Local Adiabatic Mixing of Kramers Pairs of Majorana Bound States

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We consider Kramers pairs of Majorana bound states under adiabatic time evolution. This is important for the prospects of using such bound states as parity qubits. We show that local adiabatic perturbations can cause a rotation in the space spanned by the Kramers pair. Hence the quantum information is unprotected against local perturbations, in contrast to the case of single localized Majorana bound states in systems with broken time reversal symmetry. We give an analytical and a numerical example for such a rotation, and specify sufficient conditions under which a rotation is avoided. We give a general scheme for determining when these conditions are satisfied, and exemplify it with a general model of a quasi 1D time reversal symmetric topological superconductor.

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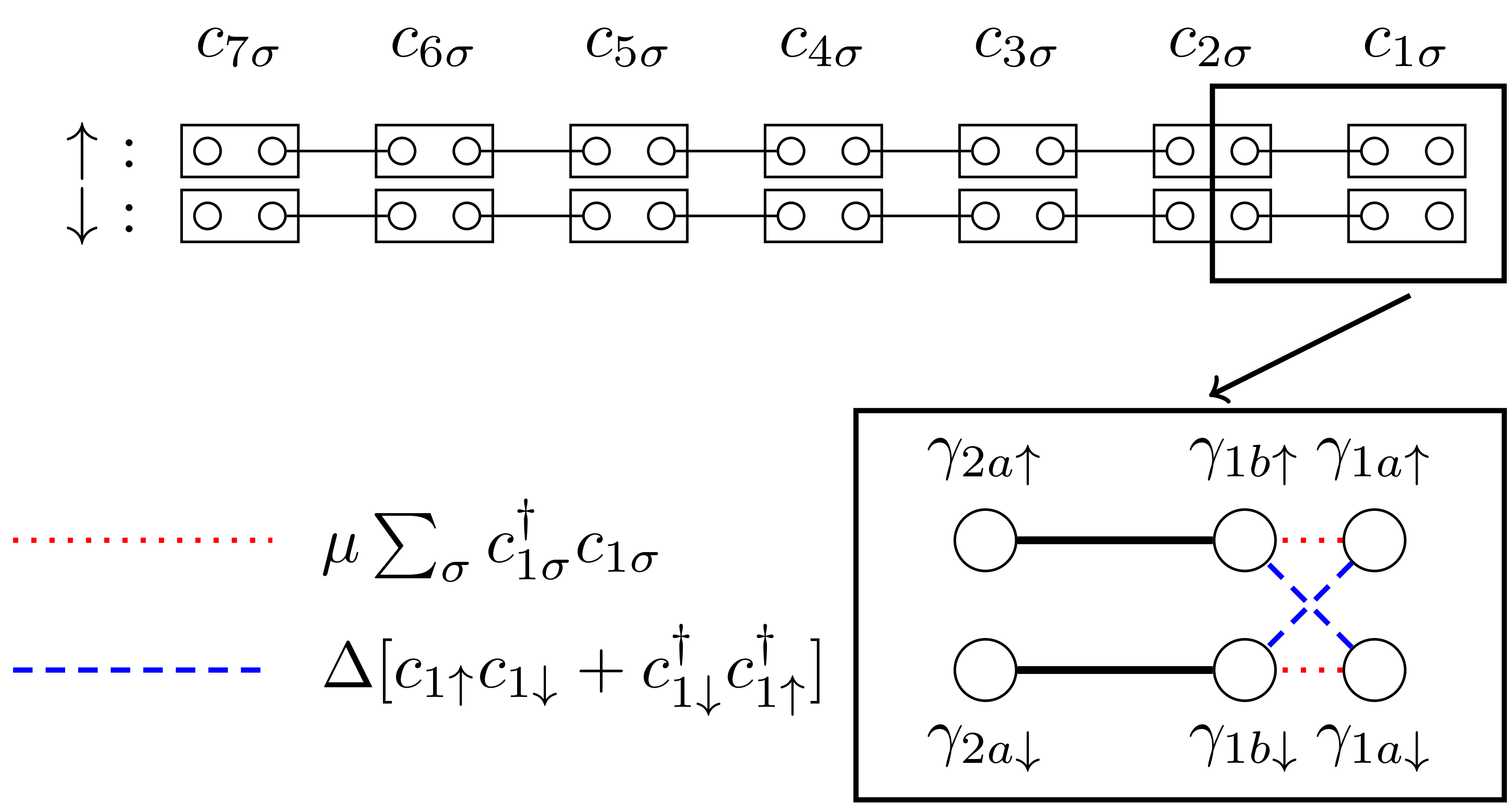
Tobias Meng

