Revealing topological superconductivity in extended quantum spin Hall Josephson junctions

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Quantum spin Hall-superconductor hybrids are promising sources of topological superconductivity and Majorana modes, particularly given recent progress on HgTe and InAs/GaSb. We propose a new method of revealing topological superconductivity in extended quantum spin Hall Josephson junctions supporting 'fractional Josephson currents'. Specifically, we show that as one threads magnetic flux between the superconductors, the critical current traces an interference pattern featuring sharp fingerprints of topological superconductivity—even when noise spoils parity conservation.

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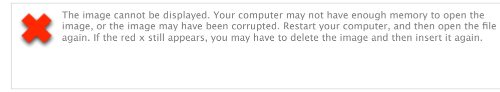
Silas Hoffman

Journal club

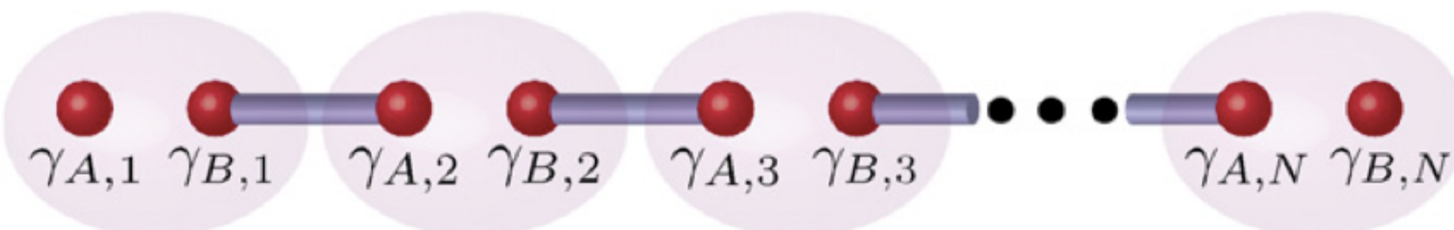
18.3.14

Obligatory Majorana Introduction

Marojanas are there own antiparticle: 

Formally equivalent to one-half an electron, e.g.: $f_i = \frac{1}{2}(\gamma_{B,i} + i\gamma^\dagger_{A,i+1})$

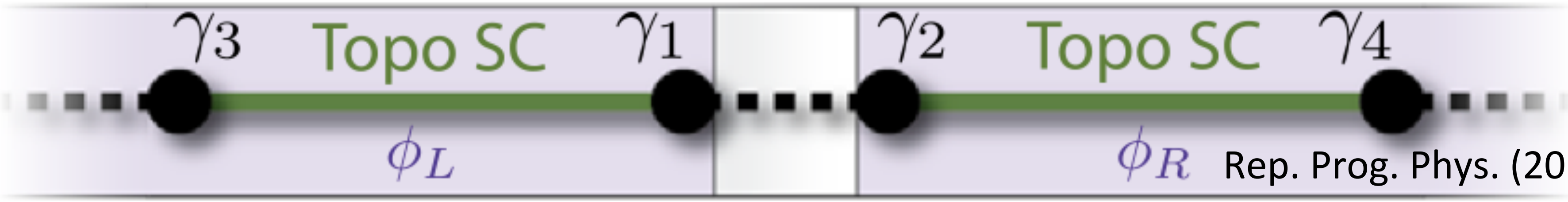
Unpaired Majoranas can be realized at the ends of 1D topological superconductors:



J. Alicea, Rep. Prog. Phys. (2012)

Because Maroranas at the ends are unconnected to the bulk, the groundstate is doubly degenerate, corresponding to the occupation or absence of an 'electron' formed from $f = \frac{1}{2}(\gamma_{A,1} + i\gamma^\dagger_{B,N})$

Majorana Signature: Unconventional Josephson Relation



Rep. Prog. Phys. (2012)

When the ends of a two 1D topological insulators are close (smaller than the coherence length of the superconductor), the Majoranas can interact (and transfer supercurrent)

