Experimental Proposal to Detect Topological Ground State Degeneracy

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One of the most profound features of topologically ordered states of matter, such as the fractional quantum Hall (FQH) states, is that they possess topology-dependent ground state degeneracies that are robust to all local perturbations. Here we propose to directly detect these topological degeneracies in an experimentally accessible setup. The detection scheme uses electrical conductance measurements in a double layer FQH system with appropriately patterned top and bottom gates. We discuss two experimental platforms; in the first, the detection of topologically degenerate states coincides with the detection of Z_N parafermion zero modes. We map the relevant physics to a single-channel Z_N quantum impurity model, providing a novel generalization of the Kondo model. Our proposal can also be adapted to detect the Z_N parafermion zero modes recently discovered in FQH line junctions proximitized with superconductivity.

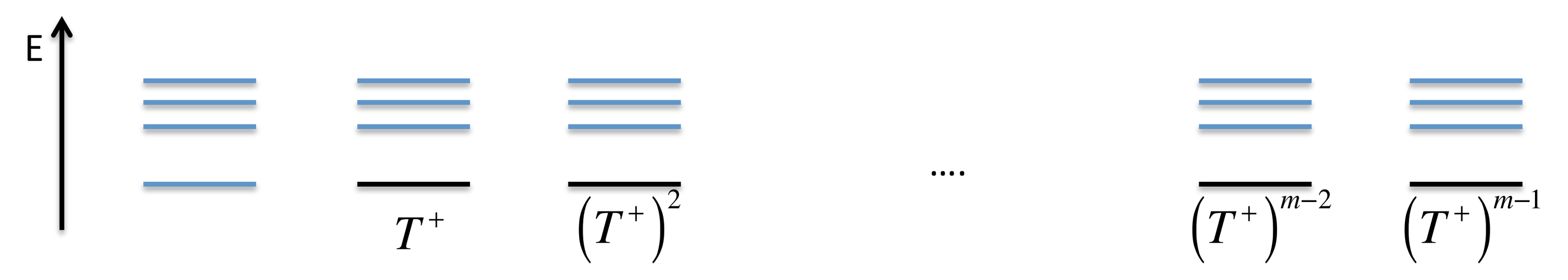
- Feature of topological order: ground state degeneracies
- Example: Majorana Fermion $\gamma = \gamma^{\dagger}$

without majorana   with majorana

- The 2 cases, with and without a majorana, have the same energy
- 2 fold degenerate ground state

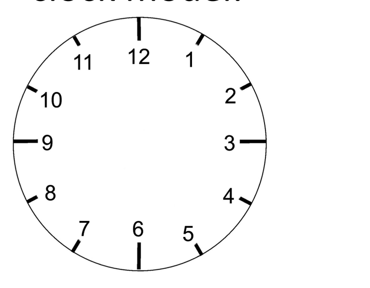
Generalization to parafermions

- Topological state
- m-fold degenerate ground state (m=2 for majoranas)



- T^{+} cycles through the ground states from left to right
- Parafermions: $(T^{+})^{m-1} = (T^{+})^{\dagger}$ $(T^{+})^{m} = 1$
- $(T^{+})^{\dagger}$ cycles through the ground states from right to left
- $(T^{+})^{l}$ jumpes from ground state n to ground state n+l
- Parafermions have special statistics, braiding, …

