Synthetic Gauge Fields in Synthetic Dimensions

PRL **112**, 043001 (2014)

A. Celi, P. Massignan, J. Ruseckas, N. Goldman, I.B. Spielman, G. Juzeliūnas, and M. Lewenstein

Abstract: We describe a simple technique for generating a cold-atom lattice pierced by a uniform magnetic field. Our method is to extend a one-dimensional optical lattice into the "dimension" provided by the internal atomic degrees of freedom, yielding a synthetic 2D lattice. Suitable laser-coupling between these internal states leads to a uniform magnetic flux within the 2D lattice. We show that this setup reproduces the main features of magnetic lattice systems, such as the fractal Hofstadter butterfly spectrum and the chiral edge states of the associated Chern insulating phases.

Journal Club

23/05/2014

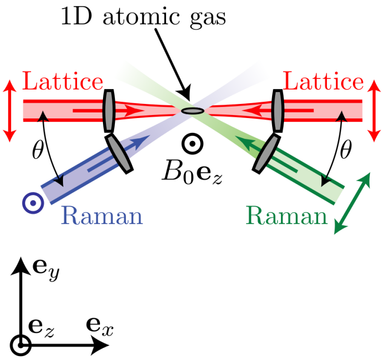
Axel U. J. Lode

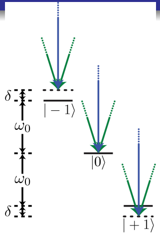
Outline

Setting

- Cold atoms are much easier to control than solid state systems
- "Quantum simulation" of otherwise inaccessible parameter regimes in solid state systems
- Scattering of edge states on different kinds of impurities
- Interaction effects
- Microscopic dynamics of chiral edge states become accessible

- 1D system of ultracold neutral atoms
- non-interacting
- magnetic field for Zeeman splits, e.g., $F = 1, m = -1, 0, +1$
- different m can be used as "synthetic dimension"
- Raman laser beams to couple the magnetic sublevels → hopping in synthetic dimension
- recoil in $\mathbf{e}_x$ direction $k_R = 2\pi\cos(\theta)/\lambda_R$





Magnetic field describing Raman lasers:

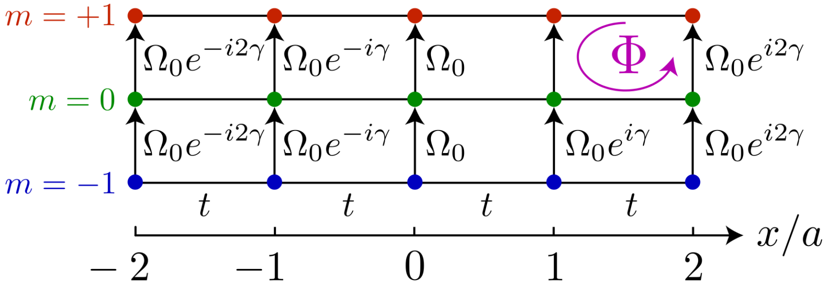
$$\mathbf{\Omega}_T = \delta \mathbf{e}_z + \Omega_R \left[\cos\left(2k_R x\right) \mathbf{e}_x - \sin\left(2k_R x\right) \mathbf{e}_y\right]$$

Atom-laser Hamiltonian:

$$H_{\mathrm{al}} = \mathbf{\Omega}_T \cdot \mathbf{F} = \delta F_z + (F_+ e^{ik_R x} + F_- e^{-ik_R x})\Omega_R/2$$

Note: $F_\pm = F_x \pm iF_y$, where $F_+ \left|m\right\rangle = g_{F,m} \left|m+1\right\rangle$
$g_{F,m} = \sqrt{F\left(F+1\right) - m\left(m+1\right)}$;

Raman beams couple states with $m = -F, \ldots, F$ sequentially. Phase depending on x is collected!



Resulting Hamiltonian:

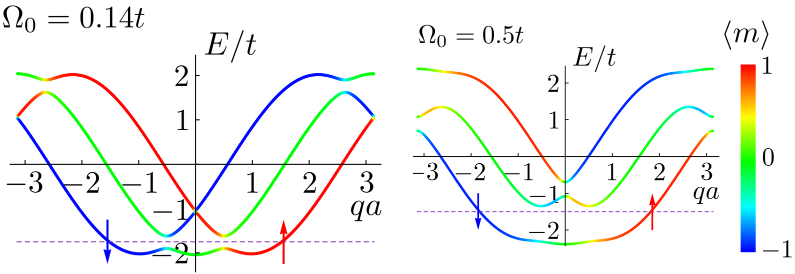
$$H = \sum_{n,m} \left(-t a^\dagger_{n+1,m} + \Omega_{m-1} e^{-i\gamma n} a^\dagger_{n,m-1} \right) a_{n,m} + \mathrm{H.c.}$$

Flux $\Phi \equiv \frac{\gamma}{2\pi} = \frac{k_R a}{\pi}$; Coupling: $\Omega_m = \frac{\Omega_R g_{F,m}}{2}$

- Shift phase $\exp(i2k_R x)$ to x direction by gauge-transforming $a_{n,m}$ and $a^\dagger_{n,m}$
- Split Hamiltonian momentum components by Fourier transformation in x: $H = \sum_q H_q$.