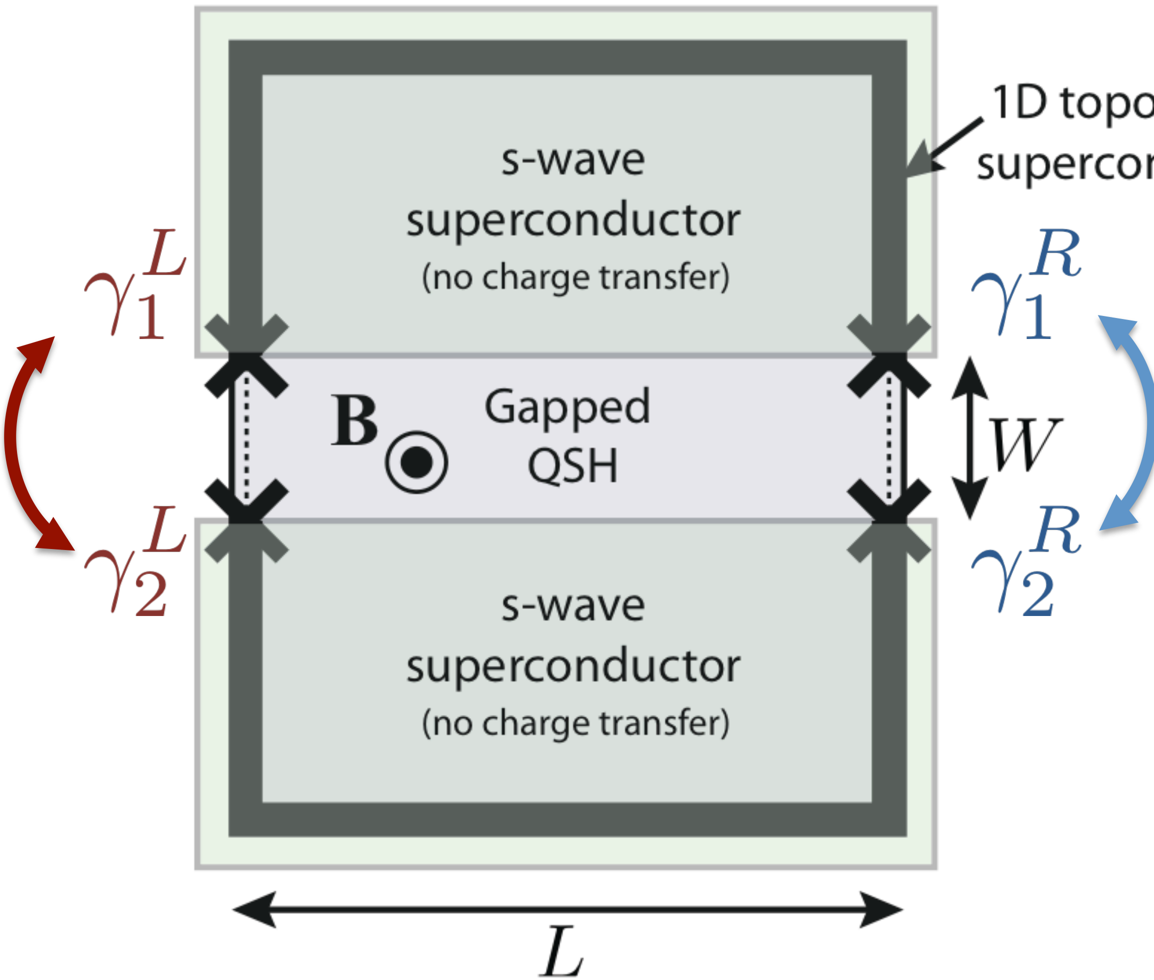
$$\boxed{H_{\text{int}} = \cos(\delta\phi/2)(1/2 - f^\dagger f) = (-1)^p \cos(\delta\phi/2)}$$

$$f = \frac{1}{2}(\gamma_1 + i\gamma_2^\dagger), \;\; p = 0, 1 \qquad I \sim \partial H_{\text{int}}/\partial\phi$$

- There is 4π periodicity and the sign of the current is determined by the occupation of the electron at the interface (parity)
- Parity switching processes (e.g. quasiparticle poisoning) restore to 2π

Is there a signature of Marojanas even in the presence of parity switching?

Setup



$$W \ll \xi, \; L \gg \xi$$

Majorana's on the left/ right side of the sample overlap

Two 1D topological Josephson junctions

$$E_{\mathbf{p}}(\phi, f) = \Delta[(-1)^{p_L}\cos(\phi/2) + (-1)^{p_R}\cos(\phi/2 + \pi f)]$$

$$\mathbf{p} = (p_L, p_R)$$

Josephson Junction Model

Using the RCSJ model:

$$I = I_{\mathbf{p}}(\phi, f) + \frac{\hbar}{2eR}\dot{\phi} + \zeta(t)$$

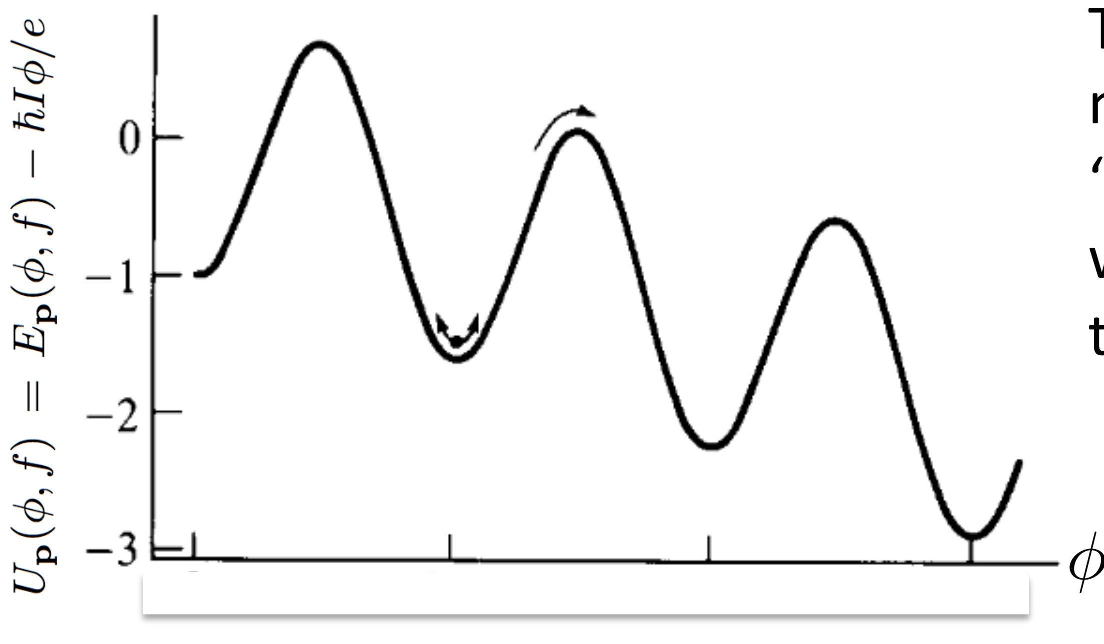
supercurrent

$$I_{\mathbf{p}}(\phi, f) = \frac{e}{\hbar}\partial_\phi E_{\mathbf{p}}(\phi, f)$$

normal

thermal noise

$$\langle \zeta(t)\zeta(t')\rangle = 2T/R\delta(t-t')$$



This is the equation of motion for a particle in a 'tilted washboard' potential with relaxation on the timescale $\tau_R \equiv \hbar^2/(4e^2R\Delta)$

When *T*=0, the critical current is the value of *I* for which no minimum exist

Fokker-Planck Analysis and Voltage

The distribution function $\mathcal{P}_{\mathbf{p}}(\phi, t)$ is determined by generalized Focker-Planck equation

$$\partial_t \mathcal{P}_{\mathbf{p}} = \frac{1}{\tau_R \Delta} \partial_\phi \left[\partial_\phi U_{\mathbf{p}}/2 + T\partial_\phi\right] \mathcal{P}_{\mathbf{p}} + \sum_{\mathbf{p}'} \left[W_{\mathbf{p}' \to \mathbf{p}} \mathcal{P}_{\mathbf{p}'} - W_{\mathbf{p} \to \mathbf{p}'} \mathcal{P}_{\mathbf{p}}\right]$$

Parity-conserving thermal fluctuations in junction held at temperature *T*

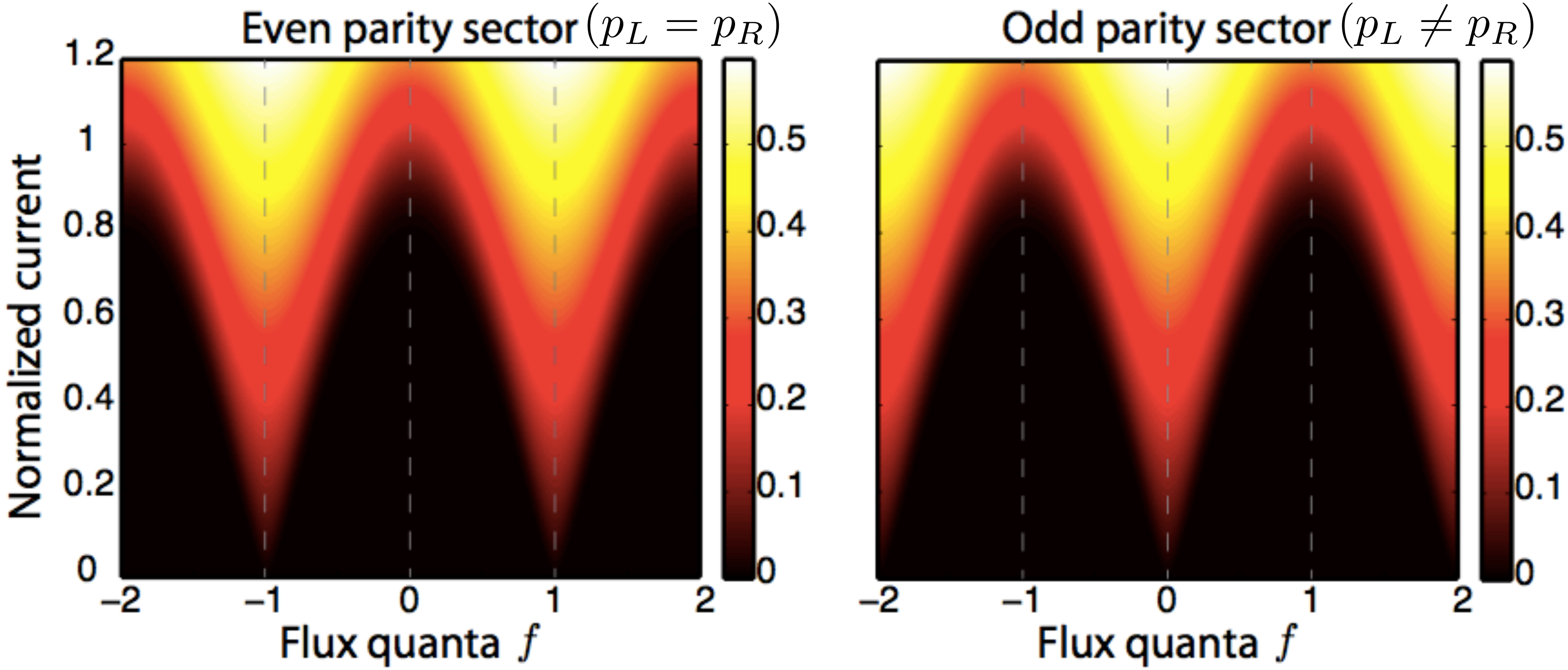
Josephson junction connected to electronic bath at temperature T_b, allowing for parity-switching on time scale $\mathcal{T}$

