Equilibrium currents in chiral systems with non-zero Chern number

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In systems with significant spin-orbit splitting in the band structure
spin is tied to the momentum.

Acceleration of the particle leads to the spin precession, which in turn affects the current

Anomalous Hall effect described in terms of the geometric phase

Anomalous velocity $e\mathbf{E} \times \mathbf{\Omega}$

This can lead to the existence of the equilibrium boundary current.

Similar currents circulate inside the system around defects or impurities.

Goals of the paper:

1. Calculate equilibrium boundary currents in materials with a nontrivial geometric phase

2. Separate the orbital magnetization (topological contribution)
   from the paramagnetic magnetization through the boundary current.

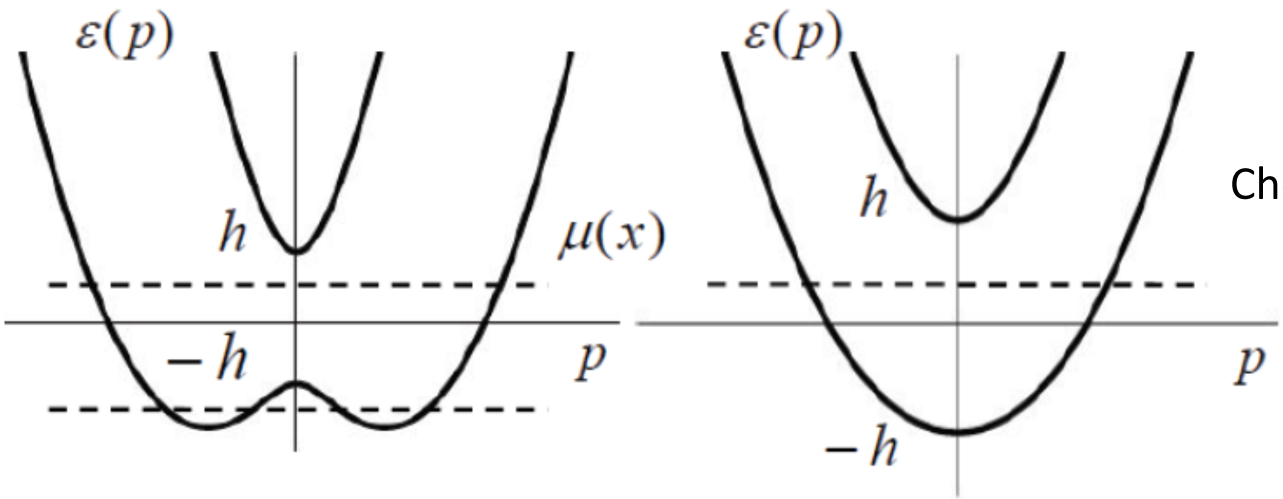
Two dimensional electron gas in a potential $U(\boldsymbol{r})$

$$H = -\frac{\hbar^2}{2m}\nabla^2 - i\hbar\lambda\,\hat{\boldsymbol{\eta}}\cdot\nabla - h\hat{\sigma}_z + U(\mathbf{r})$$

$h = \frac{eg}{2m_0 c}H_z$ $\hat{\boldsymbol{\eta}} = \mathbf{z}\times\hat{\boldsymbol{\sigma}}$ z is perpendicular to the plane

Neglect the effect of the magnetic field on the orbital motion of electrons

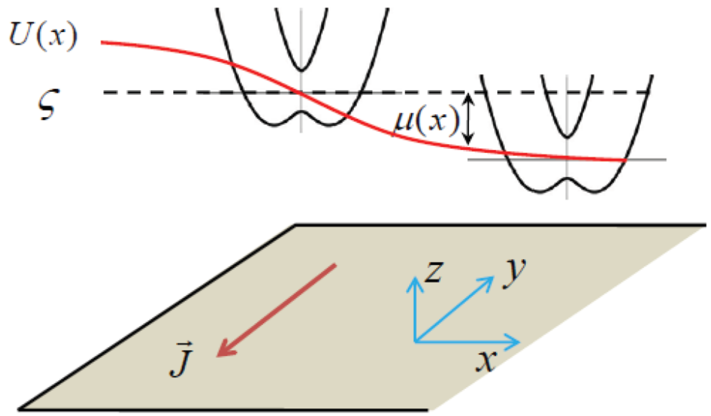
1. Large g-factors
2. System of neutral atoms, where $(e/c)\mathbf{p}\cdot\mathbf{A}$ term is absent



