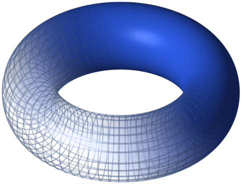
Fibonacci Anyons From Abelian Bilayer QH States

by Abolhassan Vaezi and Maissam Barkeshli (arXiv:1403.3383)



Constantin Schrade

University of Basel

May 9, 2014

Overview

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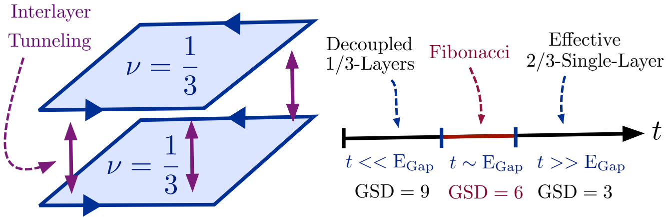

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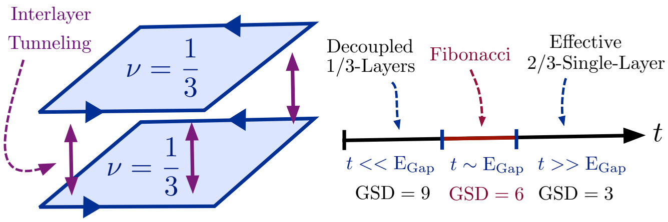

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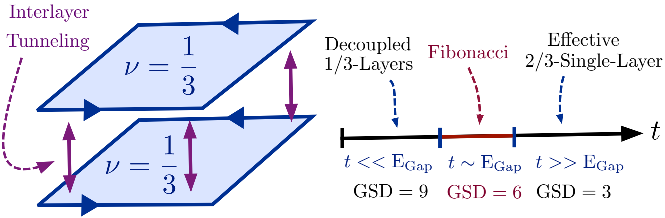

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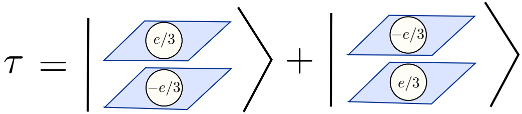

Outline

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Non-Interacting System

Single-Particle-Problem of Electrons in a Magnetic Field:

$$\mathcal{H} = \frac{(-i\vec{\nabla}-\vec{A})^2}{2m^*} \text{ with } \vec{A} = (-y, 0) \text{ and } \hbar = c = e = B = 1$$

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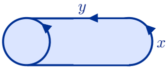
Geometry: Cylinder



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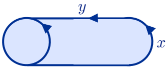

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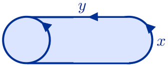

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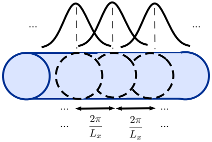
Ground State Wavefunction (n=1):

$$\Psi_k(x, y) \propto e^{ix\left(\frac{2\pi k}{L_x}\right)} \cdot e^{-\frac{\left(y-y_0(k)\right)^2}{2}} \text{ with } k \in \mathbb{Z} \text{ and } y_0(k) := -\frac{2\pi}{L_x} k$$



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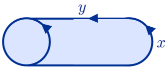

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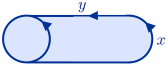


Many Body Ground State Wavefunction (n=1):

Number of Electrons

$$|\Psi\rangle \propto \sum_{k_1,\dots,k_{N_e}} a_{k_1,\dots,k_{N_e}} |\Psi_{k_1}\rangle \otimes \dots \otimes |\Psi_{k_{N_e}}\rangle$$

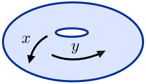
fully antisymmetrized

$$|\dots n_{-1}\ ,\ n_0\ ,\ n_1 \dots\rangle$$

either 0 or 1

Non-Interacting System

Geometry: Torus



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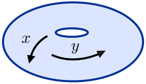
Geometry: Torus 

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Notice that $k \in \mathbb{Z}/N_s\mathbb{Z}$ with $N_s = \frac{L_x L_y}{2\pi}$ the # of single particle states.