From the ac Josephson effect:

$$V = \frac{\hbar}{2e}\langle\dot{\phi}\rangle = R\sum_{\mathbf{p}}\int_0^{4\pi} d\phi [I - I_{\mathbf{p}}(\phi, f)]\mathcal{P}_{\mathbf{p}}(\phi)$$

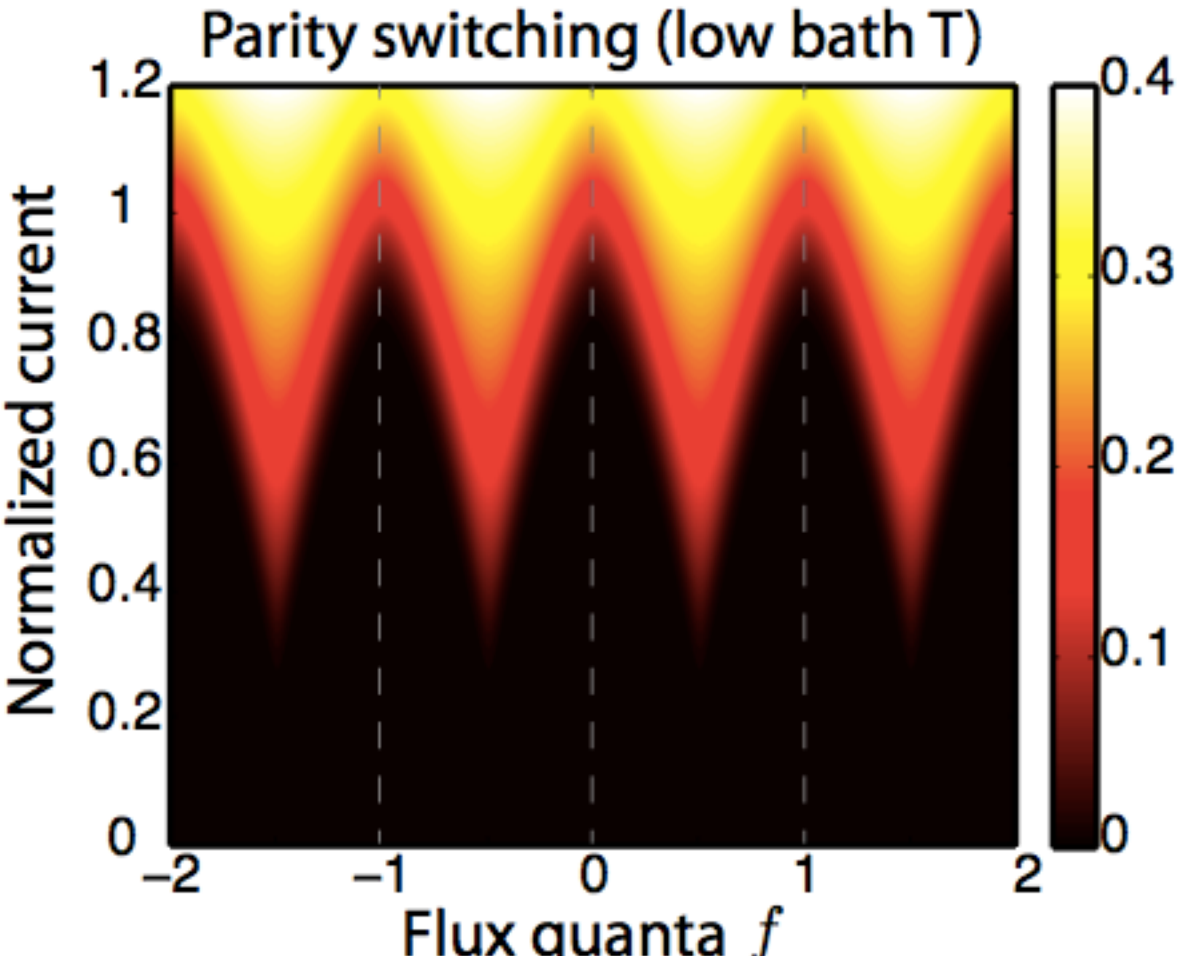
Critical current is the maximum current for which *V* is zero