The setup

- Time reversal symmetric Kitaev double chain (DIII): Kramers pairs of Majoranas

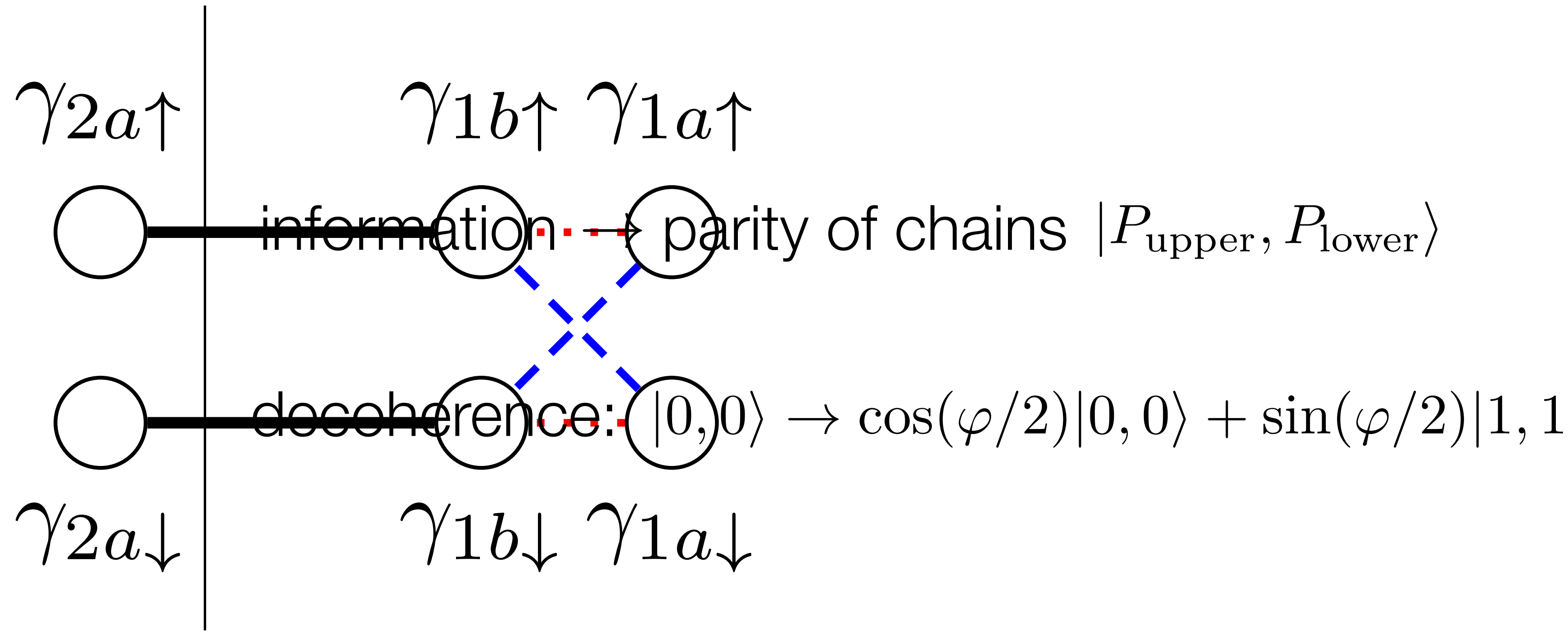


- $\mathcal{T} \gamma_{ij\uparrow} \mathcal{T}^{-1} = \gamma_{ij\downarrow} \quad , \quad \mathcal{T}^2 = -1$