$h < m\lambda^2$ $h > m\lambda^2$

Chemical potential

$\mu(x)$



The smooth potential $U(\mathbf{R})$ determines the position of the bottom of the of the band

$$e\mathbf{E}_{\text{edge}} = -\mathbf{x}(\partial U/\partial x)$$

$$\mu(\mathbf{R}) = \zeta - U(\mathbf{R})$$

Density matrix $$f_{\alpha\beta}(\mathbf{r},\mathbf{r}';t) = \langle \psi^{\dagger}_{\beta}(\mathbf{r}',t)\psi_{\alpha}(\mathbf{r},t)\rangle$$

In the Wigner representation: $$\hat{f}_{\mathbf{p}}(\mathbf{R},t) = \int d\boldsymbol{\rho}\, e^{-i\mathbf{p}\boldsymbol{\rho}}\hat{f}\left(\mathbf{R}+\frac{\boldsymbol{\rho}}{2},\mathbf{R}-\frac{\boldsymbol{\rho}}{2};t\right)$$

1. Steady state
2. Boundary potential is smooth on the scale of the electron wave length

$$p_F L_{edge} \gg 1$$

$$\frac{1}{2}\left\{\frac{\mathbf{p}}{m}+\lambda\hat{\boldsymbol{\eta}},\nabla\hat{f}_{\mathbf{p}}\right\}+i\lambda p[\hat{\eta}_{\mathbf{p}},\hat{f}_{\mathbf{p}}]-ih[\hat{\sigma}_z,\hat{f}_{\mathbf{p}}]-\boldsymbol{\nabla}U\cdot\frac{\partial\hat{f}_{\mathbf{p}}}{\partial\mathbf{p}}=0$$

$$\hat{\eta}_{\mathbf{p}} = \hat{\boldsymbol{\eta}}\cdot\mathbf{n}_p$$

Projection of the spin operator onto the direction of the momentum

Solution: In the zero order in the gradient of the potential

$$\hat{f}^{(0)}_{\mathbf{p}} = \frac{1}{2}(f_+ + f_-) + \frac{1}{2}(f_+ - f_-)\frac{\lambda p\ \hat{\eta}_{\mathbf{p}} - h\hat{\sigma}_z}{\sqrt{\lambda^2 p^2 + h^2}}$$

The two spin-split subbands:

$$f_\pm = \frac{1}{\exp\left[\frac{\varepsilon_\pm(p)+U(\mathbf{R})-\zeta}{T}\right] + 1} \qquad \varepsilon_\pm(p) = \frac{p^2}{2m} \pm \sqrt{\lambda^2 p^2 + h^2}$$

Position dependent chemical potential

$$\mu(\mathbf{R}) = \zeta - U(\mathbf{R})$$

Electrochemical potential

Solution: First order

Correction linear in gradient of the potential $$\hat{f}_{\mathbf{p}} = \hat{f}_{\mathbf{p}}^{(0)} + \hat{f}_{\mathbf{p}}^{(1)}$$

$$[\lambda p \hat{\eta}_{\mathbf{p}} - h\hat{\sigma}_z, \hat{f}_{\mathbf{p}}^{(1)}] = i\hat{\mathcal{K}}_{\mathbf{p}}$$

where

$$\hat{\mathcal{K}}_{\mathbf{p}} = \frac{1}{2}\left\{\frac{\mathbf{p}}{m} + \lambda\hat{\boldsymbol{\eta}}, \boldsymbol{\nabla}\hat{f}_{\mathbf{p}}^{(0)}\right\} - \boldsymbol{\nabla}U \cdot \frac{\partial \hat{f}_{\mathbf{p}}^{(0)}}{\partial \mathbf{p}}$$

