



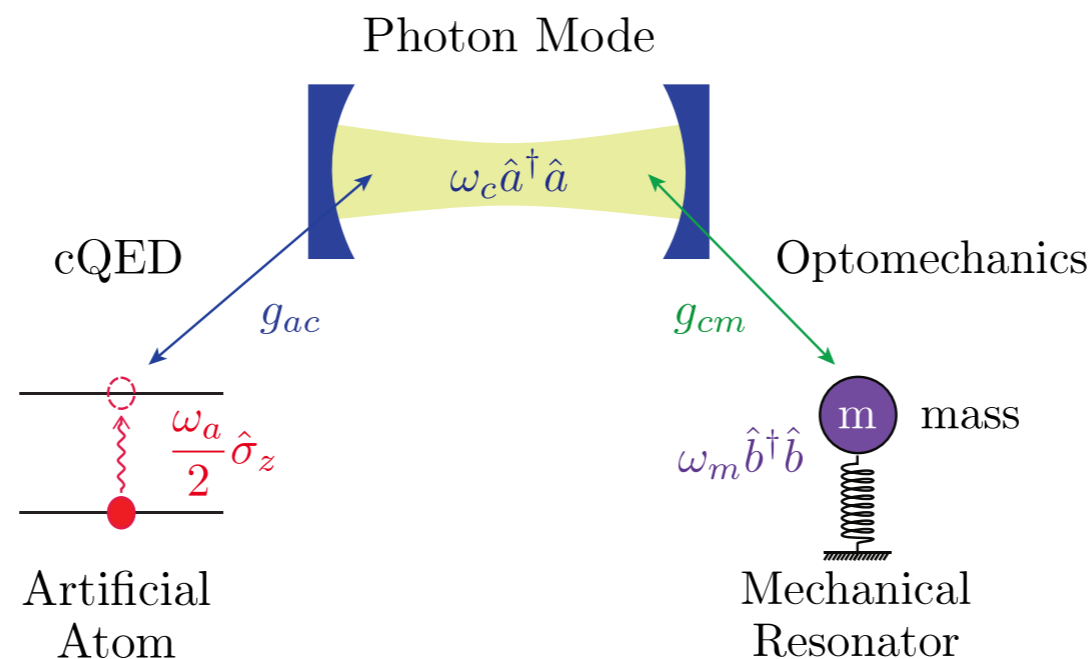
## Single-Polariton Optomechanics

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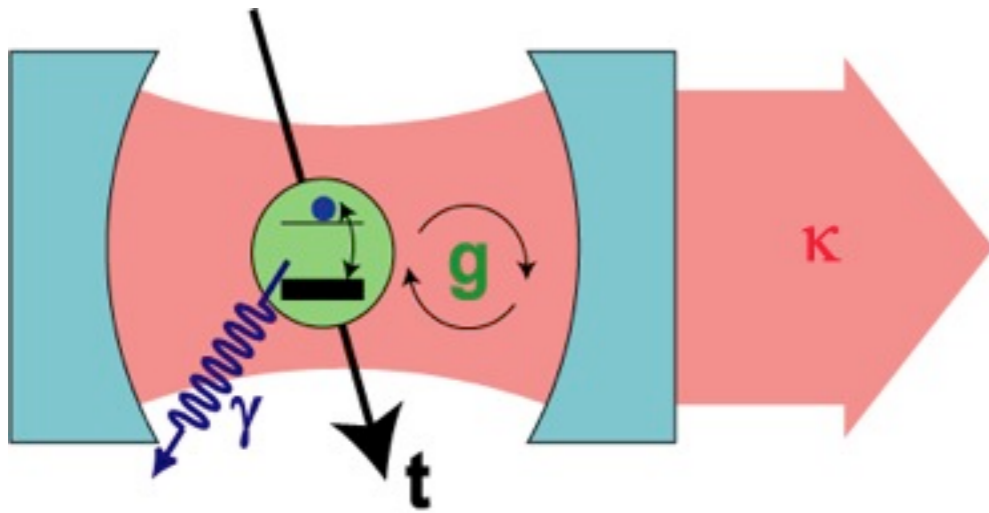
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(Received 15 July 2013; published 3 January 2014)