- also: particle-hole symmetry

The question

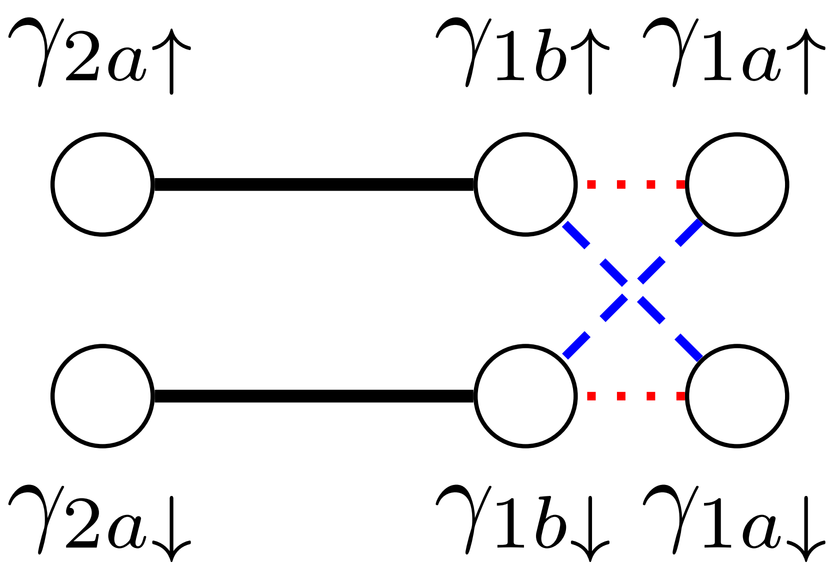
- Single Kitaev chain: Majoranas are far apart = information protected



- Now: Kramers pairs of Majoranas at same site: fate of protection?

$\Rightarrow$ can information get lost due to presence of Kramers partner?

Simple analytical model



$$H_0 = \frac{E_\mathrm{g}}{2} \sum_\sigma i\gamma_{2a\sigma}\gamma_{1b\sigma} = E_\mathrm{g} \sum_\sigma d_\sigma^\dagger d_\sigma + \mathrm{const.}$$

$$H_\mu = \frac{\mu}{2} \sum_\sigma \left(i\gamma_{1a\sigma}\gamma_{1b\sigma} + 1 \right),$$

$$H_\Delta = \frac{\Delta}{2} \left(i\gamma_{1a\uparrow}\gamma_{1b\downarrow} + i\gamma_{1b\uparrow}\gamma_{1a\downarrow} \right).$$

- Kramers pair of zero energy Majorana bound states in terms of $\gamma_{ij\sigma}$:

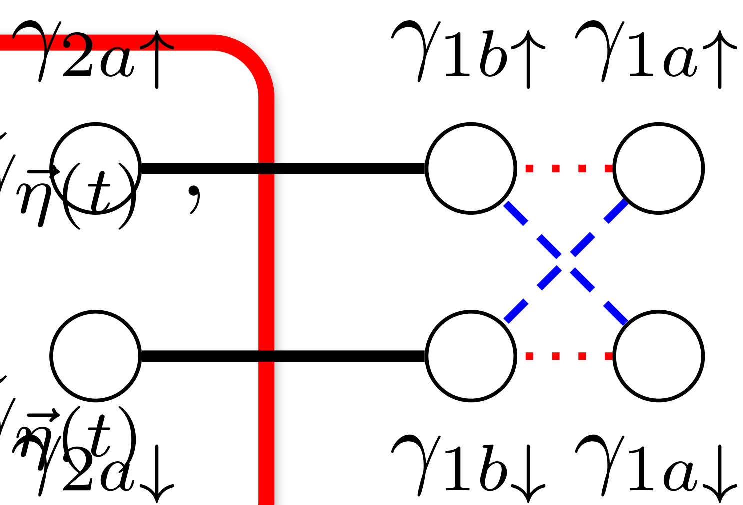
$$\boxed{\begin{aligned} \gamma(\theta,\alpha) &= \cos\theta\gamma_{1a\uparrow} + \sin\theta\left(\cos\alpha\gamma_{1b\uparrow} + \sin\alpha\gamma_{1b\downarrow}\right), \\ \tilde{\gamma}(\theta,\alpha) &= \cos\theta\gamma_{1a\downarrow} - \sin\theta\left(\cos\alpha\gamma_{1b\downarrow} - \sin\alpha\gamma_{1b\uparrow}\right). \end{aligned}}$$

