Backscattering between helical edge states via dynamic nuclear polarization

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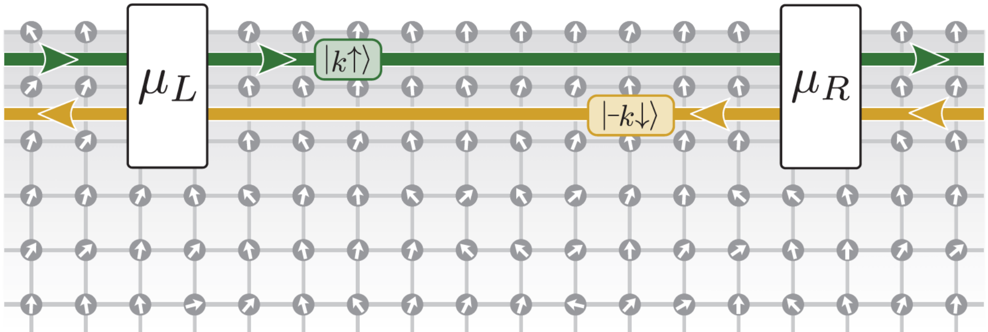
Hamiltonian



$$H = H_0 + H_{\rm flip} + H_{\rm R}$$

$$H_0 = \int dx\, \psi^\dagger_\alpha(x)\sigma^z_{\alpha\beta}\left(-iv\hbar\partial_x + \frac{1}{2}A\langle I\rangle\right)\psi_\beta(x)$$

$$H_{\rm flip} = \frac{A}{2\rho_n}\sum_i \psi^\dagger_\alpha(x_i)\left(\sigma^+_{\alpha\beta}I^-_i + \sigma^-_{\alpha\beta}I^+_i\right)\psi_\beta(x_i)$$

$$H_R = \int dx\, \psi^\dagger_\alpha(x)\sigma^y_{\alpha\beta}\{a(x), i\partial_x\}\psi_\beta(x)$$

$$\langle a(x)a(x')\rangle = V_R\delta(x-x')$$

Dynamic nuclear polarization

$$\Gamma = \frac{A^2}{8\pi\hbar^3 v^2 \rho_n} \int d\epsilon \left\{ (1/2 - \langle I(t) \rangle) \, f_+(\epsilon)[1 - f_-(\epsilon)] \right.$$
$$\left. - (1/2 + \langle I(t) \rangle) \, f_-(\epsilon)[1 - f_+(\epsilon)] \right\}$$

$$\partial_t \langle I(t) \rangle = \Gamma/\rho_n - \Gamma_d \langle I(t) \rangle / \rho_n$$

usually $\Gamma_d \ll \Gamma$

$$\langle I(t) \rangle = \langle I \rangle (1 - e^{-t/\tau_n})$$

$$\langle I \rangle = \frac{\int d\epsilon [f_+(\epsilon) - f_-(\epsilon)]/2}{\int d\epsilon \{ f_+(\epsilon) + f_-(\epsilon)[1 - 2f_+(\epsilon)]\} + \frac{\Gamma_d 8\pi\hbar^3 v^2 \rho_n}{A^2}}$$

$$\tau_n^{-1} = \frac{A^2 \int d\epsilon \, \{ f_+(\epsilon) + f_-(\epsilon)[1 - 2f_+(\epsilon)]\}}{8\pi\hbar^3 v^2 \rho_n^2} + \frac{\Gamma_d}{\rho_n}$$

Transport in short edges

$$f_\pm(\epsilon) = f_0(\epsilon - \mu_{L/R}, T) \quad f_0(\epsilon, T) = [1 + \exp(\epsilon/k_\mathrm{B}T)]^{-1}$$

$$T = 0$$

$$\langle I \rangle = \frac{1}{2}\left[1 + \frac{\Gamma_d 8\pi\hbar^3 v^2 \rho_n}{A^2(\mu_L - \mu_R)}\right]^{-1}$$

$$\Gamma_d = 0,\ T \neq 0$$

$$\langle I \rangle = \frac{1}{2}\tanh\left[(\mu_L - \mu_R)/2k_\mathrm{B}T\right]$$