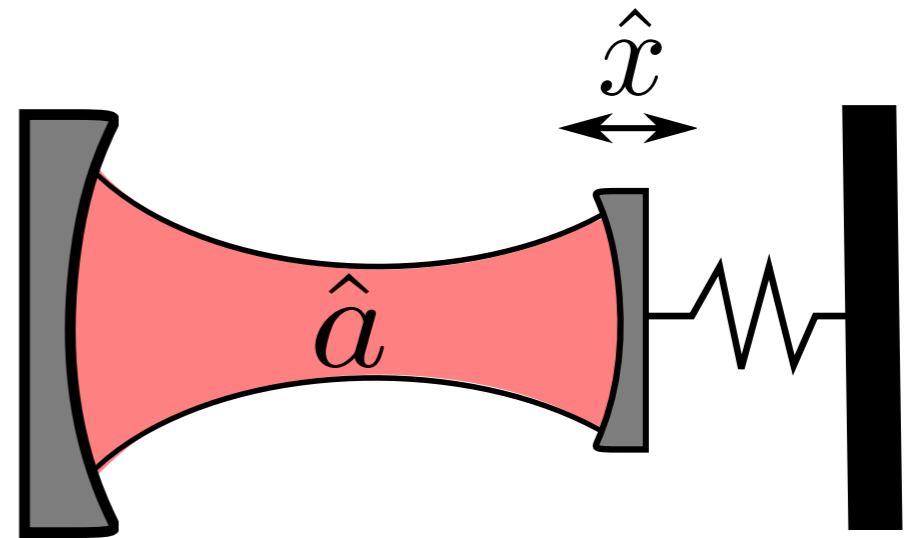
This Letter investigates a hybrid quantum system combining cavity quantum electrodynamics and optomechanics. The Hamiltonian problem of a photon mode coupled to a two-level atom via a Jaynes-Cummings coupling and to a mechanical mode via radiation pressure coupling is solved analytically. The atom-cavity polariton number operator commutes with the total Hamiltonian leading to an exact description in terms of tripartite atom-cavity-mechanics polarons. We demonstrate the possibility to obtain cooling of mechanical motion at the single-polariton level and describe the peculiar quantum statistics of phonons in such an unconventional regime.



# Motivation



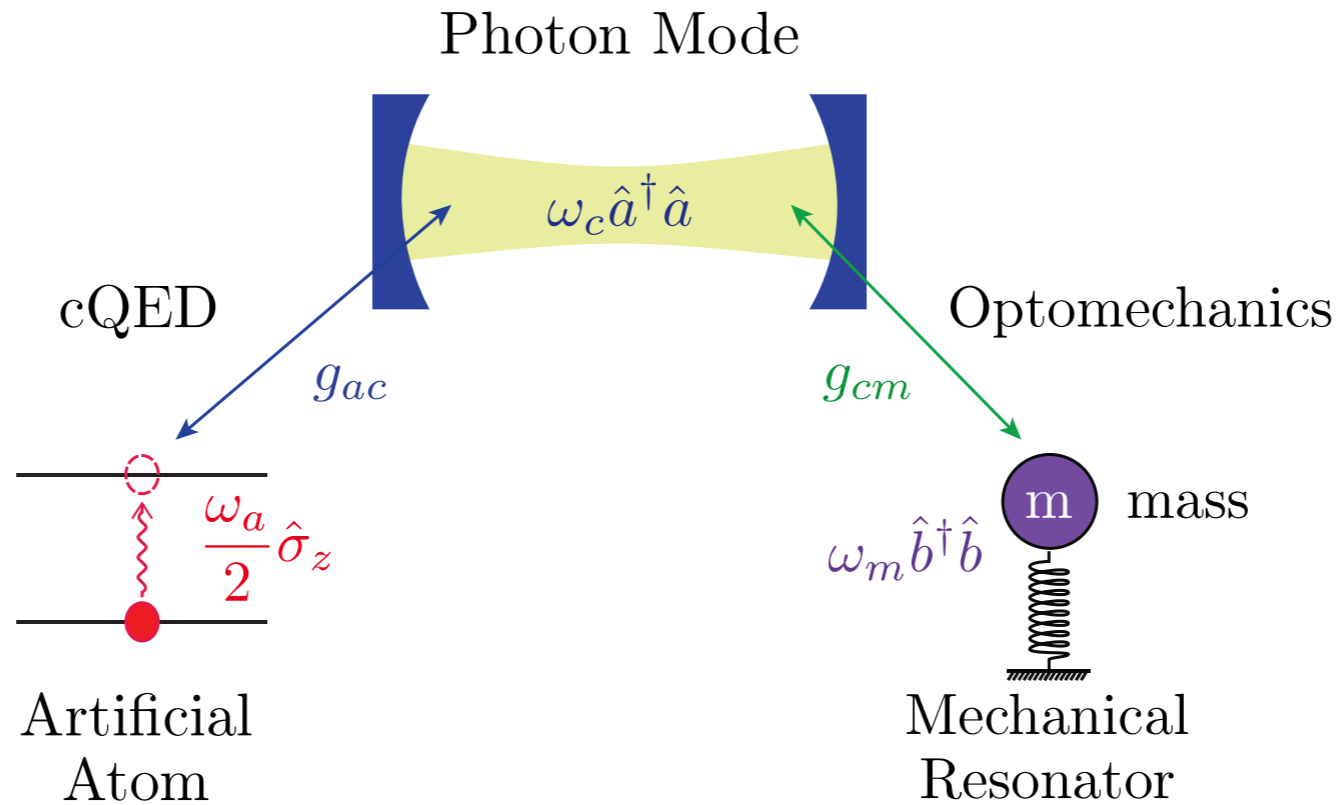
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cavity quantum electrodynamics