$$\mu = B\,\cos(\alpha) \quad , \quad \Delta = B\,\sin(\alpha) \quad , \quad \tan(\theta) = B/E_g$$

Adiabatic time reversal symmetric evolution

- More generally: look at Hamiltonian $H(\vec{\eta})$ respecting TRS, $[H(\vec{\eta}), \mathcal{T}] = 0$
- For each value of $\vec{\eta}$, there are two Majoranas: $\gamma_{\vec{\eta}}$ and $\tilde{\gamma}_{\vec{\eta}}$
- Adiabatic modification of $\vec{\eta}$ in a loop: $\vec{\eta}(t_{\text{initial}}) = \vec{\eta}(t_{\text{final}})$
- For Majoranas, this implies

$$\boxed{\begin{aligned} \gamma_{\vec{\eta}} &\to \Gamma(t) = a(t)\,\gamma_{\vec{\eta}(t)} + b(t)\,\tilde{\gamma}_{\vec{\eta}(t)}\,, \\ \tilde{\gamma}_{\vec{\eta}} &\to \tilde{\Gamma}(t) = \tilde{a}(t)\,\gamma_{\vec{\eta}(t)} + \tilde{b}(t)\,\tilde{\gamma}_{\vec{\eta}(t)} \end{aligned}}$$



$$a(t)^2 + b(t)^2 = 1 = \tilde{a}(t)^2 + \tilde{b}(t)^2 \quad , \quad a(t),\, b(t),\, \tilde{a}(t),\, \tilde{b}(t) \in \mathbb{R}$$

$$\Longrightarrow \vec{\eta} \equiv \frac{\mu}{E_g}\,,\ \frac{\Delta}{E_g}$$

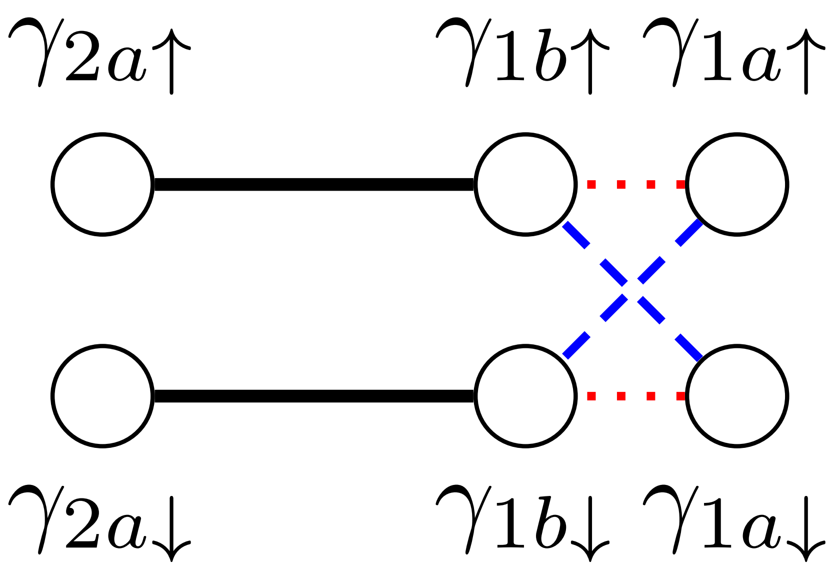
Adiabatic evolution

- Define: $a(t) = \cos(\varphi(t) - \varphi_0)\ ,\quad b(t) = \sin(\varphi(t) - \varphi_0)$
- From there: $$\frac{d}{dt}\tilde{\Gamma}(t) = i[\tilde{\Gamma}(t), H(\vec{\eta}(t)] = 0$$ $$\Rightarrow 0 = \quad \{\Gamma(t), \frac{d}{dt}\tilde{\Gamma}(t)\} = -2\dot{\varphi} + \{\gamma_{\vec{\eta}(t)}, \dot{\tilde{\gamma}}_{\eta(\vec{t})}\}$$
- Rewrite as integral over path $\mathcal{W}$ of $\vec{\eta}(t)$ in parameter space

