PHYSICAL REVIEW B **90**, 134404 (2014)

Unpaired Majorana modes on dislocations and string defects in Kitaev's honeycomb model

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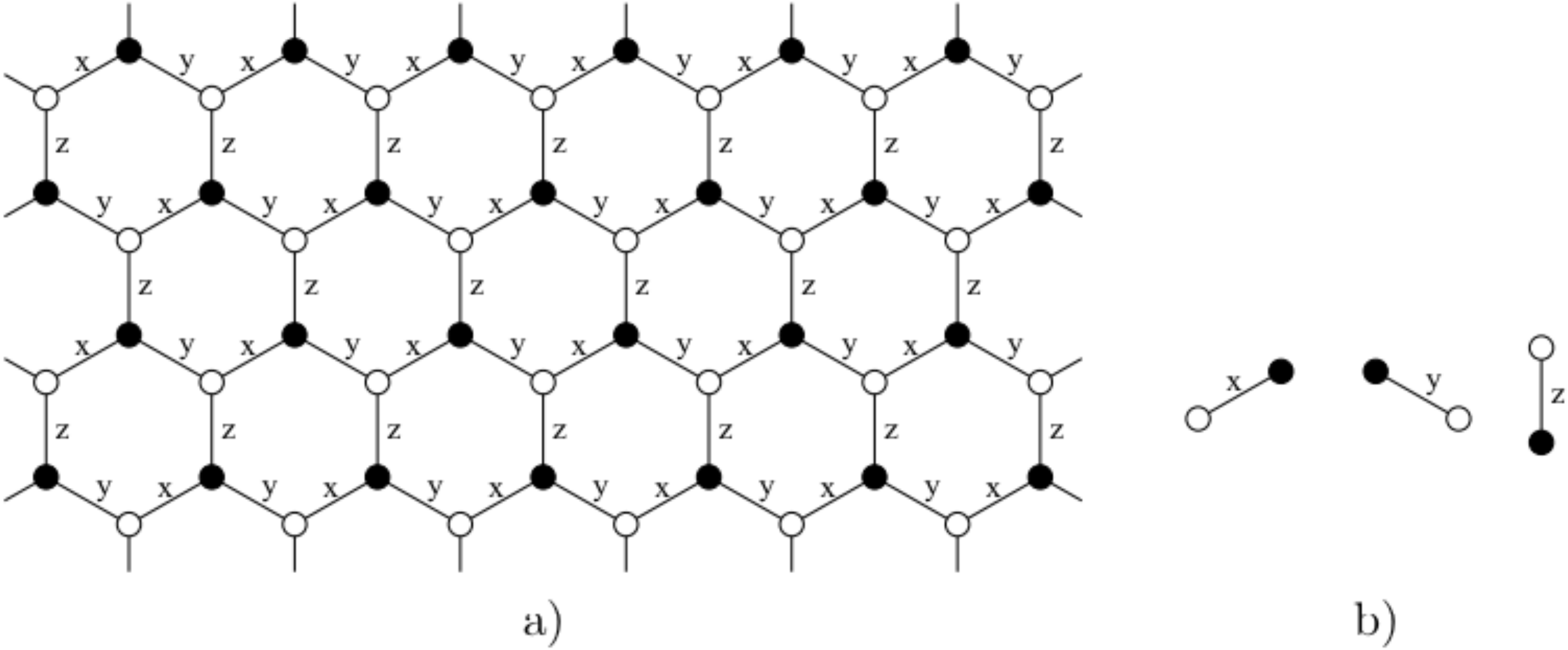
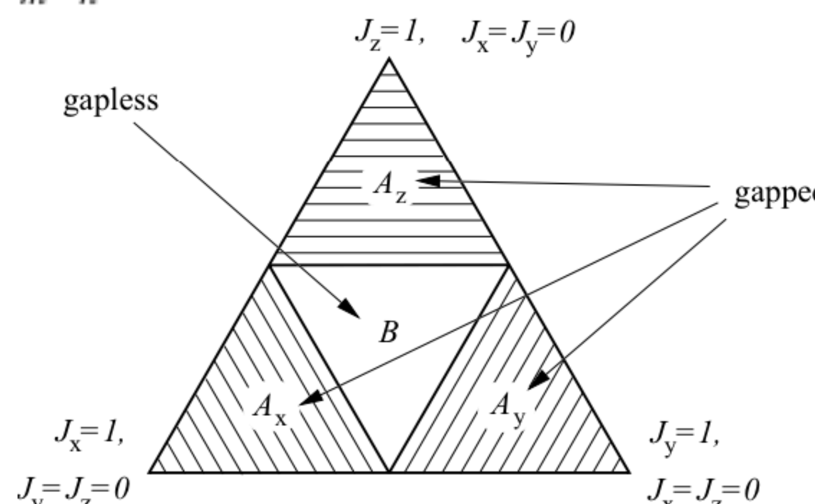
(Received 16 July 2014; published 7 October 2014)