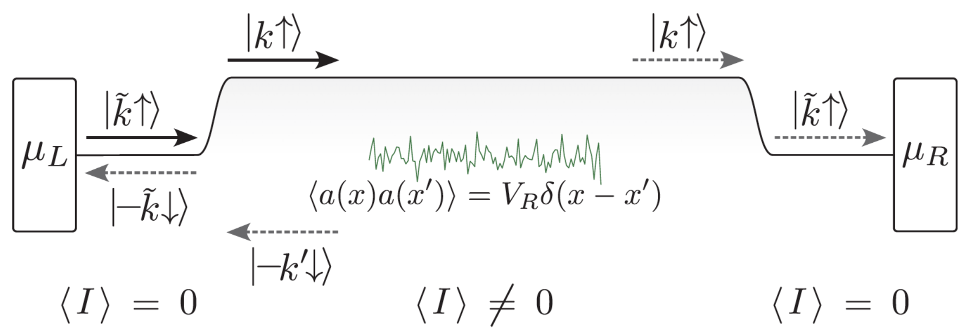
$$W_{kk'} = 2\pi V_R A^2 \langle I \rangle^2 \delta(\epsilon_k - \epsilon_{k'})/(\hbar^3 v^2 L)$$

Total backscattering rate

$$\frac{1}{\tau_R} \equiv \sum_{k'} W_{kk'} = A^2 \langle I \rangle^2 \frac{V_R}{\hbar^4 v^3}$$

$$\sigma = (e^2/\pi\hbar)\ell \text{ with } \ell = v\tau_R/2$$

conductance



$$G = \left(\frac{h}{e^2} + \frac{L}{\sigma}\right)^{-1}$$

$$L \ll \ell \qquad \delta G = -\frac{e^2}{h} A^2 \langle I \rangle^2 \frac{V_R}{\hbar^4 v^4} L$$

Transport in long edges

$\delta\mu$ is the difference in chemical potential between left and right movers

$$\langle I\rangle(x) \;=\; \frac{1}{2}\tanh\left(\frac{j\pi\hbar}{ek_{\rm B}T(x)}\right)$$

electron distribution function $f(k,x) = f_0(\xi_k, m_z, T(x)) + \delta f(k, m_z, x)$

$m_z = \pm 1/2$ is z component of spin

$\xi_k = \epsilon_k + \mathrm{sgn}(m_z) A\langle I\rangle(x) - \mu(x)$

Boltzmann equation $v_k \partial_x f_0 = -\delta f 2/\tau_R$

$$\delta f(k,x) \;=\; \frac{\tau_R}{2}\delta(\xi_k)\left[\mathrm{sign}(k) A\partial_x\langle I\rangle - \partial_x\mu - \frac{\xi_k}{T^2}\partial_x T\right]$$

Current $$j = e\int \frac{dk}{2\pi} v_k \delta f$$

Conductivity $$\sigma(x) = \frac{e^2}{\pi\hbar}\frac{\hbar^4 v^4}{2V_R}\frac{1}{A^2\langle I\rangle^2(x)}$$

If $T(x) \equiv T$

$$j^3 = \frac{e}{\pi\hbar}\frac{2\hbar^4 v^4}{V_R}\left(\frac{ek_{\mathrm{B}}T}{A\pi\hbar}\right)^2\left(-\frac{\partial\mu}{\partial x}\right)$$

