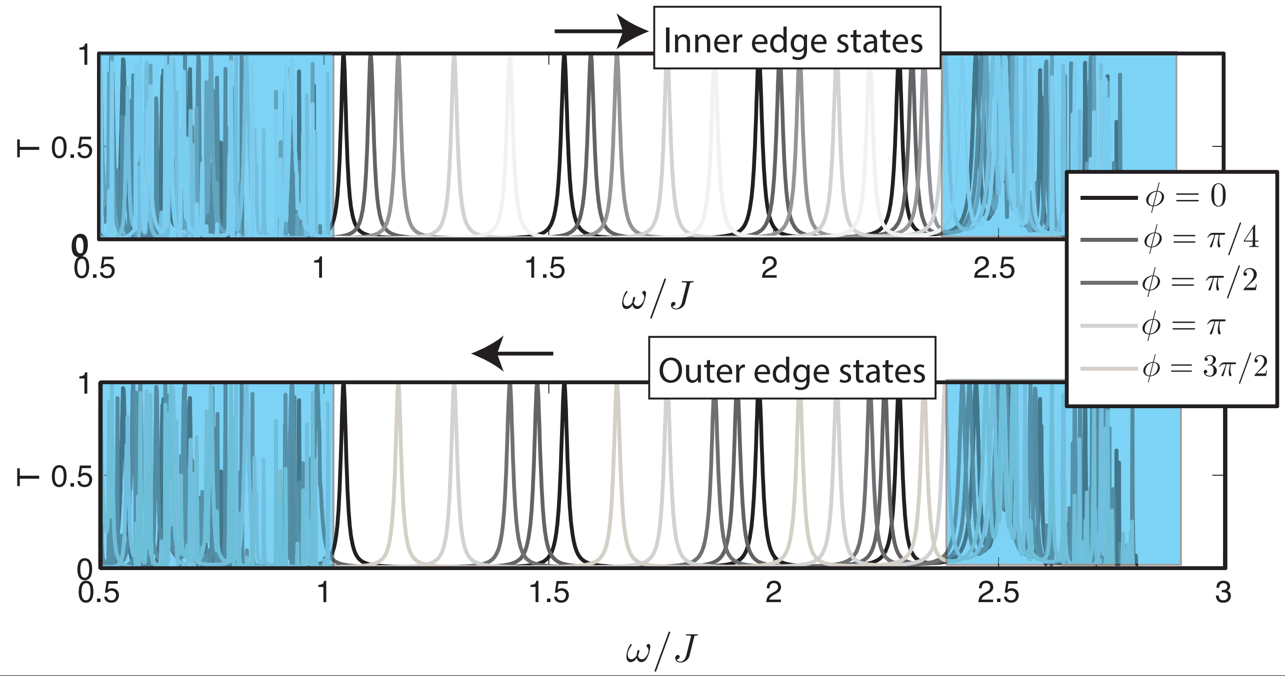
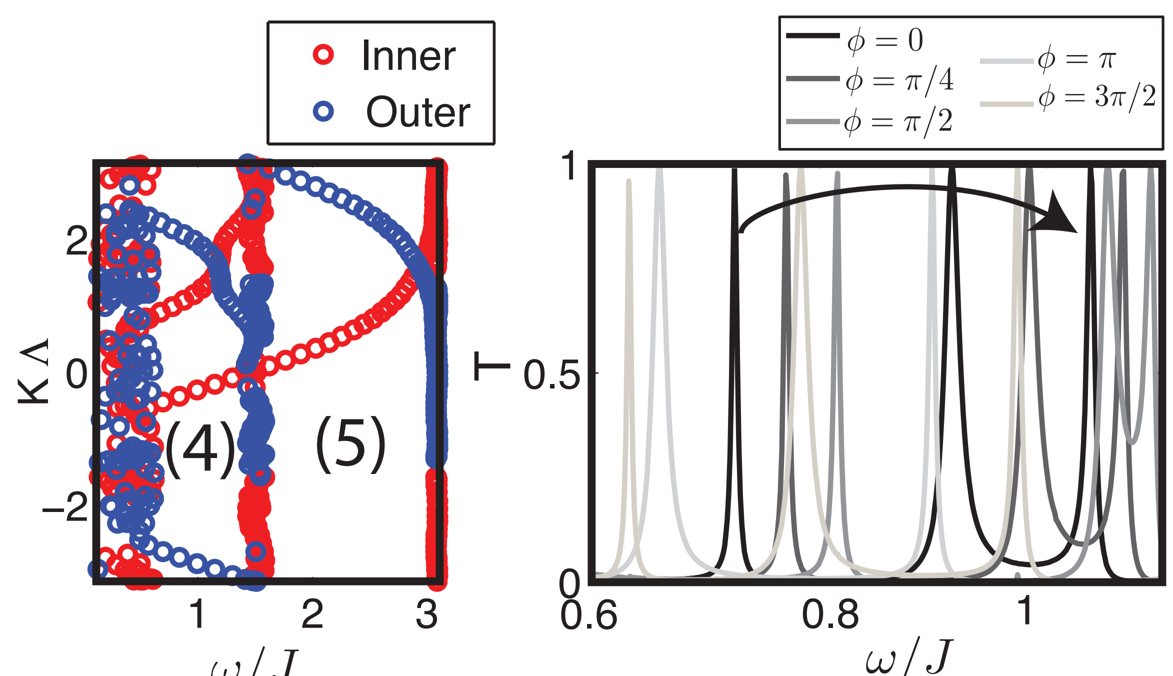
Resonators on an annulus