No parity switching



$$I_{\mathbf{p},c} = e\Delta|\cos(\pi f/2)|/\hbar \qquad I_{\mathbf{p},c} = e\Delta|\sin(\pi f/2)|/\hbar$$

Parity Switching, $T_b \ll \Delta$



Phase will always fall to parity independent minimum

This is, roughly, the minimum voltage between the odd and even parity sectors

At $T=T_b=0$, the maximum current minimum of the potential admits a solution:

$$I_c = e\Delta/\hbar \max\{\cos^2(\pi f/2), \sin^2(\pi f/2)\}$$

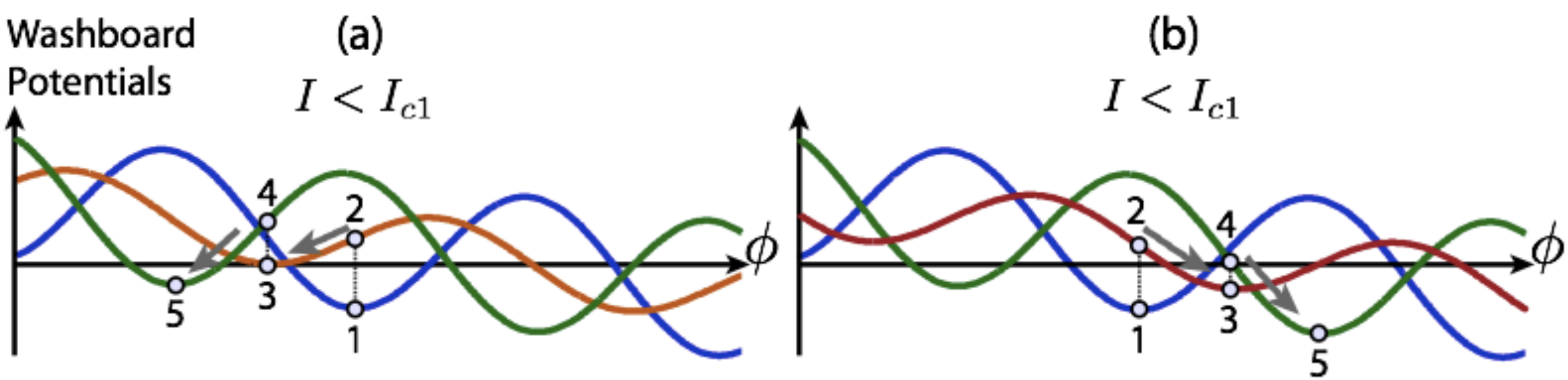
Parity Switching, T_b>>Δ

The parity can fluctuate, independent of initial and final energy, on time scale τ

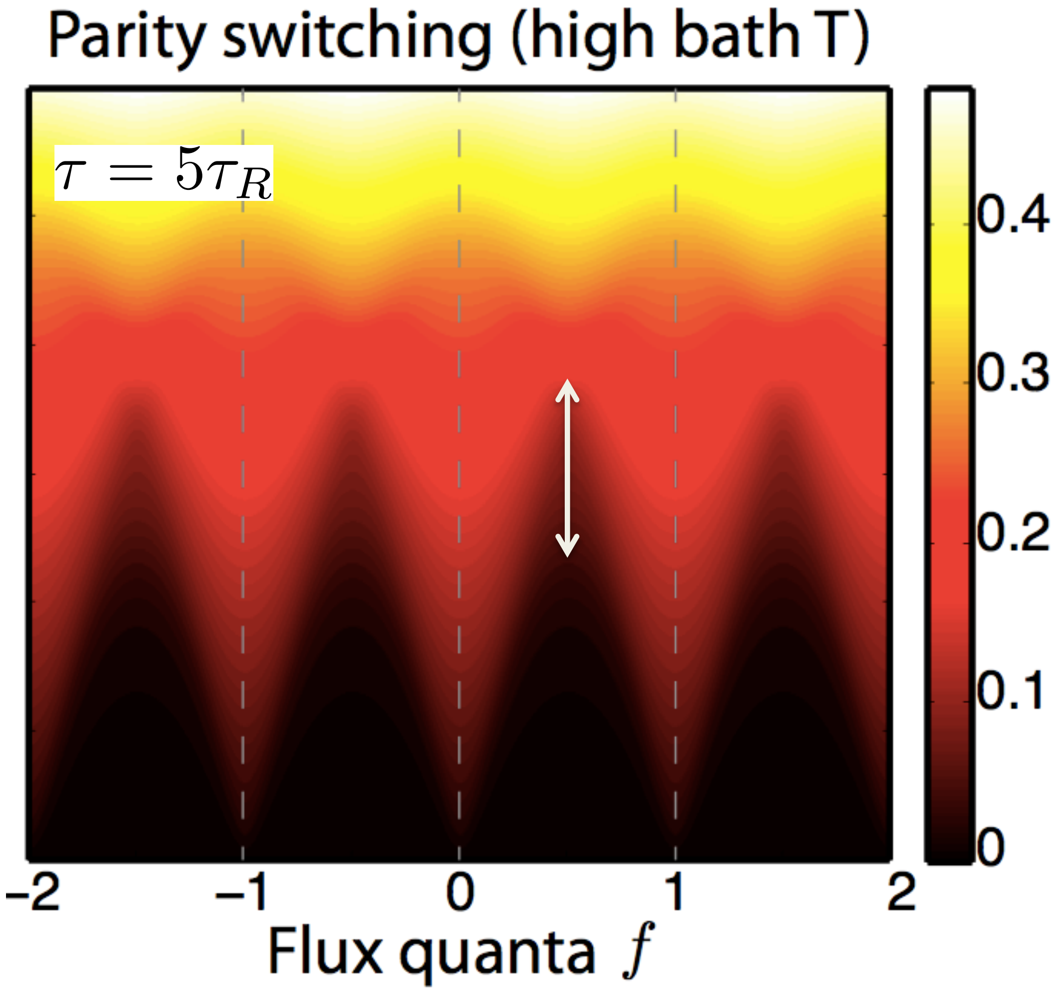
Two critical currents become important:

$$I_{c1} = \min_{\mathbf{p}} I_{\mathbf{p},c},\ \ I_{c2} = \max_{\mathbf{p}} I_{\mathbf{p},c}$$

If $I < I_{c1}$ all parities have minimum energies, but the phase may still fluctuate even for T=0



Parity Switching, $T_b >> \Delta$, $I < I_{c1}$



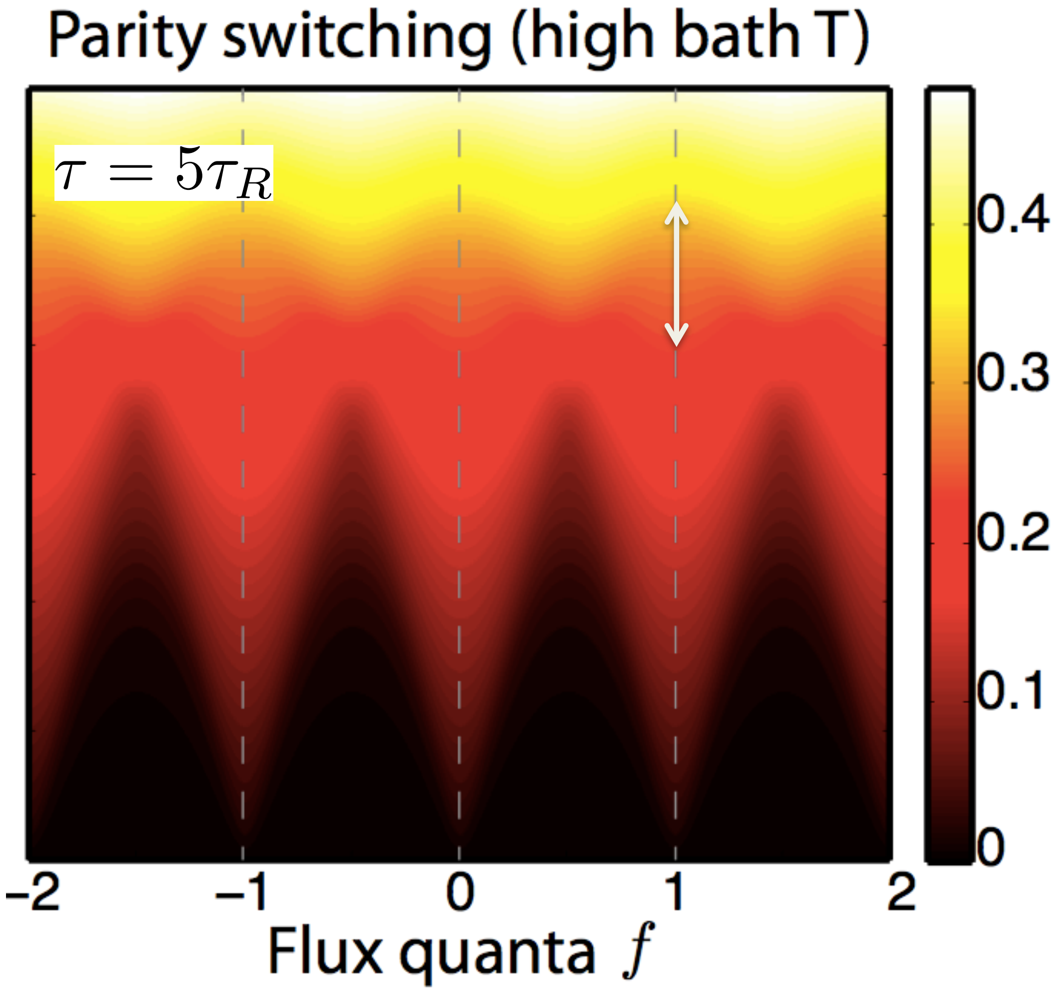
Relaxation time due to *parity flip*

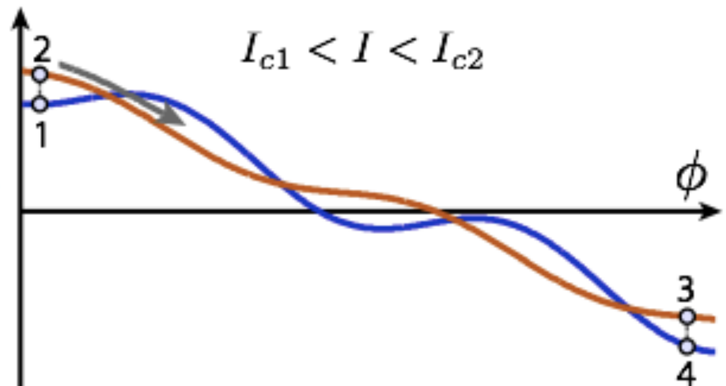
$$\tau_{\rm rel} \sim \max_{\mathbf{p}} \frac{\hbar}{eR\sqrt{I_{\mathbf{p},c}^2 - I^2}}$$

If $\tau \gg \tau_{\rm rel}$ then the mean phase change is zero (telegraph noise)

As $I \sim I_{\mathbf{p},c}$, $\tau \ll \tau_{\rm rel}$ and there is a net diffusion of phase, smearing the critical current line