optomechanics

# Model



$$\begin{aligned}
 \hat{H}_{\text{tot}} = & \omega_c \hat{a}^\dagger \hat{a} + \frac{\omega_a}{2} \hat{\sigma}_z + ig_{ac} (\hat{\sigma}_+ \hat{a} - \hat{\sigma}_- \hat{a}^\dagger) \\
 & + \omega_m \hat{b}^\dagger \hat{b} - g_{cm} \hat{a}^\dagger \hat{a} (\hat{b} + \hat{b}^\dagger),
 \end{aligned} \tag{1}$$

# Atom-cavity-mechanics polaritons

$$\hat{H}_{\text{tot}} = \omega_c \hat{a}^\dagger \hat{a} + \frac{\omega_a}{2} \hat{\sigma}_z + ig_{ac}(\hat{\sigma}_+ \hat{a} - \hat{\sigma}_- \hat{a}^\dagger) + \omega_m \hat{b}^\dagger \hat{b} - g_{cm} \hat{a}^\dagger \hat{a} (\hat{b} + \hat{b}^\dagger), \quad (1)$$

$\hat{N}_{\text{polariton}} = \hat{a}^\dagger \hat{a} + \hat{\sigma}_+ \hat{\sigma}_-$  is conserved.

$\hat{H}_{\text{JC}} = \omega_c \hat{a}^\dagger \hat{a} + \frac{\omega_a}{2} \hat{\sigma}_z + ig_{ac}(\hat{\sigma}_+ \hat{a} - \hat{\sigma}_- \hat{a}^\dagger)$  for  $\omega_a = \omega_c$

$\hat{H}_{\text{JC}} |\pm^{(n)}\rangle = \omega_{\pm}^{(n)} |\pm^{(n)}\rangle$  with  $\omega_{\pm}^{(n)} = [(n - 1/2)\omega_c \pm \frac{\Omega^{(n)}}{2}]$  and  $\Omega^{(n)} = 2\sqrt{n}g_{ac}$

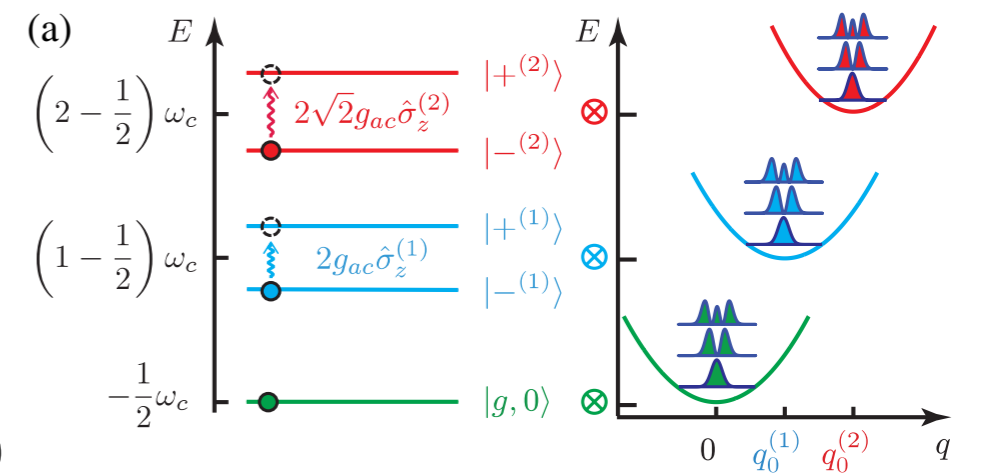
Eigenstates are polaritons:  $|\pm^{(n)}\rangle = (1/\sqrt{2})(|g, k = n\rangle \pm i|e, k = n - 1\rangle)$