system with $\alpha = 1/4$ and third gap: winding number is ***one***

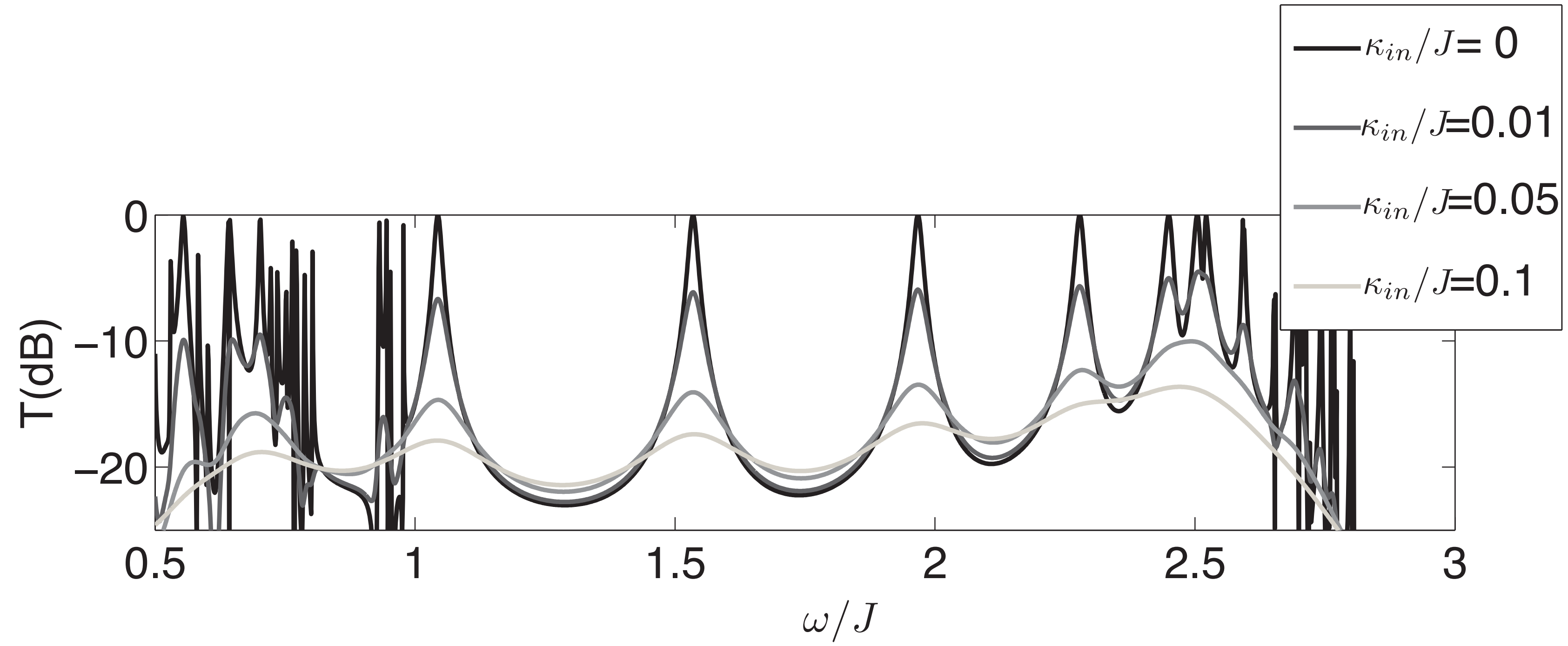


system with $\alpha = 1/6$ and fourth gap: winding number is ***two***

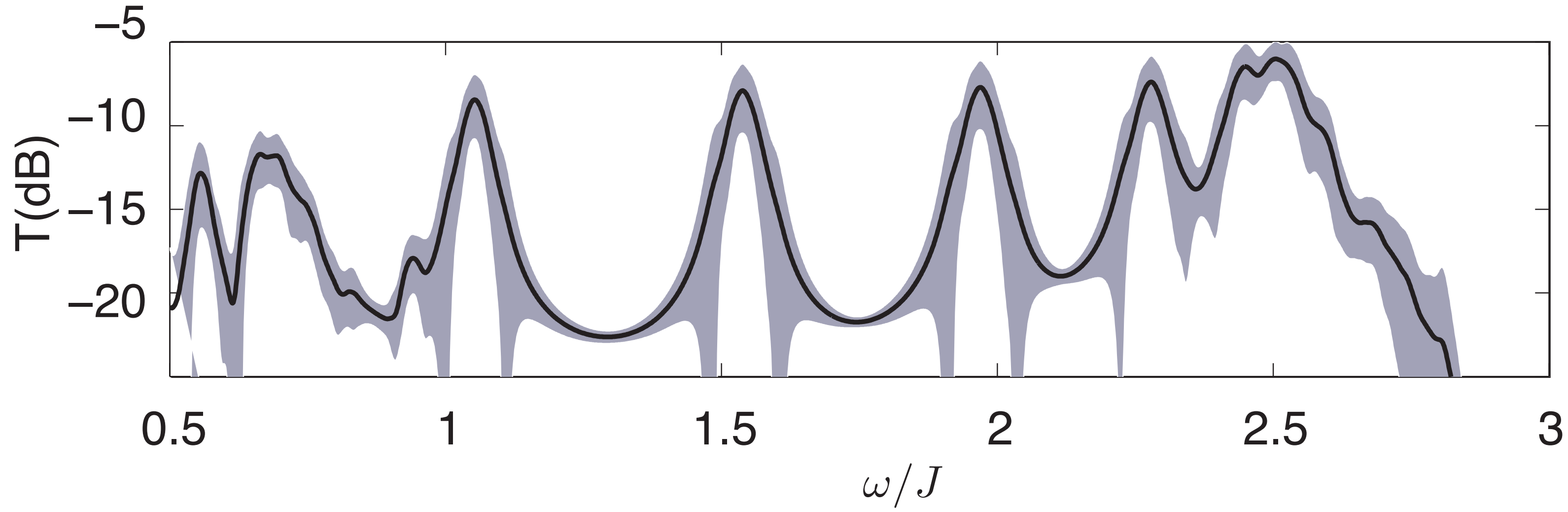


Resonators on an annulus

i) Effects of loss:
propagation loss
$-i\kappa_{in}\hat{a}_i^\dagger\hat{a}_i$
➡ decrease of contrast



ii) Effects of disorder:
frequency mismatch of adjacent resonators
$U_i\hat{a}_i^\dagger\hat{a}_i$
➡ broadening



Transmission spectrum still resolvable in presence of weak loss and disorder

Conclusion

If winding number of the edge state is t

➡ edge spectrum shifts by t peaks when $\phi = 0 \to 2\pi$

Direction of movement of the peak

➡ sign of the winding number of the edge states

Topology in non-electronic systems

Integer topological invariants can be measured
by transmission spectroscopy