Parity Switching, $T_b >> \Delta$, $I_{c1} < I < I_{c2}$

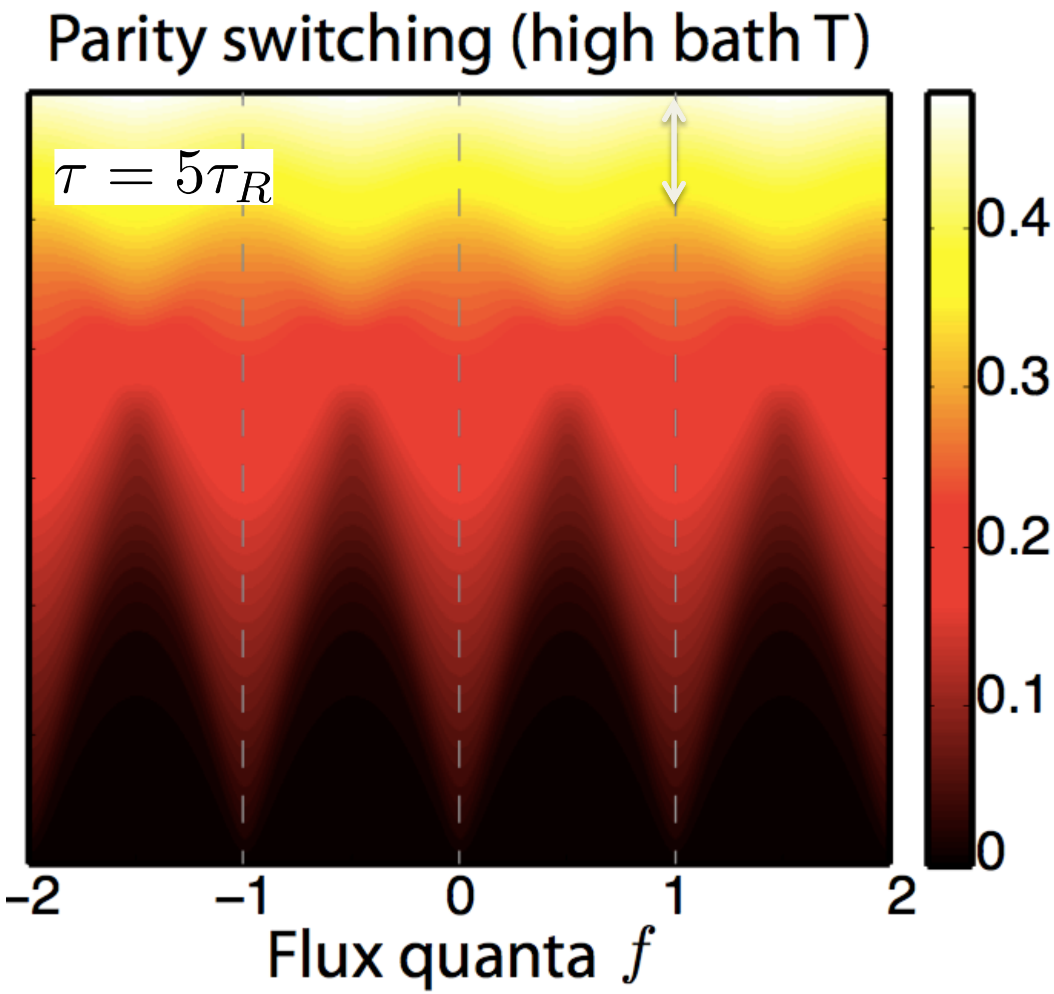


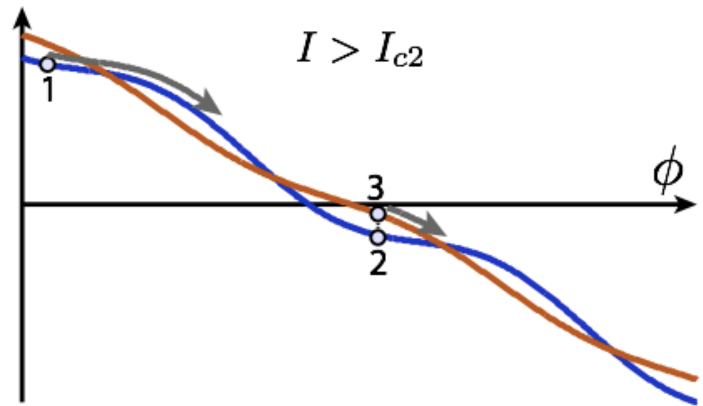


In the sectors with no minima, the phase slides down the potential, so there will always be a voltage

In the two sectors with minima which contribute no average current if $\tau \gg \tau_{\text{rel}}$ while the critical current line is smeared when $I \sim I_{c2}$