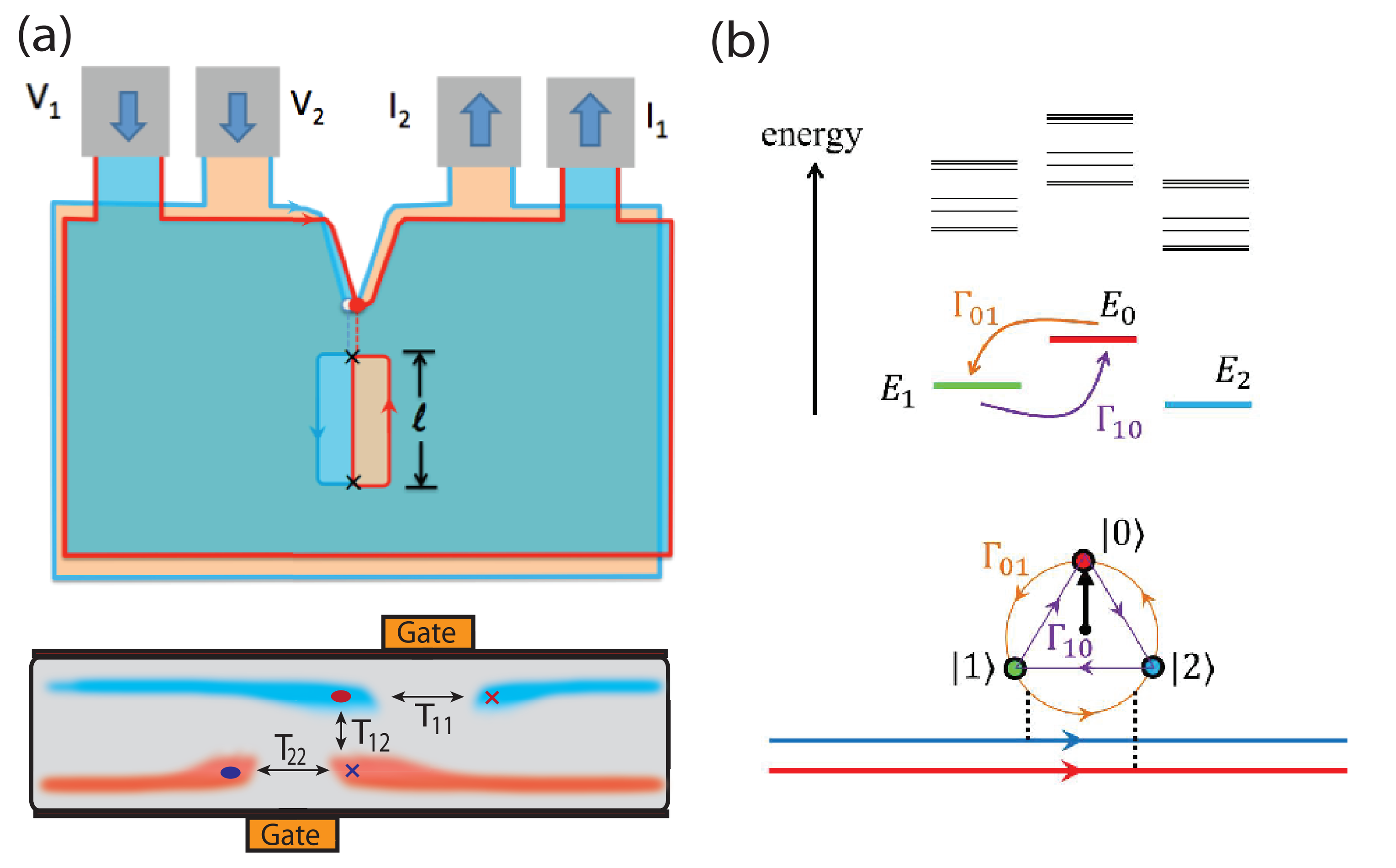
clock model:



Physical realization

- $1/m$ Laughlin FQH states have m^{g} topologically degenerate states on manifold with genus g
- Sphere: g=0, Donut: g=1
- Bilayer FQH state

- Each layer 1/m Laughlin state
- Two gates form 2 antidots
- Antidots aligned to form topological line junction
- 4 voltage and current probes
- Structure has genus g=1
- Quantum point contact
- Measure: inter layer current
- l: length of line junction



- Bilayer 1/m FQH state with topological line junction eq. to single layer 1/m Laughlin state on surface with non-trivial genus
- Endpoints of line junction become non-Abelian twist defects localizing parafermions

- Quasiparticle tunneling between layers cycles through the degenerate ground states