We study the gapped phase of Kitaev's honeycomb model (a Z_2 spin liquid) on a lattice with topological defects. We find that some dislocations and string defects carry unpaired Majorana fermions. Physical excitations associated with these defects are (complex) fermion modes made out of two (real) Majorana fermions connected by a Z_2 gauge string. The quantum state of these modes is robust against local noise and can be changed by winding a Z_2 vortex around one of the dislocations. The exact solution respects gauge invariance and reveals a crucial role of the gauge field in the physics of Majorana modes. To facilitate these theoretical developments, we recast the degenerate perturbation theory for spins in the language of Majorana fermions.

DOI: 10.1103/PhysRevB.90.134404 PACS number(s): 75.10.Kt, 03.65.Vf, 75.10.Jm



$$H = -J_x \sum_{x\text{links}} \sigma_m^x \sigma_n^x - J_y \sum_{y\text{links}} \sigma_m^y \sigma_n^y - J_z \sum_{z\text{links}} \sigma_m^z \sigma_n^z,$$



[3] A. Kitaev, Ann. Phys. (NY) **321**, 2 (2006).

10 Odds and ends

What follows are some open questions, as well as thoughts of how the present results can be extended.

1. Duan, Demler, and Lukin [42] proposed an optical lattice implementation of the Hamiltonian (4). It would be interesting to find a solid state realization as well. For example, the anisotropic exchange could be simulated by interaction of both lattice spins with a spin-1 atom coupled to a crystal field.

2. The weak translational symmetry breaking in the Abelian phase has some interesting consequences. A particularly unusual phenomenon takes place when the lattice has a dislocation. A particle winding around the dislocation changes its type: $e \leftrightarrow m$. Since $m = e \times \varepsilon$, the fermionic parity appears not to be conserved. To restore the conservation law, we must assume that the dislocation carries an unpaired Majorana mode. Therefore, Abelian phases can also be used for the implementation of quantum memory.

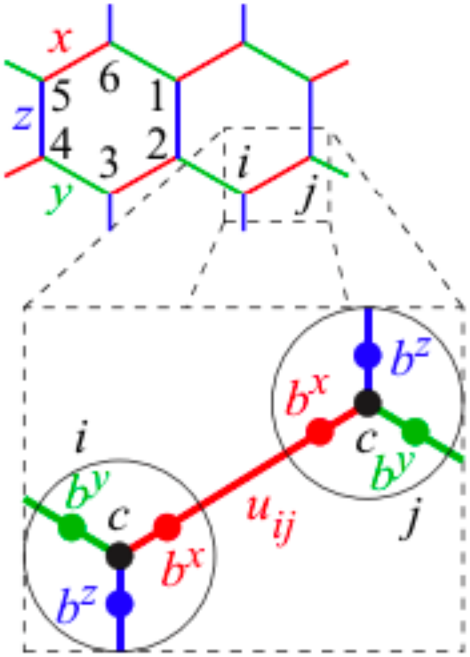
3. Chiral phases ($\nu \neq 0$) require that the time-reversal symmetry be broken. In the present model, this is achieved by applying a magnetic field. However, a spontaneous breaking of time-reversal symmetry occurs in the presence of odd cycles in the lattice. For example, one can replace each vertex of the honeycomb lattice by a triangle. In this case, a gapped $\nu = \pm 1$ phase is realized without external magnetic field [70].