$$\varphi = \oint_{\mathcal{W}} \vec{A}\, d\vec{\eta}\quad ,\quad \vec{A} = \frac{1}{2}\{\gamma_{\vec{\eta}}, \nabla_{\vec{\eta}}\tilde{\gamma}_{\vec{\eta}}\}$$

(= Berry phase & Berry connection)

Back to example



$$H_0 = \frac{E_\mathrm{g}}{2} \sum_\sigma i\gamma_{2a\sigma}\gamma_{1b\sigma} = E_\mathrm{g} \sum_\sigma d_\sigma^\dagger d_\sigma + \text{const.}$$

$$H_\mu = \frac{\mu}{2} \sum_\sigma \left(i\gamma_{1a\sigma}\gamma_{1b\sigma} + 1 \right),$$

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$$\mu = B\cos(\alpha) \quad , \quad \Delta = B\sin(\alpha) \quad , \quad \tan(\theta) = B/E_g$$

- Path $\mathcal{W}$ corresponds to $\alpha : 0 \to 2\pi$

$$\boxed{\Rightarrow \quad \varphi = \pi \sin^2\left(\arctan\left(\frac{\sqrt{\mu^2+\Delta^2}}{E_g}\right)\right)}$$

„ low frequency noise that leads to a fluctuating time dependence of the Hamiltonian would lead to a decoherence time that grows only algebraically large with E_G. "

Numerical study

As a second example for mixing we present numerical calculations of the Berry curvature [16] for a continuous 1D TRS p-wave superconductor. The Hamiltonian then reads,

$$H = \left(\frac{p^2}{2m} - \mu(x)\right)\tau_z + p(\boldsymbol{\alpha}\cdot\boldsymbol{\sigma})\tau_z + p(\boldsymbol{v}_\Delta\cdot\boldsymbol{\sigma})\tau_x + \Delta\tau_x, \tag{9}$$

with an associated operator spinor $\boldsymbol{\Psi} = (\Psi_\uparrow, \Psi_\downarrow, \Psi_\downarrow^\dagger, -\Psi_\uparrow^\dagger)^T$. The symmetries of this Hamiltonian are $\mathcal{P} = \sigma_y\tau_y K$ and $\mathcal{T} = i\sigma_y K$, making DIII the relevant symmetry class. The Hamiltonian describes a 1D system with p-wave pairing, $\boldsymbol{v}_\Delta$, s-wave pairing, Δ, and spin-orbit interaction, $\boldsymbol{\alpha}$. If $\boldsymbol{\alpha} = 0$, $\Delta = 0$ and $\boldsymbol{v}_\Delta = (v_x, 0, 0)^T$, this model is the continuum version of two Kitaev wires with opposite spin and p-wave pairing. If $\boldsymbol{\alpha}$ and Δ are non-zero, the two spin directions mix.