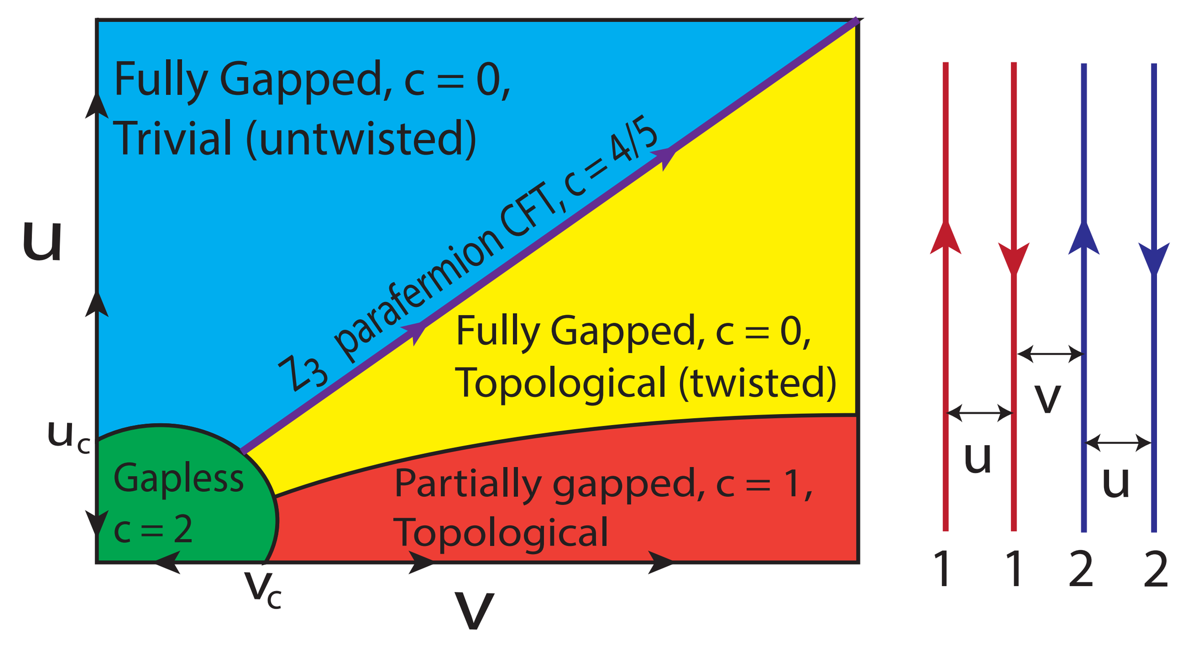
model

$$H = \int dx \left(H_0 + H_{tun} \right)$$

$$H_0 = m\pi v_0 \sum_{\alpha,\beta,I,J} n_{\alpha I} \lambda_{IJ}^{\alpha\beta} n_{\beta J} \qquad n_{\alpha I} = \frac{1}{2\pi} \partial_x \phi_{\alpha I}$$

$$H_t = \sum_{I,J} T_{IJ} \cos\left(m(\phi_{LI} + \phi_{RJ}) \right) \qquad \left[\phi_{R/L,I}(x), \phi_{R/L,J}(y) \right] = \pm i \frac{\pi}{m} sign(x-y)$$

- Minimization of cosine terms not for all fields possible: either inter layer T_{12}, T_{21} (v-terms) or intra layer T_{22}, T_{11} (u-terms)



m=3
$T_{11}=T_{22}=u$
$T_{12}=v$
$T_{21}=0$

Effective model for the topological phase

- Consider now the QPC and an effective model for tunneling between the outer edges through the line junction
- The ground state degeneracy is now lifted due to finite size by exp. small factors $e^{-l/loc}$
  label the ground states E_n