[3] A. Kitaev, Ann. Phys. (NY) **321**, 2 (2006).

Each spin can be mapped to 2 Dirac modes, and so four Majoranas

$$\tilde{\sigma}^x = ib^x c, \qquad \tilde{\sigma}^y = ib^y c, \qquad \tilde{\sigma}^z = ib^z c.$$

These correspond to a larger Hilbert space, so we must project onto physical subspace.



For the spins

$$\sigma^x \sigma^y \sigma^z = i$$

For the Majoranas

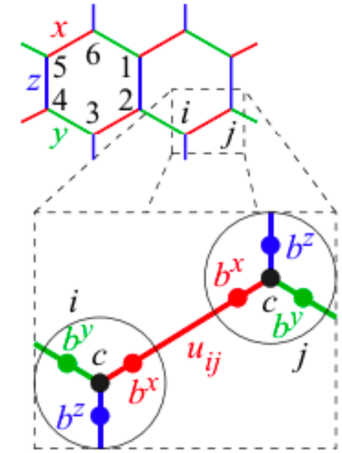
$$\tilde{\sigma}^x \tilde{\sigma}^y \tilde{\sigma}^z = ib^x b^y b^z c = iD.$$

So we require

$$D_j = b_j^x b_j^y b_j^z c_j;$$

$$D_j|\xi\rangle = |\xi\rangle \text{ for all } j.$$

The Hamiltonian becomes one of hopping Majoranas



$$H = i \sum_{\langle mn \rangle} J_{\alpha_{mn}} u_{mn} c_m c_n,$$

Where

$$u_{mn} = -u_{nm} \equiv i b_m^\alpha b_n^\alpha.$$

These operators have eigenvalues +/-1 and commute with the Hamiltonian

We can therefore replace these with their eigenvalues, and get a quadratic Hamiltonian

However, they do not commute with the D operators, the projectors onto the physical subspace

These can be thought of as generators of a Z2 gauge symmetry

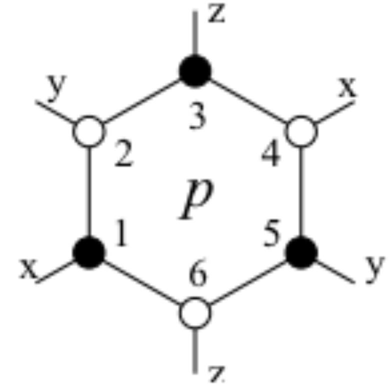
$$b_n^\alpha \mapsto D_n^\dagger b_n^\alpha D_n = -b_n^\alpha,$$

$$c_n \mapsto D_n^\dagger c_n D_n = -c_n,$$

$$u_{mn} \mapsto D_n^\dagger u_{mn} D_n = -u_{mn}.$$

We thus define the Z_2 flux on a plaquette with sites $1,2,\ldots,n$ on the boundary, going counterclockwise, as

$$W = \left(\sigma_1^{\alpha_{12}}\sigma_2^{\alpha_{12}}\right)\left(\sigma_2^{\alpha_{23}}\sigma_3^{\alpha_{23}}\right)\ldots\left(\sigma_n^{\alpha_{n1}}\sigma_1^{\alpha_{n1}}\right). \qquad (11)$$