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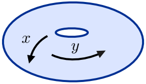

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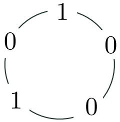
Many Body Picture:

$0\ 0\ 1\ 0\ 1 \xrightarrow{\text{Shift}} 1\ 0\ 0\ 1\ 0 \sim$



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$$H = \sum_i \sum_{r>s} U_{r,s} c_{i+s}^\dagger c_{i+r}^\dagger c_{i+r+s} c_i.$$

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Now choose $V(x) \propto \nabla_x^2 \delta(x)$ as the "$\frac{1}{3}$-Laughlin-Potential":

$$U_{r,s} = (r^2 - s^2) \; e^{-2\pi^2 \frac{(r^2+s^2)}{L_x^2}} .$$

Thin Torus Limit

Now assume that $L_x << \ell_B = 1$. This is the Thin Torus Limit!

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Most relevant are the $U_{1,0}$- and $U_{2,0}$-terms:

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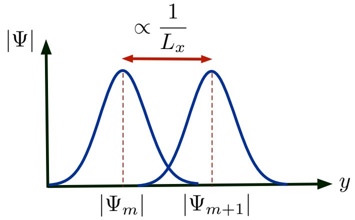
Physically this limit means that the single-particle wavefunctions $\Psi_m(x, y)$ and $\Psi_{m+1}(x, y)$ have hardly any overlap:



Ground States in the Thin Torus Limit

Thin Torus Hamiltonian:

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The ground state is threefold degenerate:

$$|GS_1\rangle = |100100100\cdots\rangle \equiv [100]$$
$$|GS_2\rangle = |010010010\cdots\rangle \equiv [010]$$
$$|GS_3\rangle = |001001001\cdots\rangle \equiv [001]$$

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Place two ground states next to each other to generate an excited state. The excitations are localized at the domain walls!

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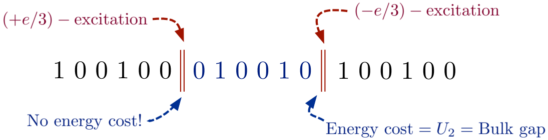

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$(+e/3)$ – excitation

$(-e/3)$ – excitation

1 0 0 1 0 0 ‖ 0 1 0 0 1 0 ‖ 1 0 0 1 0 0

No energy cost!

Energy cost $= U_2 =$ Bulk gap

What is the charge of the excitation?

$6\ e^-$ → 1 0 0 1 0 0 1 0 0 1 0 0 1 0 0 1 0 0

$7\ e^-$ → 1 0 0 <u>0 1</u> 1 0 0 <u>0 1</u> 1 0 0 <u>0 1</u> 1 0 0

→ $(-\frac{e}{3})$ – charge at the domain wall!

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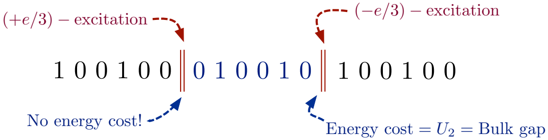

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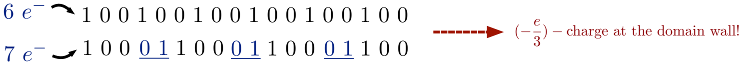

$$|GS_i\rangle \rightarrow \left|GS_{(i+k) \bmod 3}\right\rangle \text{ gives charge } q = k\frac{e}{3}.$$

Bilayer System

Consider the following $\nu = \frac{1}{3}$-bilayer system with interlayer-tunneling:

$\mathcal{H} = \mathcal{H}_{\uparrow} + \mathcal{H}_{\downarrow} + \mathcal{H}_t$
$\mathcal{H}_\sigma = \sum_i U_{1,0}\ n_{i,\sigma} n_{i+1,\sigma} + U_{2,0}\ n_{i,\sigma} n_{i+2,\sigma}$ with $\sigma \in \{\uparrow, \downarrow\}$
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Also note the global $\mathbb{Z}_2$-layer exchange symmetry ($c_{i\uparrow} \leftrightarrow c_{i\downarrow}$)!