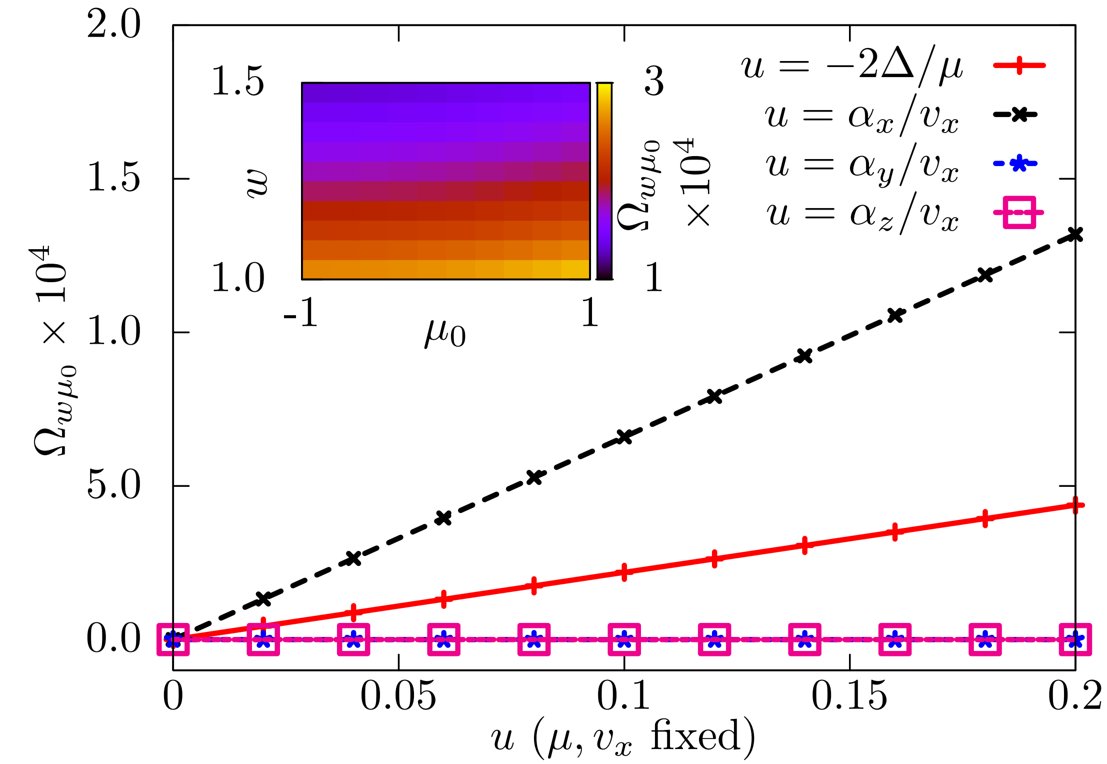


FIG. 2. Berry curvature for the Hamiltonian (9) with $m = 1$, $\mu = 10, \mu_0 = 0$, $\boldsymbol{v}_\Delta = (5, 0, 0)^T$ and $\Delta = \boldsymbol{\alpha} = 0$ and the parameters on the x-axis take the corresponding non-zero values. The inset shows a larger area of the Berry curvature for the case $\alpha_x/v_\Delta = 0.2$. The unit of length for all plots is 1. The total wire has a length of 40 and x_0 is measured from the middle of the wire.

$$\mu(x) = \mu_0 + \mu\tanh\left(\tfrac{x - x_0}{w}\right)$$

$\mu(x<0) < \mu_c :$ trivial

$\mu(x>0) > \mu_c :$ topological

$$\mu_0 \approx \mu_c$$

adiabatic cycle of

μ_0 , w

$$\Longrightarrow \Omega_{\eta_i\eta_j} = \partial_{\eta_i}A_{\eta_j} - \partial_{\eta_j}A_{\eta_i}$$

How to avoid Majorana mixing

- "No mixing of time reversal partner if TRS sectors are decoupled“

- Needed: additional conserved quantum number distinguishing the sectors

$$\begin{aligned} [H(\boldsymbol{\eta}), \Pi] &= 0, \\ [\mathcal{P}, \Pi] &= 0, \\ \{\mathcal{T}, \Pi\} &= 0, \end{aligned}$$