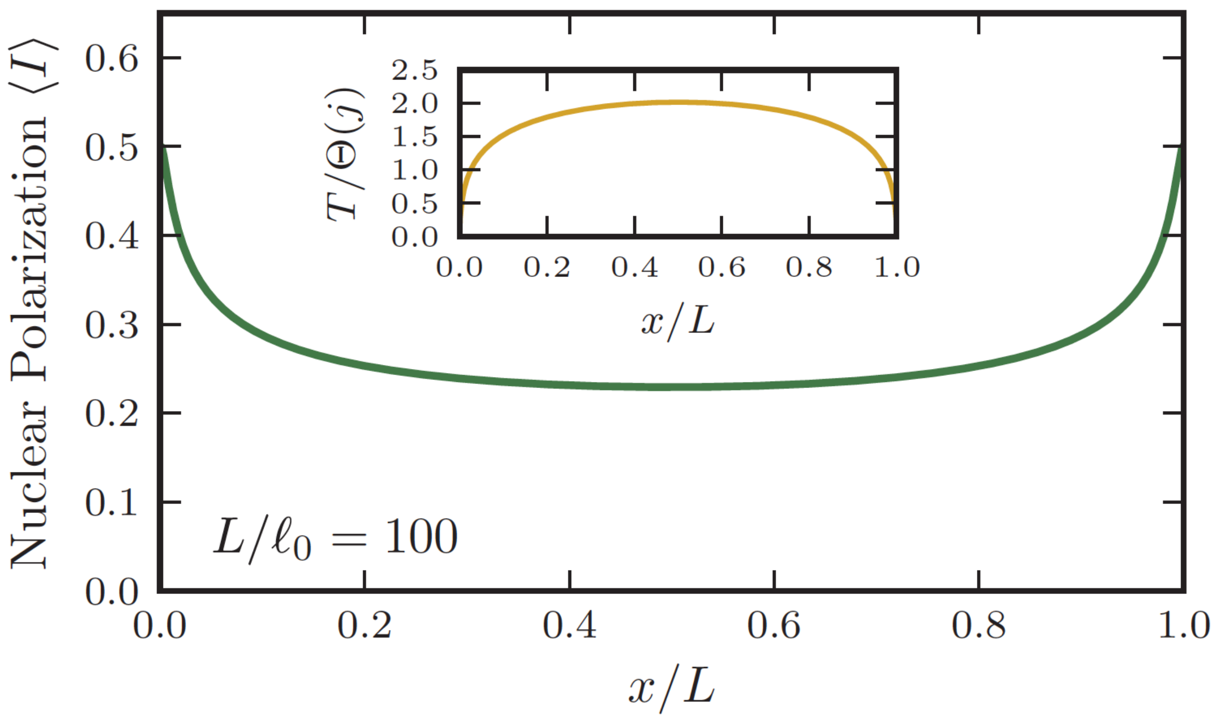
if $\sigma/L \ll e^2/h, \quad j \propto T^{2/3}\mu^{1/3}$

Heat current j_Q

$$j_Q = -\kappa \partial_x T, \text{ where } \kappa = \sigma T (k_B/e)^2 (\pi^2/3)$$

$$\partial_x j_Q = j^2/\sigma$$

$$-\sigma(x)\partial_x \left[\sigma(x)\partial_x T^2(x)\right] \; = \; j^2 \left(\frac{e}{k_B}\right)^2 \frac{6}{\pi^2}$$



$$T(0) = T(L) = 0,$$
$$\ell_0 = (\hbar v)^4/(2V_R A^2),$$
$$\overline{x} \equiv x/\ell_0,$$
$$t(\overline{x}) \equiv T(x)/\Theta(j)$$
$$\Theta(j) = j\pi\hbar/ek_B$$

Conclusions

- We have studied the mechanism by which spin-polarized edge states in a 2D topological insulator may lead to a dynamic polarization of nuclear spins through the hyperfine interaction.

- The resulting effective local Zeeman field in combination with Rashba potential disorder leads to a channel for backscattering between helical edge states.

- For short edges we find a reduction in the conductance $\delta G \propto -A^2 \langle I \rangle^2 V_R$

- For long edges at finite temperature we have uncovered a nonlinear current voltage relation $j \propto T^{2/3} \mu^{1/3}$

- The persistence of the proposed scattering mechanism to T=0 implies that it could be a limiting factor on transport in electronics based on topologically protected edge states in two dimensions.