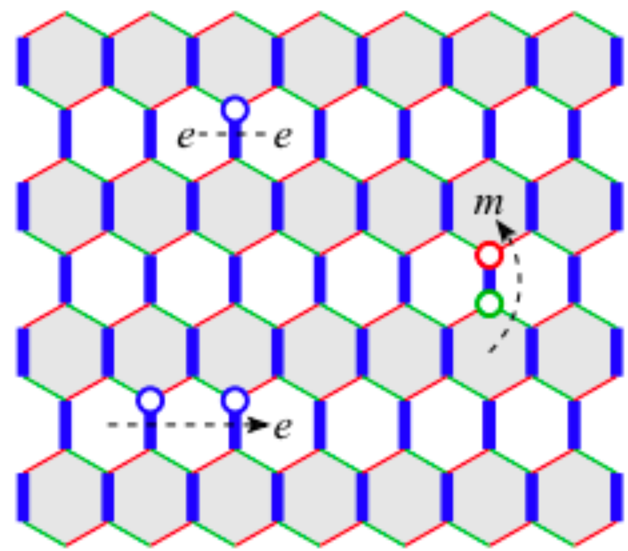
and after using the normalization condition $c_n^2 = 1$, we obtain the flux in terms of Z_2 gauge variables:

$$W = (-i)^n\, u_{12}u_{23}\ldots u_{n1}, \qquad (12)$$

$J_z \gg J_x,\, J_y$:

Excitations come in two forms, fermions and Z_2 vortices. Fermion excitations ψ are associated with breaking the alignment of spins on strong bonds and thus have a high energy cost of approximately $2J_z$, so we shall refer to them as high-energy fermions. Low-energy excitations are Z_2 vortices, $W = -1$,

creating a $e \times m$ vortex pair is accompanied by creation or annihilation of a fermion ψ with a high energy cost of $2J_z$.



Deeply in the gapped phase where z links dominate we may drop the x and y terms in the Hamiltonian as a starting point:

$$H_0 = -J_z \sum_{z\text{links}} \sigma_m^z \sigma_n^z = J_z \sum_{z\text{links}} i u_{mn} c_m c_n .$$

The parity of the ψ fermions is

$$\pi_\psi = \prod_{z\text{links}} (\psi_{mn}\psi_{mn}^\dagger - \psi_{mn}^\dagger \psi_{mn}) = \prod_{z\text{links}} (-i u_{mn} c_m c_n) = \prod_{z\text{links}} \sigma_m^z \sigma_n^z .$$

The parity of the exm fermions is

$$\pi_{e\times m} = \prod_{p\in e} W_p = \prod_{p\in m} W_p = \prod_{zlinks} \sigma_m^z \sigma_n^z$$

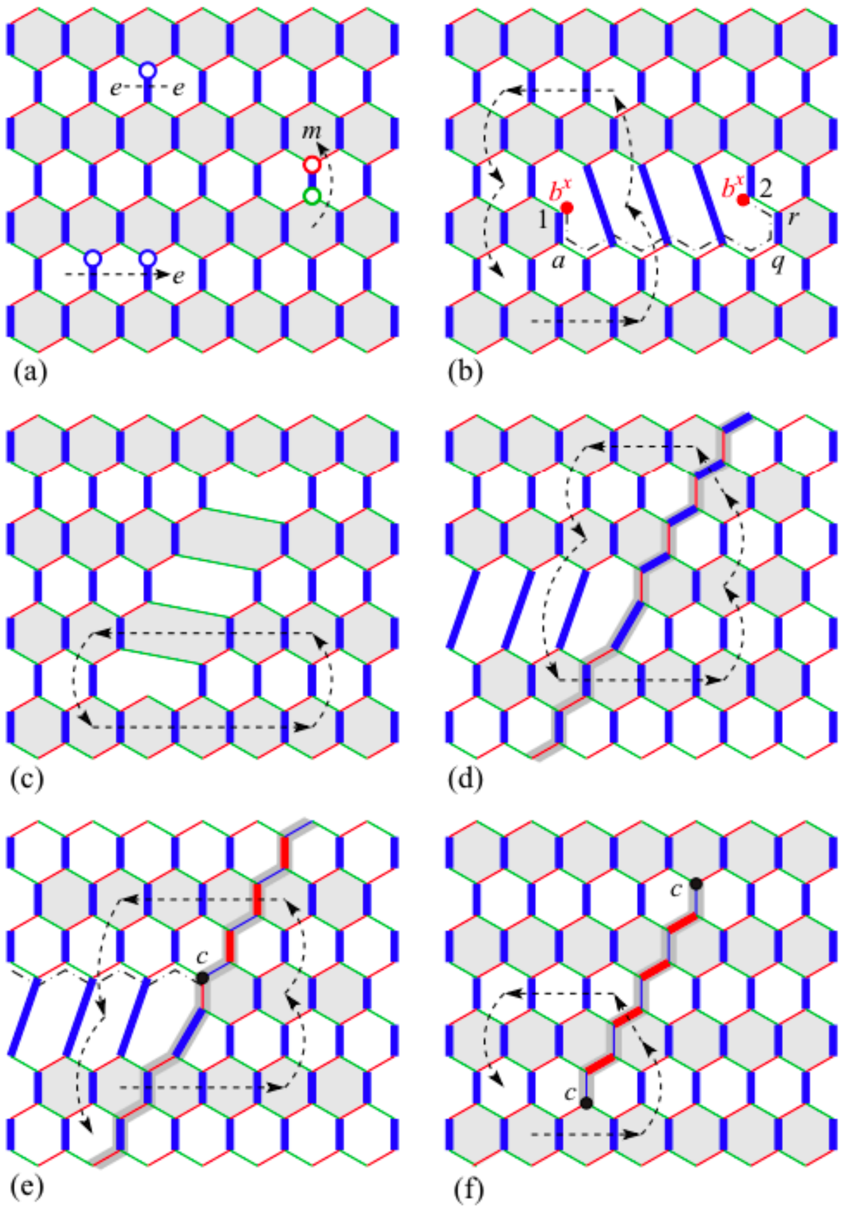
Clearly $\pi_\psi \pi_{e\times m} = 1$. In the Majorana mapping, this corresponds to the gauge invariance condition