Weak tunneling limit ($t \ll U_2 \ll U_1$)

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$$c_{i,+} := \frac{c_{i\uparrow} + c_{i\downarrow}}{\sqrt{2}} \text{ at } E = -t$$
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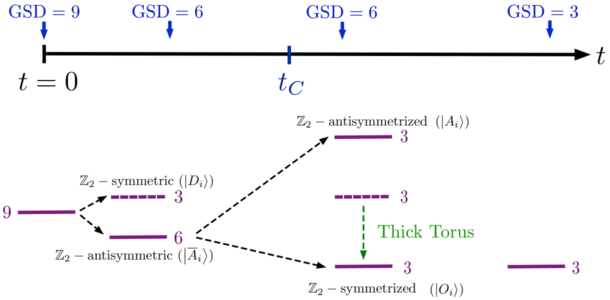
This Hamiltonian describes the particle-hole conjugate of the $\nu = \frac{1}{3}$-state, which is the $\nu = \frac{2}{3}$-state!

Intermediate tunneling regime ($U_2 \ll t \ll U_1$)



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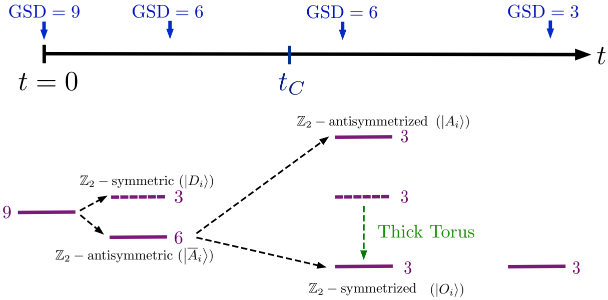

Note: Even for $t < t_c$ the states

$$|D_1\rangle = \begin{bmatrix} 100 \\ 100 \end{bmatrix} \quad |D_2\rangle = \begin{bmatrix} 010 \\ 010 \end{bmatrix} \quad |D_3\rangle = \begin{bmatrix} 001 \\ 001 \end{bmatrix}$$

are not effected by tunneling and split from the states $\{|\overline{A}_i\rangle\}$.

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$$\mathcal{H}_{high} = \left\{ \begin{bmatrix} 110 \\ 000 \end{bmatrix} , \begin{bmatrix} 000 \\ 110 \end{bmatrix} \right\}$$

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Integrating out the high-energy deg. of freedom yields to 2^{nd} order:

$$H_{\text{eff}} = -J \sum_i S_i^x - \frac{U_{2,0}}{2} \sum_i S_i^z S_{i+1}^z + 1, \quad J = 4{t_0^{\perp}}^2 / U_{1,0}.$$

Intermediate tunneling regime ($U_2 \ll t \ll U_1$)

The phase transition at $t = t_C$ is of Ising type!

2nd Order Perturbation theory: U_1 is the dominant energy scale.

$$\mathcal{H}_{low} = \left\{ |\uparrow\rangle \equiv \begin{bmatrix} 100 \\ 010 \end{bmatrix}, \ |\downarrow\rangle \equiv \begin{bmatrix} 010 \\ 100 \end{bmatrix} \right\}$$

$$\mathcal{H}_{high} = \left\{ \begin{bmatrix} 110 \\ 000 \end{bmatrix}, \begin{bmatrix} 000 \\ 110 \end{bmatrix} \right\}$$

Only these states can be "connected" by vertical tunneling

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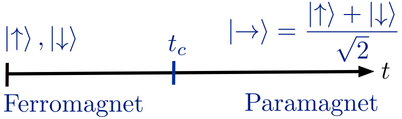

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Ground States in the Thick Torus limit:

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Ground States in the Thick Torus limit:

$\mathbb{Z}_2$-symmetric states:

$$|D_1\rangle = \begin{bmatrix} 100 \\ 100 \end{bmatrix} \equiv [200] \quad |D_2\rangle = \begin{bmatrix} 010 \\ 010 \end{bmatrix} \equiv [020] \quad |D_3\rangle = \begin{bmatrix} 001 \\ 001 \end{bmatrix} \equiv [002]$$

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$\mathbb{Z}_2$-symmetrized states:

$$|O_1\rangle = \begin{bmatrix} 100 \\ 010 \end{bmatrix} + \begin{bmatrix} 010 \\ 100 \end{bmatrix} \equiv [110]$$

$$|O_2\rangle = \begin{bmatrix} 100 \\ 001 \end{bmatrix} + \begin{bmatrix} 001 \\ 100 \end{bmatrix} \equiv [101]$$

$$|O_3\rangle = \begin{bmatrix} 010 \\ 001 \end{bmatrix} + \begin{bmatrix} 001 \\ 010 \end{bmatrix} \equiv [011]$$

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Neutral excited states:

Intermediate tunneling regime ($U_2 \ll t \ll U_1$)

Neutral excited states:

To create an excitation with a (+e/3) charge in in one layer and a (-e/3) in the other layer we need to place the following patterns next to each other:

$$\cdots\ 2\ 0\ 0\ 2\ 0\ 0\ 2\ 0\ 0\ \cdots \longrightarrow \cdots\ 0\ 1\ 1\ 0\ 1\ 1\ 0\ 1\ 1\ \cdots$$

$$\cdots\ 0\ 1\ 1\ 0\ 1\ 1\ 0\ 1\ 1\ \cdots \longrightarrow \cdots\ 2\ 0\ 0\ 2\ 0\ 0\ 2\ 0\ 0\ \cdots$$

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Intermediate tunneling regime ($U_2 \ll t \ll U_1$)

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Neutral excited states:

The patterns that create neutral excitations are encoded in an Adjacency Matrix:

$$A_{\tau=(e/3,-e/3)} = \begin{array}{c} \begin{matrix} [200] & [020] & [002] & [110] & [101] & [011] \end{matrix} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array} \begin{matrix} [200] \\ [020] \\ [002] \\ [110] \\ [101] \\ [011] \end{matrix}$$

Fact: $i \times j = \sum_k (A_i)_{jk}\ k \quad \Rightarrow \quad \tau \times \tau = 1 + \tau$

Intermediate tunneling regime ($U_2 \ll t \ll U_1$)

Overview of all excited states:

Intermediate tunneling regime ($U_2 \ll t \ll U_1$)

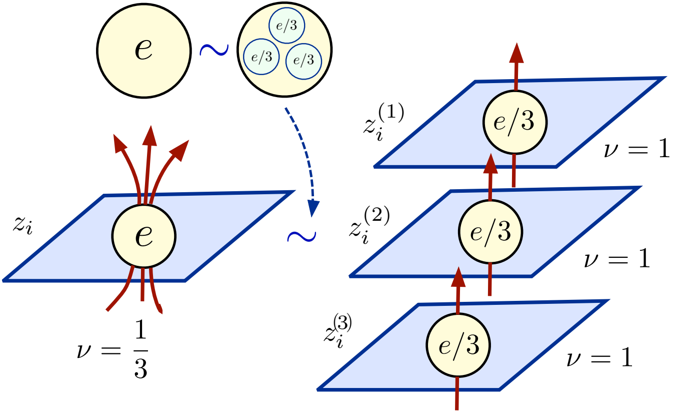
Overview of all excited states:

	Label	Charge (mod e)	Topological Spin	Quantum Dim.
1	V_0	0	0	1
2	V_1	$2e/3$	$1/3$	1
3	V_2	$e/3$	$1/3$	1
4	τ	0	$\pm 2/5$	F
5	$V_1\tau$	$2e/3$	$1/3 \pm 2/5$	F
6	$V_2\tau$	$e/3$	$1/3 \pm 2/5$	F