pho-bit  
qu-ton

$$\hat{H}_{\text{tot}} = \sum_{n \in \mathbb{N}} \left\{ (n - 1/2)\omega_c \mathbb{1}^{(n)} + \frac{\Omega^{(n)}}{2} \hat{\sigma}_z^{(n)} - g_{cm} \left( \frac{1}{2} \hat{\sigma}_x^{(n)} + \left( n - \frac{1}{2} \right) \mathbb{1}^{(n)} \right) (\hat{b} + \hat{b}^\dagger) \right\} + \omega_m \hat{b}^\dagger \hat{b},$$

coupling      displacement

(2)



$$q_0^{(n)} = \sqrt{2}g_{cm}(n - 1/2)/\omega_m$$

displaced position

# Atom-cavity-mechanics polaritons

$$\hat{H}_{\text{tot}} = \sum_{n \in \mathbb{N}} \left\{ (n - 1/2) \omega_c \mathbb{1}^{(n)} + \frac{\Omega^{(n)}}{2} \hat{\sigma}_z^{(n)} - g_{cm} \left( \frac{1}{2} \hat{\sigma}_x^{(n)} + \left( n - \frac{1}{2} \right) \mathbb{1}^{(n)} \right) (\hat{b} + \hat{b}^\dagger) \right\} + \omega_m \hat{b}^\dagger \hat{b}, \quad (2)$$

$$\hat{H}^{(n)} = \frac{\Omega^{(n)}}{2} \hat{\sigma}_z^{(n)} + \omega_m \hat{b}_n^\dagger \hat{b}_n - \frac{g_{cm}}{2} (\hat{\sigma}_-^{(n)} \hat{b}_n^\dagger + \hat{\sigma}_+^{(n)} \hat{b}_n) \quad \text{RWA}$$
~~$$- g_{cm} \frac{\sqrt{2}}{2} q_0^{(n)} \hat{\sigma}_x^{(n)} + \left( \omega_0^{(n)} - \frac{\omega_m}{2} q_0^{(n)2} \right) \mathbb{1}^{(n)}, \quad (3)$$~~