$$\pi_\psi \pi_{e\times m} = \prod_n D_n = +1.$$

The conservation of fermionic parity has been tied to Z_2 gauge invariance by Pedrocchi *et al* . [28]

[28] F. L. Pedrocchi, S. Chesi, and D. Loss, Phys. Rev. B **84**, 165414 (2011).

Multiple types of dislocation are considered



8-2:

$$\Psi = \frac{b_1^x + iU_{12}b_2^x}{2}, \quad \Psi^\dagger = \frac{b_1^x - iU_{12}b_2^x}{2}.$$

$$U_{12} = u_{1a}u_{ab}\ldots u_{qr}u_{r2},$$

$$\pi_\psi \, \pi_{e\times m} \, \pi_{12} = 1.$$

Two unpaired modes at each end, so a Dirac mode can be gapped by local perturbations

5-7:

Unpaired c type Majoranas at ends of dislocations

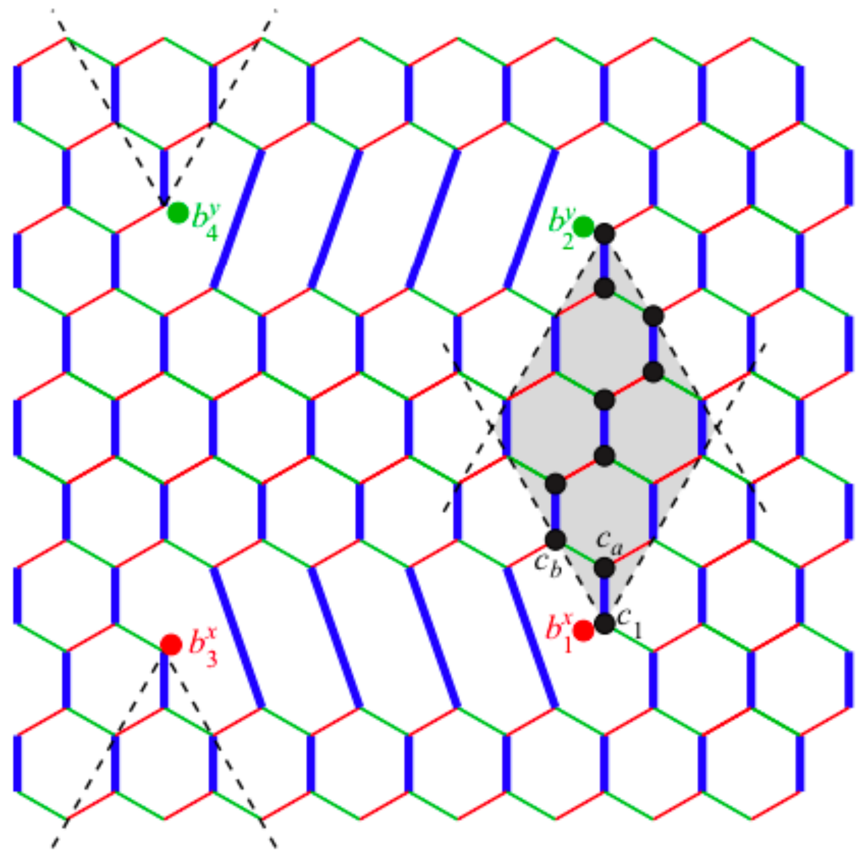
What effects do local perturbations have on the dangling Majorana modes?