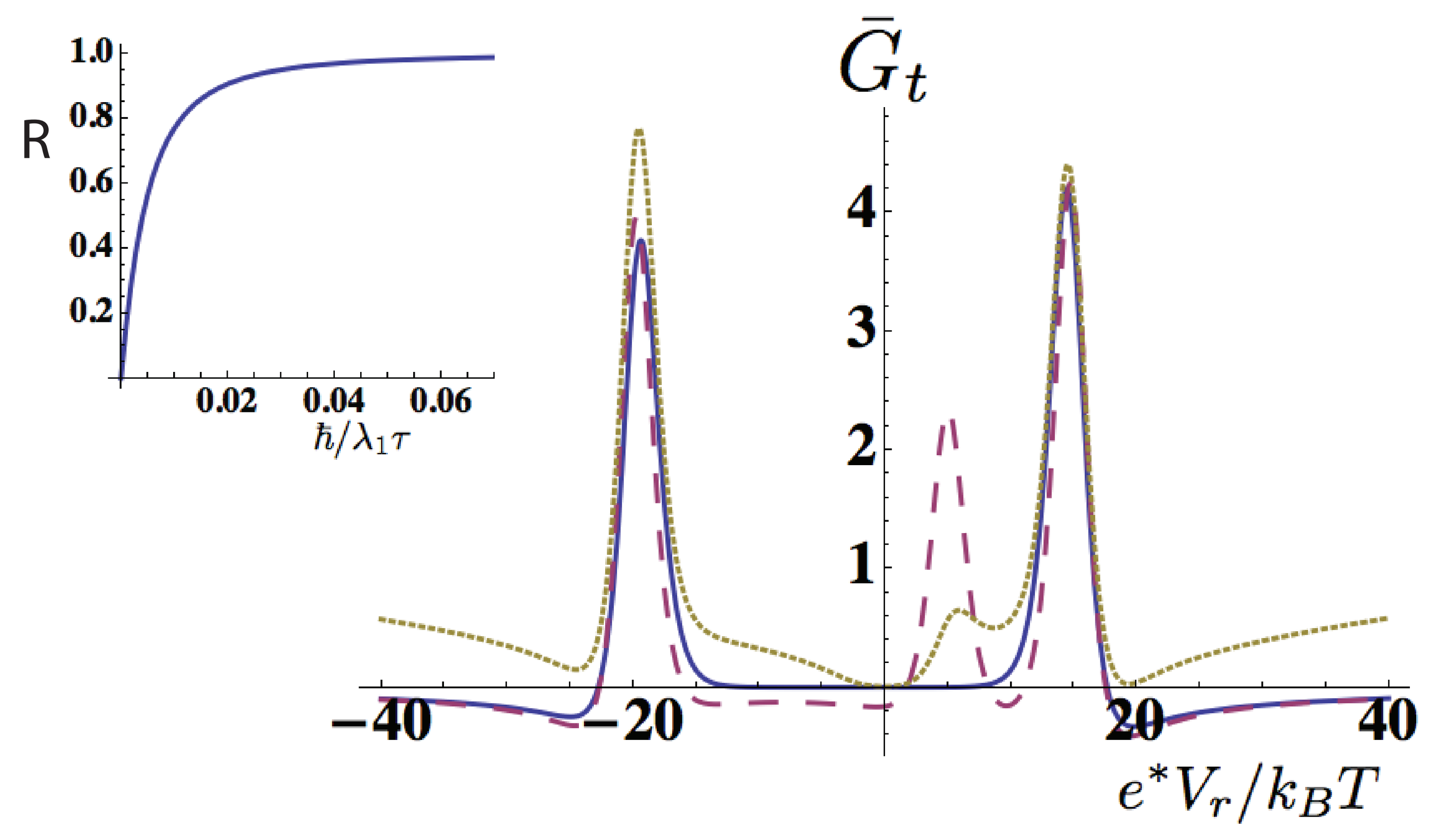
$$H(t) = \int dx \left(H_0 + H_b(t) \right) + \sum_{n=1}^{m} E_n |n\rangle\langle n|$$

$V_r(t)$ is a time dep. potential between the layers

$$H_b(t) = \delta(x) \sum_{l=-\infty}^{\infty} \lambda_l e^{ileV_r(t)/\hbar} e^{il(\phi_{R1}-\phi_{R2})(t,x)} \left(T^{+} \right)^{l} \qquad \left(T^{+} \right)^{l} |n\rangle = |n+l\rangle$$

Weak tunneling & master equation approach

- Effective model is solved for weak tunneling using master equation
- Include also temp. dep. relaxation rate $1/\tau$ to go from one layer to the other



- Condition for a steady state current through the QPC: cycling through all the ground states
- Cycling requires energy (for m=3)

$$eV_r > \max_{n=0,1,2}(E_{n+1} - E_n) \quad \text{or} \quad -eV_r > \min_{n=0,1,2}(E_n - E_{n+1})$$

- These two lead to 2 resonance peaks
- If relaxation rate $1/\tau$ or frequency of the ac field $V_r(t)$ larger than transition rates: additional resonances when $eV_r = E_{n+1} - E_n$

This gives 3 resonances in the case m=3

More generally, in a 1/m FQH bilayer state, at strong enough 1/τ or $V_r(t)$ frequency, the number of resonance peaks is equal to the number of topological ground states m (if only include single particle quasi-tunneling)