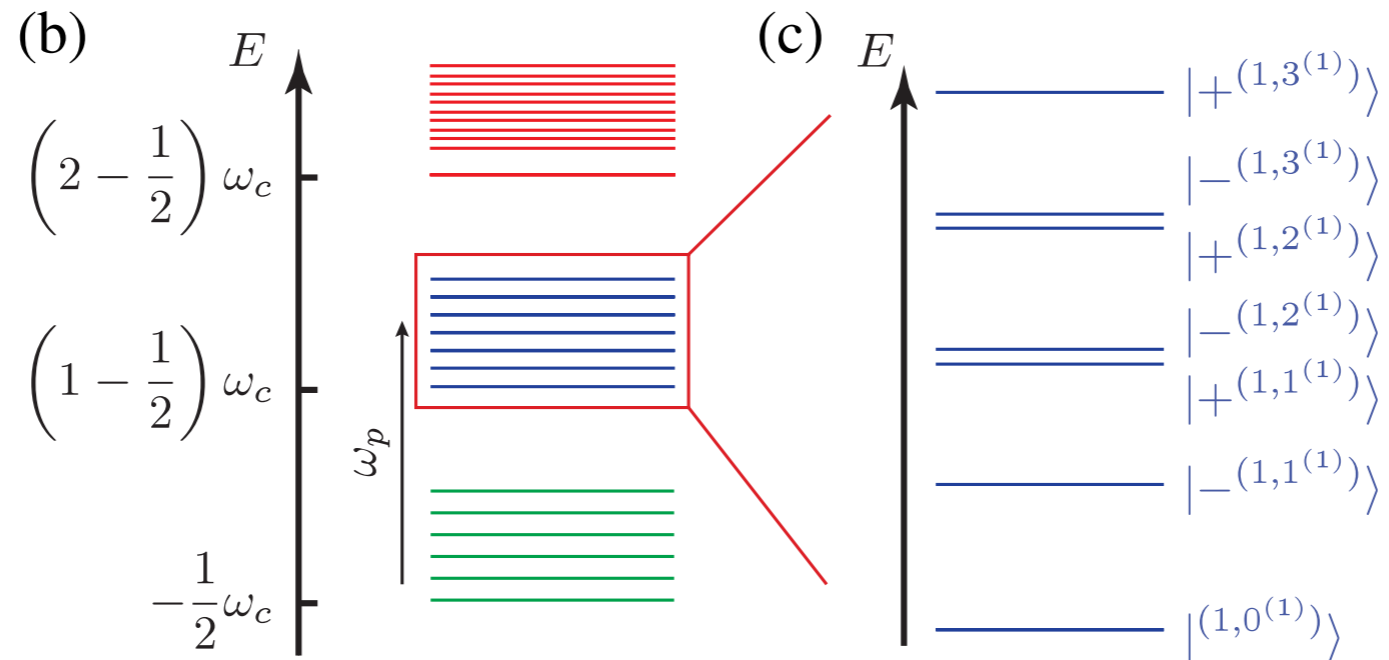
$g_{cm} \ll \omega_m$

$$q_0^{(n)} = \sqrt{2} g_{cm} (n - 1/2) / \omega_m$$

displaced position

effective Jaynes-Cummings model  
 $m^{(n)}$  new effective polaron number  
 $n$  previous polariton number

# Atom-cavity-mechanics polaritons



$$\hat{H}^{(n)} |\pm^{(n,m^{(n)})}\rangle = \omega_0^{(n)} - \frac{\omega_m}{2} q_0^{(n)2} + \left(m - \frac{1}{2}\right) \omega_m \pm \nu^{(n,m)}, \quad (4)$$

where

$$\nu^{(n,m)} = \sqrt{\left(\frac{\Omega^{(n)} - \omega_m}{2}\right)^2 + \frac{m^{(n)}}{4} g_{cm}^2}. \quad (5)$$

effective Jaynes-Cummings model

$m^{(n)}$  new effective polaron number

$n$  previous polariton number

# Master equation

$$\begin{aligned} \frac{d\hat{\rho}(t)}{dt} = & -i[\hat{H}_{\text{tot}} + \hat{V}_p(t), \hat{\rho}] + \gamma_{ac}L[\hat{a}]\hat{\rho} + \gamma_{ac}L[\hat{\sigma}_-]\hat{\rho} \\ & + n_{\text{th}}\gamma_mL[\hat{b}^\dagger]\hat{\rho} + (n_{\text{th}} + 1)\gamma_mL[\hat{b}]\hat{\rho}, \end{aligned} \quad (6)$$

$$\hat{V}_p(t) = iF_p(\hat{a}^\dagger e^{i\omega_p t} - \hat{a}e^{-i\omega_p t})$$

coherent drive

$$L[\hat{o}]\hat{\rho} = \hat{o}\hat{\rho}\hat{o}^\dagger - \frac{1}{2}(\hat{o}^\dagger\hat{o}\hat{\rho} + \hat{\rho}\hat{o}^\dagger\hat{o})$$

Simulation parameters (photon blockade regime):

$$\begin{aligned} \omega_c/\omega_m = 10^2, \quad \omega_a = \omega_c, \quad g_{ac}/\omega_m = \\ 1/2, \quad g_{cm}/\omega_m = 10^{-1}, \quad Q_m = \omega_m/\gamma_m = 10^4, \quad Q_{ac} = \\ \omega_{a,c}/\gamma_{ac} = 10^4, \quad F_p/\gamma_{ac} = 1, \quad \text{and } n_{\text{th}} = 3.45. \end{aligned}$$