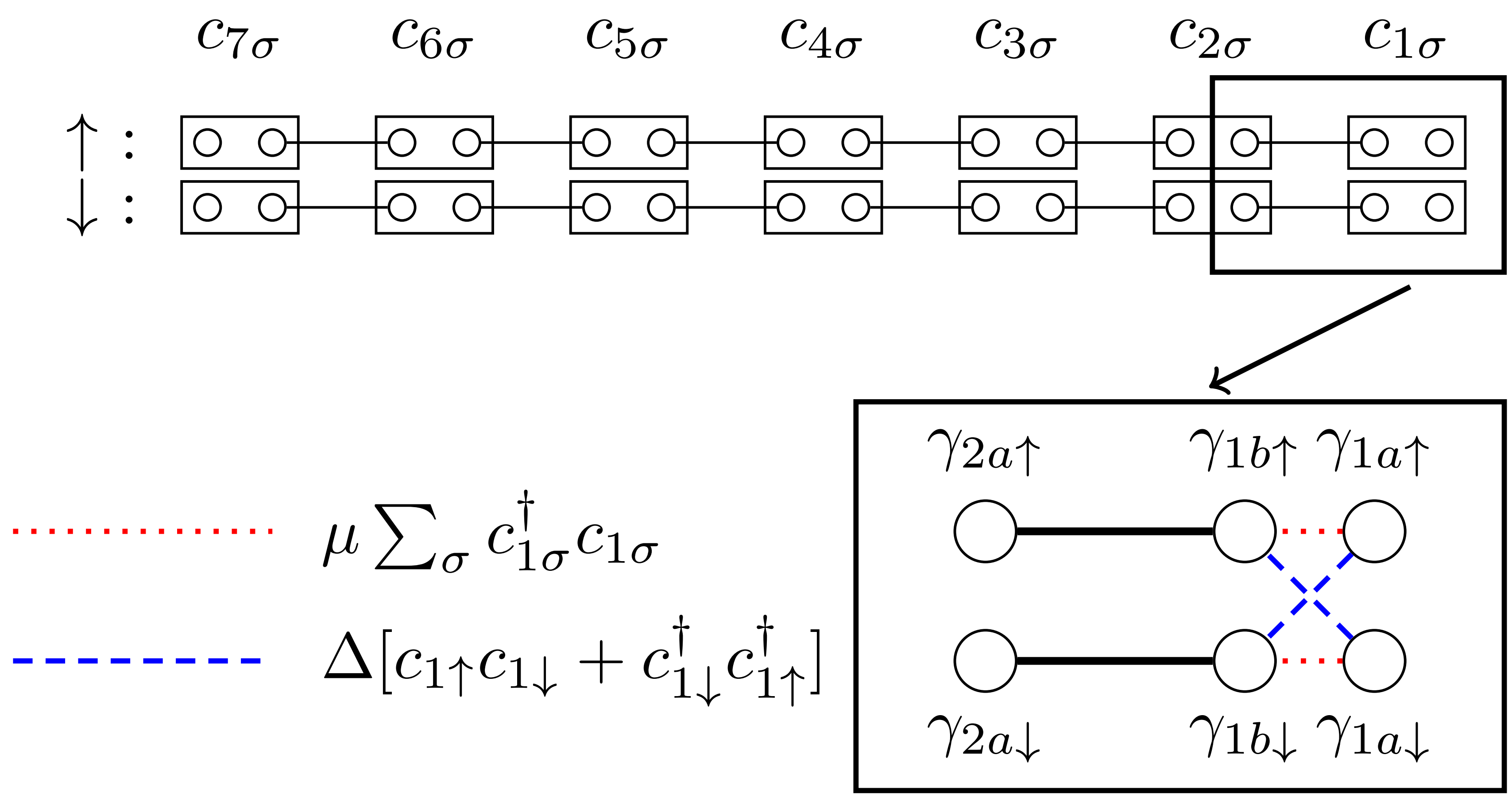
$$\Pi|\gamma_{\boldsymbol{\eta}}\rangle = |\gamma_{\boldsymbol{\eta}}\rangle \text{ and } \Pi|\tilde{\gamma}_{\boldsymbol{\eta}}\rangle = -|\tilde{\gamma}_{\boldsymbol{\eta}}\rangle$$

- Corresponds to fine tuned situations - for numerical study:

  no s-wave pairing, $\Delta = 0$, and "spin-orbit perpendicular to p-wave", $\vec{\alpha} \perp \vec{v}_\Delta$

Conclusions of arXiv:1405.5104

- With Kramers partners: decoherence due to local, adiabatic fluctuations



- Decoherence can be avoided for special situations