Correction to the density matrix:

$$\hat{f}_{\mathbf{p}}^{(1)} = -\frac{\lambda \boldsymbol{\nabla}U \cdot [\lambda p(\mathbf{n}_p \times \boldsymbol{\sigma}) + h\hat{\boldsymbol{\sigma}}]}{4(\lambda^2 p^2 + h^2)^{3/2}}\Big[f_+ - f_- \\ -(f'_+ + f'_-)\sqrt{\lambda^2 p^2 + h^2}\Big],$$

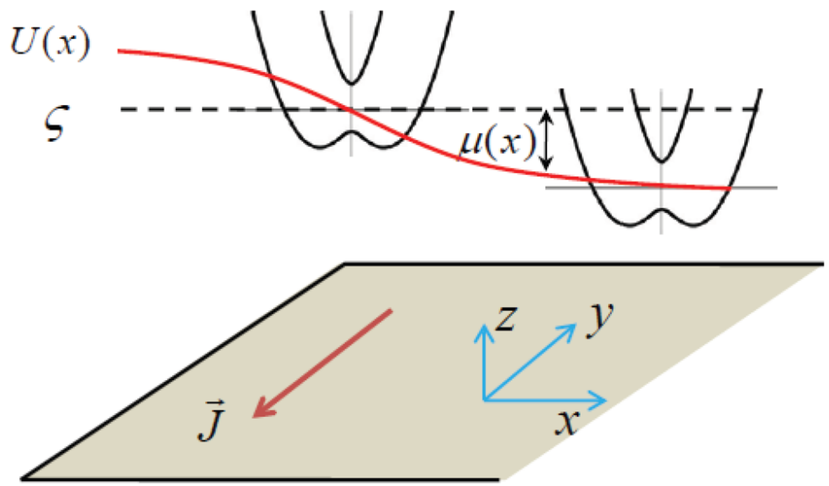
Derivative

Electric current propagating along the edge of the system:

$$\mathbf{j} = e\mathrm{Tr}\sum_{\mathbf{p}}\left(\frac{\mathbf{p}}{m} + \lambda\boldsymbol{\eta}\right)\hat{f}_{\mathbf{p}} - \frac{eg}{4m_0}\hat{\mathbf{z}}\times\nabla\mathrm{Tr}\sum_{\mathbf{p}}\hat{\sigma}_z\hat{f}_{\mathbf{p}}$$

Each contribution vanishes when electrons are present in both upper and lower subbands $h < \mu(\mathbf{R})$

Nonzero when only the lowest subband is populated $\mu(\mathbf{R}) < h$



$$e\mathbf{E}_{\mathrm{edge}} = -\mathbf{x}(\partial U/\partial x)$$

First contribution:

$$j_y^{(1)}(x) = -\frac{e}{4\pi}\lambda^2 h \frac{\partial U}{\partial x} \int\limits_0^\infty \frac{pdp}{(\lambda^2 p^2 + h^2)^{3/2}}$$
$$\times \left(f_+ - f_- - (f'_+ + f'_-)\sqrt{\lambda^2 p^2 + h^2} \right).$$

Berry curvature: $$\mathbf{\Omega}_\beta(\mathbf{p}) = \mathbf{z}\frac{\beta\lambda^2 h}{(\lambda^2 p^2 + h^2)^{3/2}}$$

$$j_y^{(1)}(x) = \frac{e}{4\pi}\frac{\partial U}{\partial x}\begin{cases} 0, & h < \mu(x), \\ 1 - h/\mathcal{H}(\mu), & -h < \mu(x) < h, \\ -2h/\mathcal{H}(\mu), & \mu(x) < -h, \end{cases}$$

$$\mathcal{H}(\mu) = \sqrt{h^2 + m^2\lambda^4 + 2m\lambda^2\mu(x)}$$

Second contribution:

$$\mathbf{j}^{(2)}(\mathbf{r}) = c\nabla\times\mathbf{M}_{\rm para}$$

This term is proportional to the extra g-factor:

$$M_{\rm para} = \frac{g\mu_B}{2}\mathrm{Tr}\sum_{\mathbf{p}}\hat{\sigma}_z\hat{f}^{(0)}_{\mathbf{p}} =$$

$$= -\frac{ge}{2m_0c}\int\frac{d^2p}{(2\pi)^2}\frac{h(f_+ - f_-)}{\sqrt{\lambda^2p^2+h^2}}$$

$$j_y^{(2)}(x) = \frac{eg}{8\pi}\frac{m}{m_0}\frac{\partial U}{\partial x}\begin{cases} 0, & h<\mu(x), \\ h/\mathcal{H}(\mu), & -h<\mu(x)<h, \\ 2h/\mathcal{H}(\mu), & \mu(x)<-h. \end{cases}$$

$$j_y^{(1)}(x) = \frac{e}{4\pi}\frac{\partial U}{\partial x}\begin{cases} 0, & h<\mu(x), \\ 1-h/\mathcal{H}(\mu), & -h<\mu(x)<h, \\ -2h/\mathcal{H}(\mu), & \mu(x)<-h, \end{cases}$$

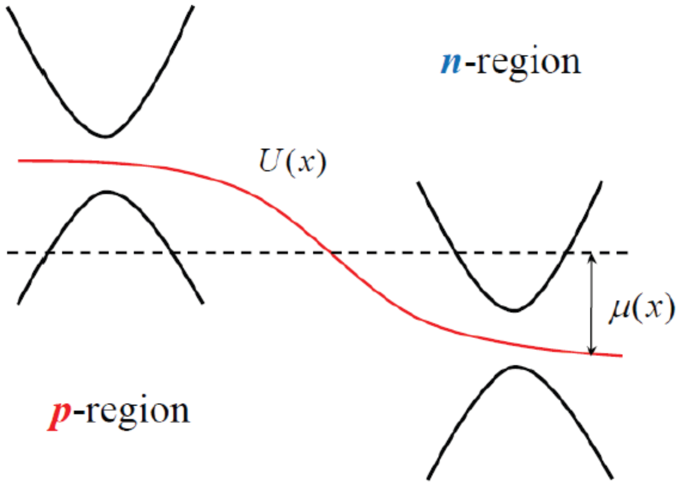
The net current

$$J_y = \int_{-\infty}^{\infty} j_y(x)dx$$

$$J_y^{(1)} = \frac{e}{4\pi}\begin{cases} 0, & h<\zeta, \\ \frac{h[\mathcal{H}(\zeta)-h]}{m\lambda^2}-\zeta, & -h<\zeta<h, \\ 2h\frac{\mathcal{H}(\zeta)}{m\lambda^2}, & \zeta<-h, \end{cases}$$

$$J_y^{(2)} = -\frac{egh}{8\pi}\frac{m}{m_0}\begin{cases} 0, & h<\zeta, \\ (\mathcal{H}(\zeta)+m\lambda^2-h)/(m\lambda^2), & -h<\zeta<h, \\ 2\mathcal{H}(\zeta)/(m\lambda^2), & \zeta<-h. \end{cases}$$

Surface of the topological insulator:



$$j_y^{(1)}(x) = -\frac{e}{4\pi}\frac{\partial U}{\partial x}\Theta(h - |\mu(x)|)$$

$$j_y^{(2)}(x) = \frac{egh}{8\pi m_0 \lambda}\frac{\partial U}{\partial x}\begin{cases} 1, & h < \mu(x), \\ 0, & -h < \mu(x) < h, \\ -1, & \mu(x) < -h. \end{cases}$$

Resume:

Our findings point to a novel way to experimentally observe topological contribution (orbital magnetization) and to separate it from the standard paramagnetic Magnetization, via the difference in the dependence of the currents on parameters.