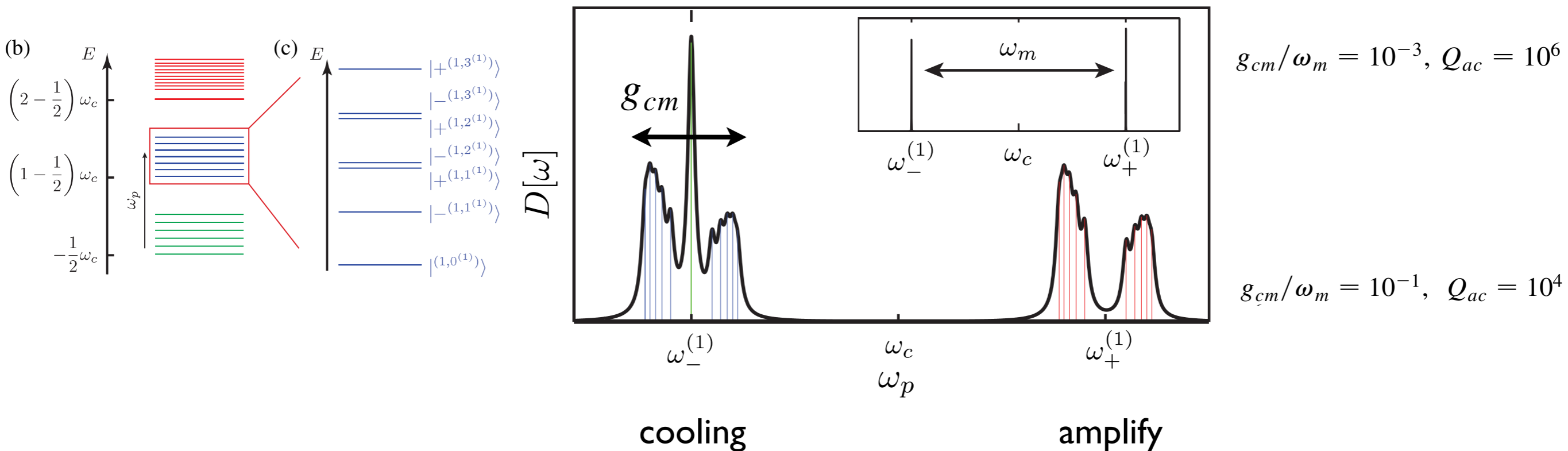
$$|g, k = 0\rangle\langle g, k = 0| \otimes \hat{\rho}_m$$

initial state

# Density of states

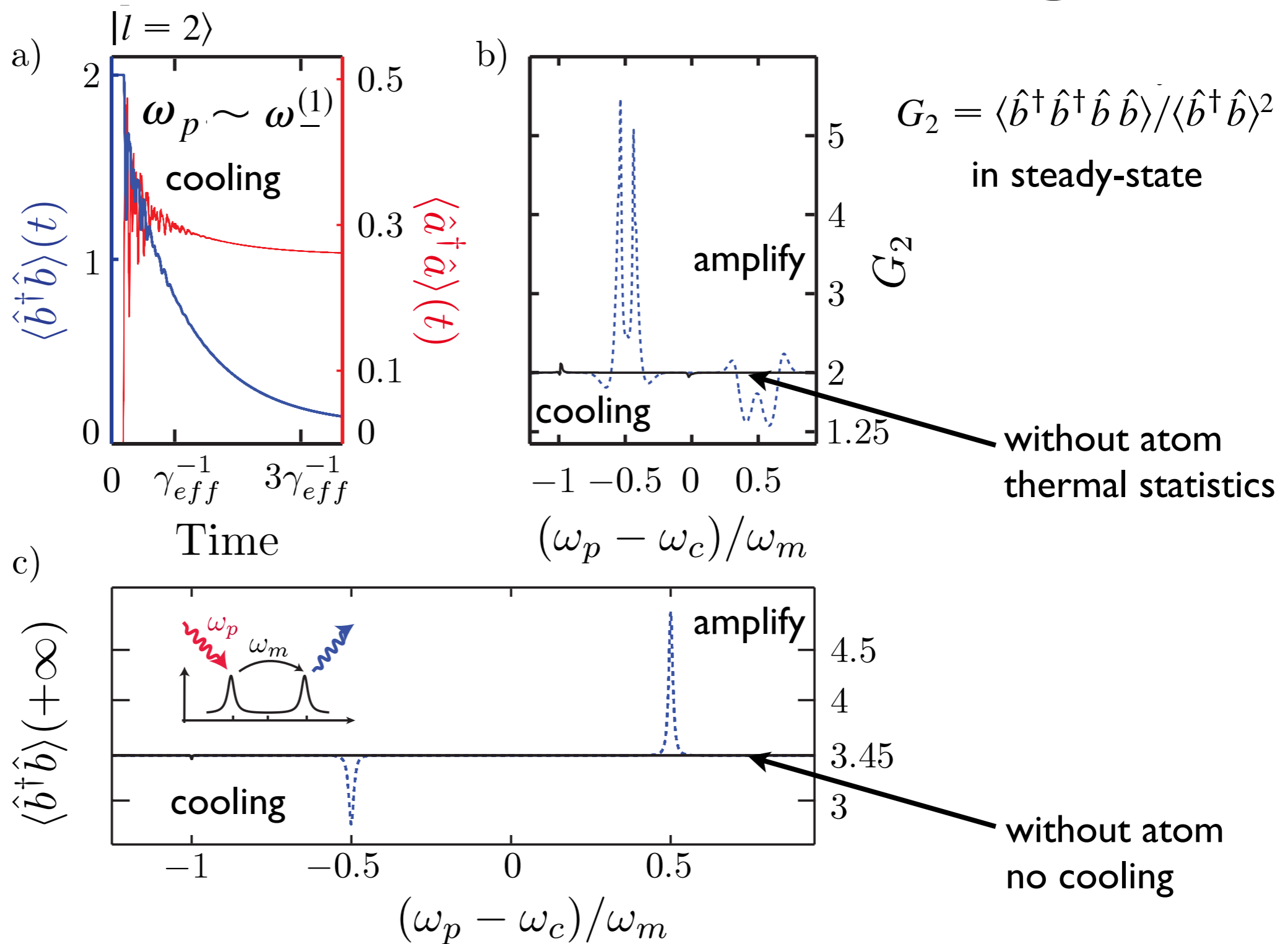
$$D[\omega] = \sum_{\substack{s', s = \pm \\ m', m \in \mathbb{N}}} |\langle s'^{(1, m')} | \hat{V}_p | s^{(0, m)} \rangle|^2 \delta(\omega - (\omega_{s'^{(1, m')}} - \omega_{s^{(0, m)}})).$$