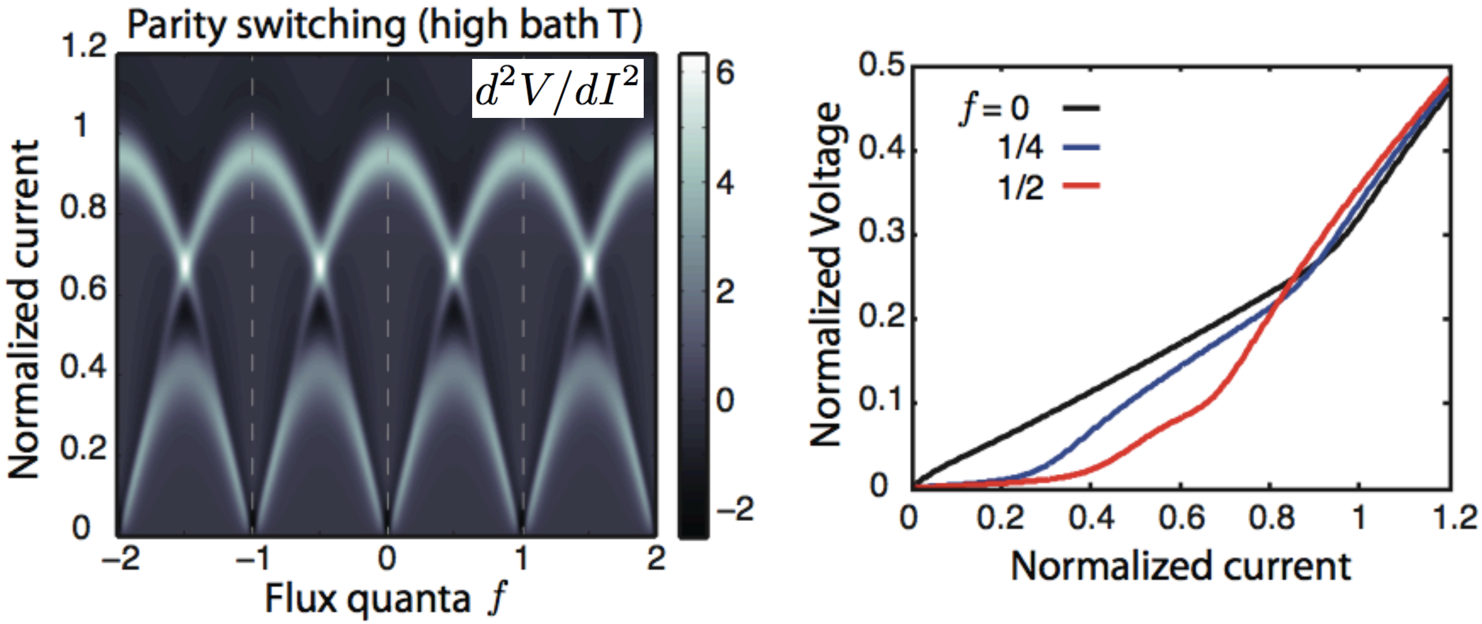
Parity Switching, $T_b \gg \Delta$, $I_{c2} < I$





There are no more minimums, and the phase slides down the potential, averaging the velocity of the individual potentials

Conclusions



- If there is quasiparticle poisoning, the most dramatic results occur when the temperature is large
  - (Near) zero critical current at f=0
  - Emergence of multiple critical currents