For example, a magnetic field: Then the four dangling Majorana modes b_m^α in Fig. 4 are coupled to the rest of the system by the term $-h_m^\alpha \sigma_m^\alpha = -i h_m^\alpha b_m^\alpha c_m$.

Integrating out the c anyons involved in high energy links, they find

$$H_{\text{eff}} = \frac{h_1^x h_2^x}{J_z} \sum_{\text{paths}} \frac{J_x^{n_x} J_y^{n_y}}{J_z^{n_x+n_y}} \, i U_{12} b_1^x b_2^x .$$

Decays exponentially with the number of links (distance) between them



Deformation of the lattice can be achieved without deforming the lattice Using strong field perturbations

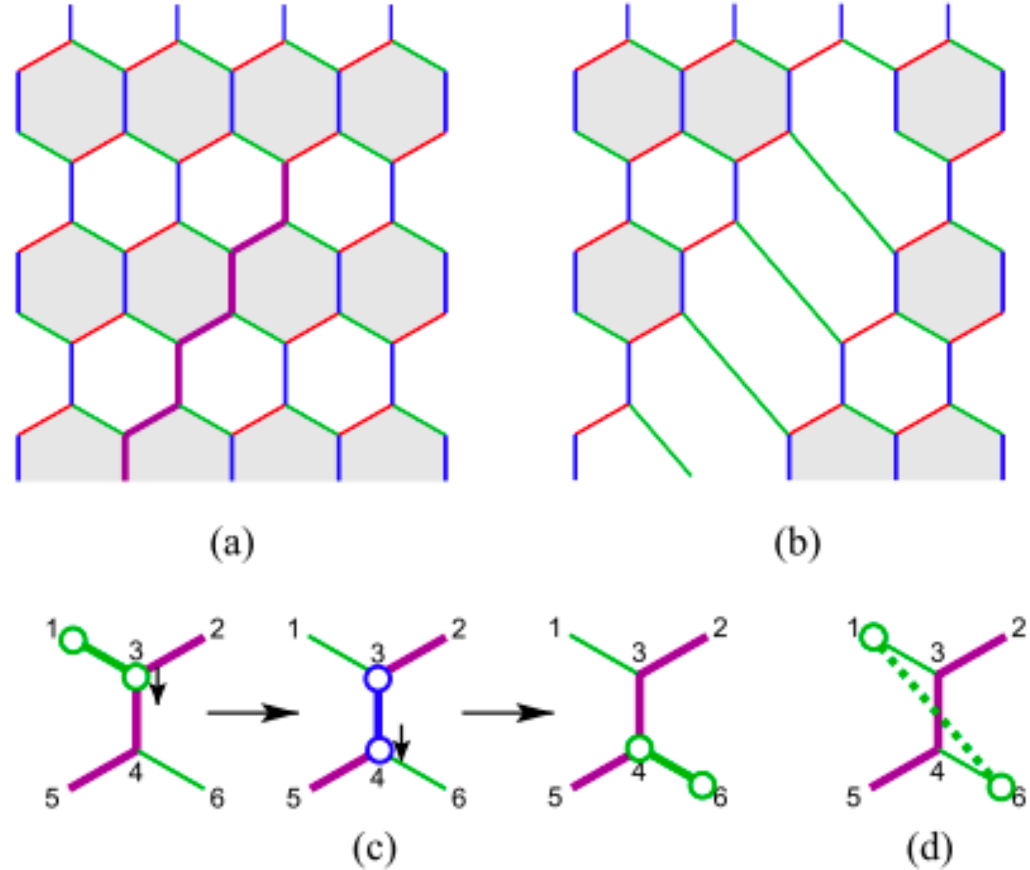


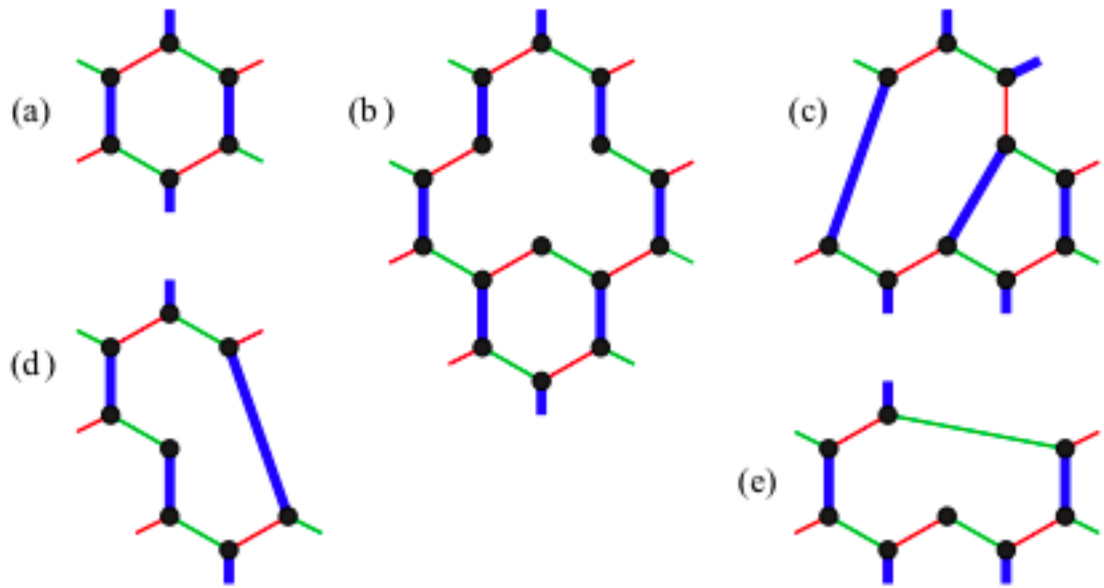
FIG. 5. (Color online) (a) Magnetic field $h_x \gg J_\alpha$ is applied to the spins along the purple line C. The original alternating z-y flavors of the bonds are preserved. (b) The effective description of such set up results in a defect that behaves as an 8–2 dislocation. (c) In order to keep the spins aligned with the applied field, three bond terms from the Kitaev Hamiltonian must be applied together as shown. Thick green and blue bonds with open circles at the ends indicate applications of $J_y \sigma_m^y \sigma_n^y$ and $J_z \sigma_m^z \sigma_n^z$ respectively. The three operations in (c) are equivalent to connecting sites 1 and 6 directly with an effective y bond (d).

Ground state is typically vortex free

The 8-2 dislocations disturb this: Flux is bound to octagonal plaquettes at ends

This is true for those that carry an unpaired Majorana, and those that do not

The 5-7 dislocations carry no vortex in both cases



Conclusions

Multiple types of dislocation were considered for high Jz honeycomb lattice models

For those that cause twists, unpaired Majorana mode is seen explicitly from mapping

Delocalized Dirac mode is stable against local perturbations

Physical dislocations of the lattice are not required, a field can be used