(7)



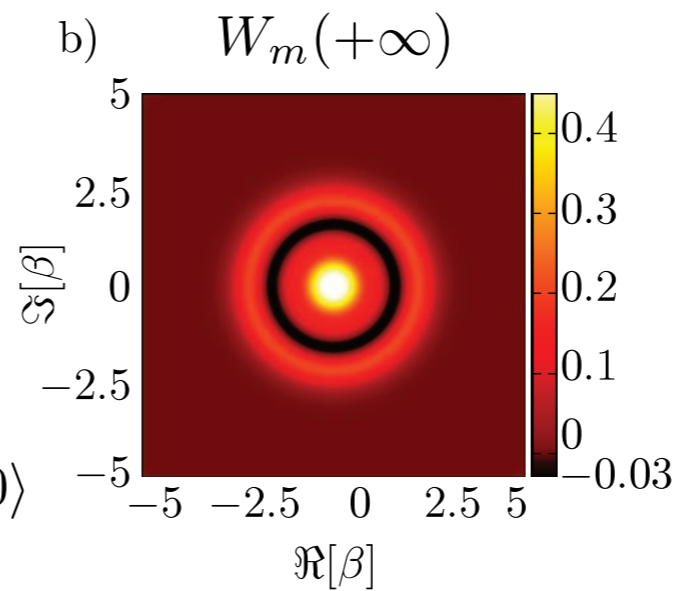
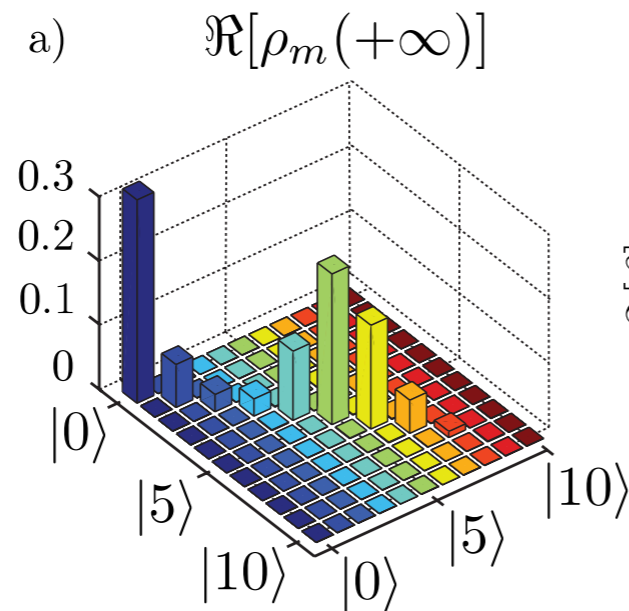