Outline

Parton Construction

Idea: Understanding the FHQE from the IQHE



Parton Construction

Uses:

- The low-energy effective theory is a Chern Simons Theory

Parton Construction

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- The low-energy effective theory is a Chern Simons Theory
- Finding Wavefunctions

$$\Psi_{\nu=1}\left(\left\{z_i^{(1)}\right\}\right)\Psi_{\nu=1}\left(\left\{z_i^{(2)}\right\}\right)\Psi_{\nu=1}\left(\left\{z_i^{(3)}\right\}\right)$$

$$\sim\prod_{i<j}\left(z_i^{(1)}-z_j^{(1)}\right)\prod_{i<j}\left(z_i^{(2)}-z_j^{(2)}\right)\prod_{i<j}\left(z_i^{(3)}-z_j^{(3)}\right)$$

$$z_i\equiv z_i^{(1)}\equiv z_i^{(2)}\equiv z_i^{(3)}$$

$$\Downarrow$$

$$\Psi_{\nu=\frac{1}{3}}\left(\{z_i\}\right)\sim\prod_{i<j}\left(z_i-z_j\right)^3$$

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Write the electron operator as $c = f_1 f_2 f_3$ for (e/3)-partons f_i.

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$$\mathcal{L}' = i \; f_a^\dagger \partial_t f_a + \frac{1}{2m} \; f_a^\dagger \left(\partial_i - iQ_p A_i\right)^2 f_a$$

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To dismiss the "unphysical states" we implement a "gauge constraint".

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Step 1: Consider

$$f_a \mapsto W_{ab} f_b \text{ with } W \in U(3)$$

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We want to construct a parton Lagrangian that is invariant under SU(3) gauge transformations.

Parton Construction

Step 2: Introduce some gauge field

$$a_\mu(x) \in \text{Lie}(SU(3)) = \text{Traceless Hermitian Matrices}$$

and define a new theory

$$\mathcal{L} = i\ f_a^\dagger \left(\delta_{ab}\partial_t - i(a_0)_{ab}\right) f_b + \frac{1}{2m}\ f_a^\dagger \left(\partial_i - iQ_p A_i - ia_i\right)^2_{ab} f_b$$

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Integrating out the partons yields an effectively Lagrangian

$$\mathcal{L}_{eff} = \frac{\epsilon_{\mu\nu\lambda}}{4\pi}\text{tr}\left[a_\mu\partial_\nu a_\lambda + \frac{2}{3}a_\mu a_\nu a_\lambda\right] + \vec{j}\cdot\vec{a} + \text{Terms that couple to } \vec{A}$$

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So for a $\nu = 1/3$ bilayer system we get a $SU(3)_1 \times SU(3)_1$ CS theory:

$$\mathcal{L} = \frac{\epsilon_{\mu\nu\lambda}}{4\pi}\sum_\sigma \mathrm{tr}\left[A^\sigma_\mu \partial_\nu A^\sigma_\lambda + \frac{2}{3} A^\sigma_\mu A^\sigma_\nu A^\sigma_\lambda\right] + j_\sigma \cdot A^\sigma$$

Higgs transition

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$$= -t \ \frac{1}{3!} \ \frac{1}{3!} \epsilon^{abc} \epsilon^{a'b'c'} \chi_{aa'} \chi_{bb'} \chi_{cc'} \quad \text{with} \quad \chi_{ij} := f_{i\uparrow}^\dagger f_{j\downarrow}$$

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The low energy effective theory after the transition is a $SU(3)_2$-CS theory.

Higgs transition

Consequences:

Higgs transition

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- There is a continuous quantum phase transition between the abelian 1/3-bilayer state and a non-abelian state described by a $SU(3)_2$-CS-Theory.

Higgs transition

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- There is a continuous quantum phase transition between the abelian 1/3-bilayer state and a non-abelian state described by a $SU(3)_2$-CS-Theory.
- The fusion rules on the thin torus coincide with those of the quasiparticles in a $SU(3)_2$-CS-Theory.

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