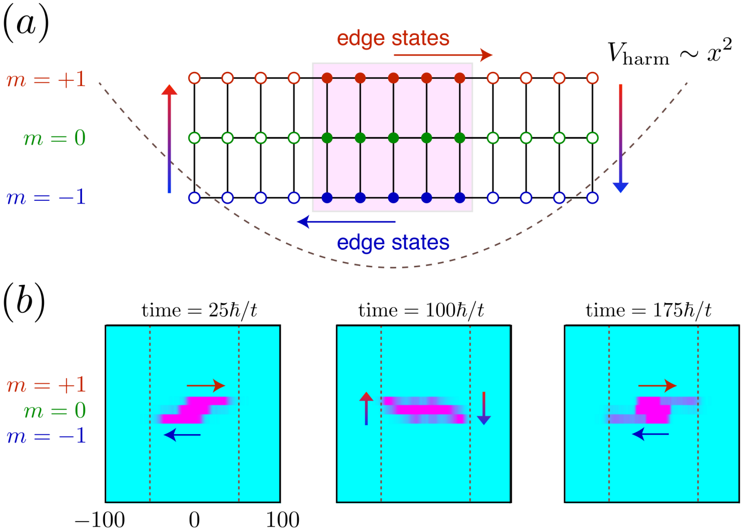
$$
\begin{aligned}
b^\dagger_{q,\,m} &= N^{-1/2} \sum_{n=1}^{N} a^\dagger_{n,\,m} e^{i(q+\gamma m)n} \\
H_q &= \sum_{m=-F}^{F} \varepsilon_{q+\gamma m} b^\dagger_{q,\,m} b_{q,\,m} + \left(\Omega_m b^\dagger_{q,\,m+1} b_{q,\,m} + \mathrm{H.c.} \right) \\
\varepsilon_k &= -2t\cos(k); \qquad q \equiv 2\pi l/N; \qquad l \in \{1, \dots, N\}
\end{aligned}
$$

F=1 spectra

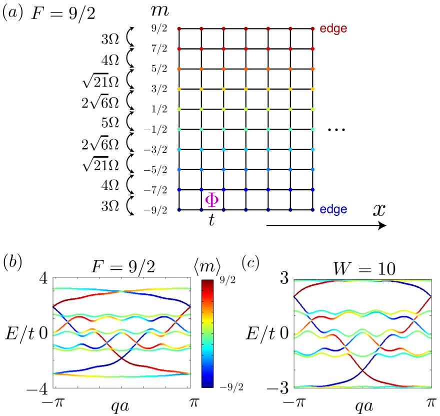


Flux: $\Phi = \frac{1}{2\pi}$

F=1 – Edge state dynamics



F=9/2 spectra

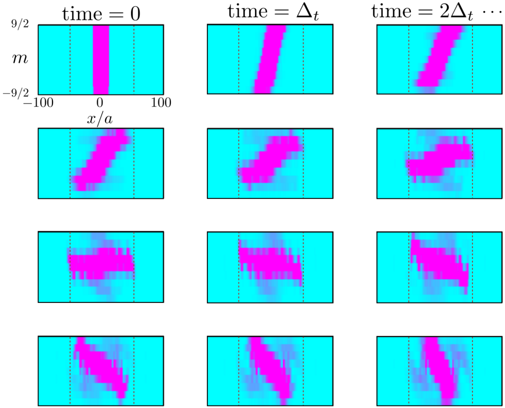


Flux: $\Phi \equiv \frac{1}{5}$

Non-uniform hopping in synthetic dimension!

$$t_{m\to m+1} = \Omega g_{F,m} = \Omega\sqrt{F(F+1) - m(m+1)}$$

F=9/2 Edge state dynamics



Flux: $\Phi \equiv \frac{1}{5}$

Non-uniform hopping in synthetic dimension!

$$t_{m\to m+1} = \Omega g_{F,m} = \Omega\sqrt{F(F+1) - m(m+1)}$$

$$\Delta_t = 37.5\frac{\hbar}{J}$$

- Dynamics of 2D lattices pierced by magnetic fields can be tackled using a 1D lattice of ultracold atoms with accessible internal states coupled by laser beams
- Quantum simulation of complicated solid state systems in a well-controlled and clean environment
- In situ probing of chiral edge modes and their dynamics
- Probing the effect of diffent kinds of defects or impurities amenable
- Probing the effect of Interactions feasible