# Atom-assisted cooling

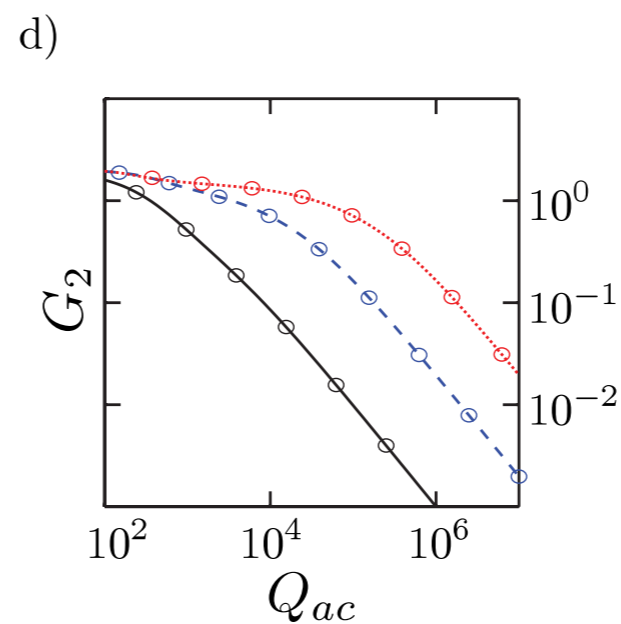
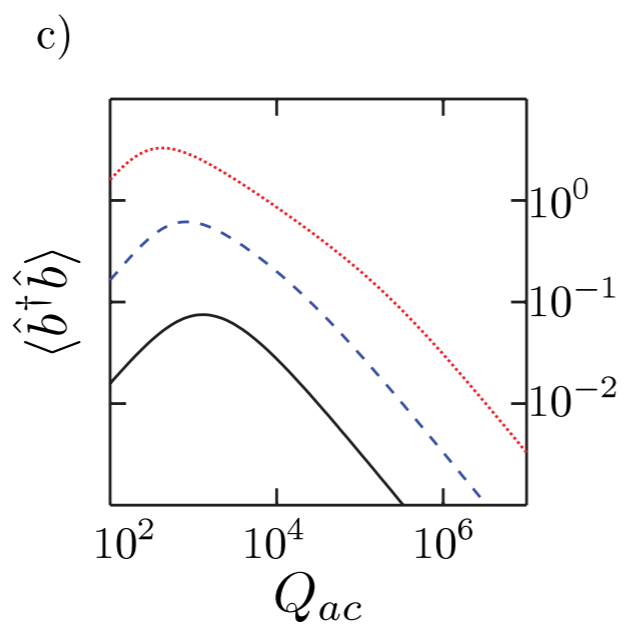


# Mechanical steady-state

$\omega_p \sim \omega_+^{(1)}$   
 amplify  
 coherent pump



non-classical mechanical  
 steady-states



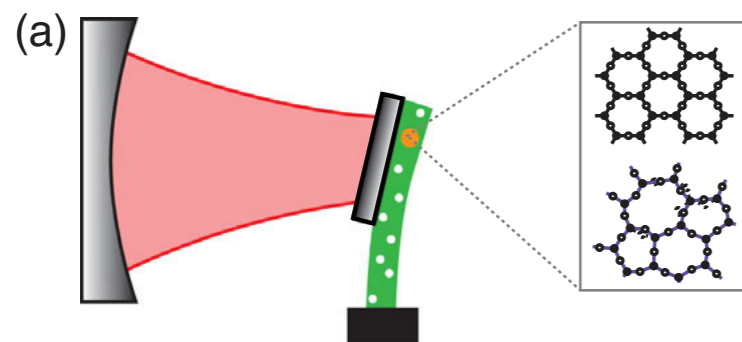
excitation of polarons and  
 emission of phonons with  
 sub-Poissonian statistics

$\omega_p \sim \omega_+^{(1)}$   
 amplify  
 incoherent pump

$$Q_m = 10^1, 10^2, \text{ and } 10^3$$

# Conclusions

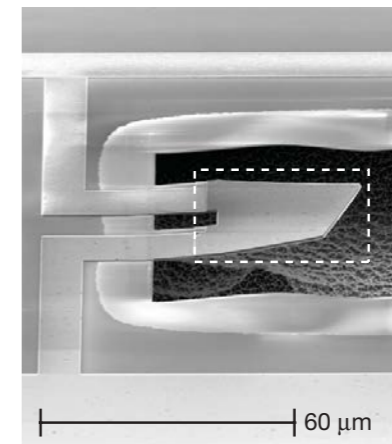
- atom-cavity-mechanics polaron eigenstates
- atom enhances single-phonon cooling
- atom leads to strong bunching of phonons
- atom yields non-classical mechanical states
- atom leads to single-phonon emission



defects in silica toroids



GaAs resonators



microwave resonators