Tuning topological superconductivity in helical Shiba chains by supercurrent

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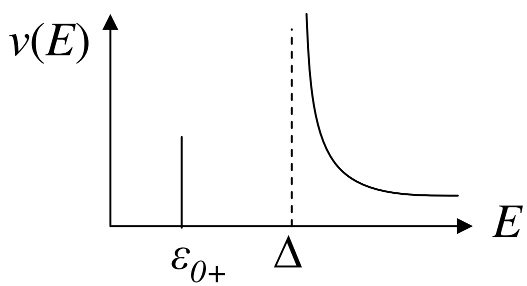
Arxiv:1406:4288

S-wave superconductor with impurities

Usual impurities affect electric charge. =>
Same scattering for both electrons of the Cooper pair.

Magnetic impurity can flip the spin of electron. =>
Scattering can lead to the state with parallel spins. Pauli principle. =>
Two electrons of a Cooper pair can not be at the same point. Pair is destroyed.

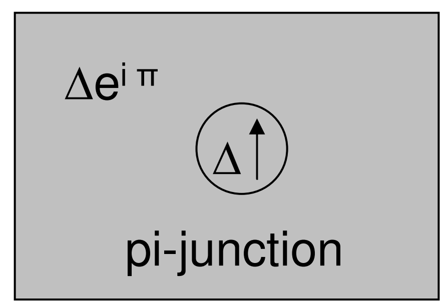
Single magnetic impurity: exchange interaction $-J(r)\vec{\sigma}\vec{S}$



Yu, Shiba, Rusinov

Zero harmonics

$$\varepsilon_{0\pm} = \pm|\Delta|\frac{1-(\pi\nu_0 SJ)^2}{1+(\pi\nu_0 SJ)^2}$$



Balatsky et al, RMP, 2006

Superconductivity prevents from ferromagnetic ordering.

RKKY in the superconductor. Helical magnetic order with the period:

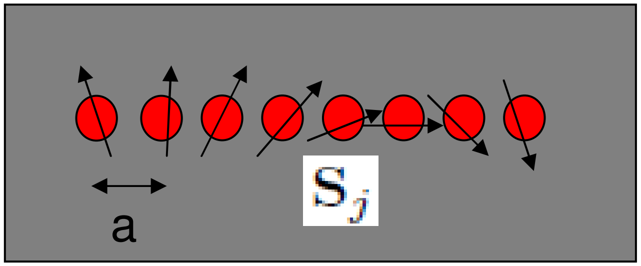
$$d_{3D} \approx \xi \frac{1}{(\xi p_F)^{2/3}} \qquad 1/p_F < d_{3D} < \xi$$

Anderson, Suhl, 1959

Anisotropy field chooses direction of easy magnetization. Magnetic domains.

Pientka et al., PRB 88, 155420 (2013)

1. Consider a superconducting system with a regular 1d lattice of magnetic atoms deposited on top of it.



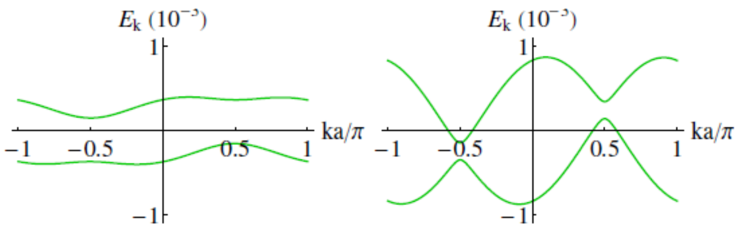
2. Shiba states form a band.

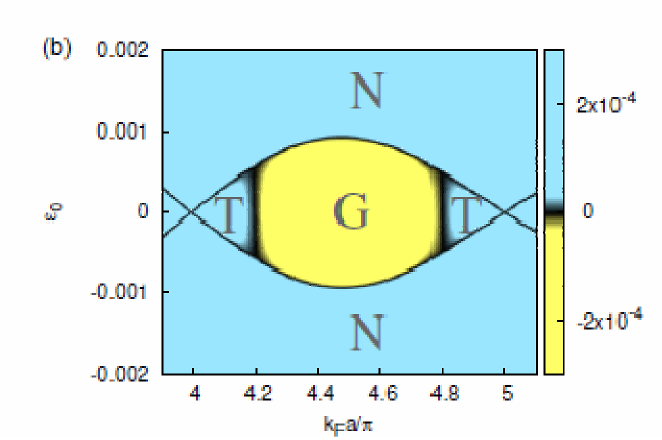
3. Helical magnetic order. $\hat{\mathbf{S}}_j = (\cos 2k_h aj \sin\theta, \sin 2k_h aj \sin\theta, \cos\theta)$

$\hat{\mathbf{S}}_j = \mathbf{S}_j/S,\ S = |\mathbf{S}_j|$

4. Combination of the superconductivity and helical magnetic order might lead to the topological superconductivity.

$\xi_0/a \ll 1$





$\xi_0 = a/5,\ k_h a = \pi/8,\ \Delta = 1$

$\varepsilon_0 \approx |\Delta|(1 - |\pi\nu_0 SJ|)$

Rontynen and Ojanen, arxiv: 1406.4288

Phase diagram in the presence of the supercurrent.

BdG equation in the Nambu basis

$$[E - \xi_k \tau_z - \mathbf{\Delta} \cdot \tau]\Psi(\mathbf{r}) = -J \sum_j \mathbf{S}_j \cdot \sigma \delta(\mathbf{r}_j)\Psi(\mathbf{r})$$

$$\xi_k = \frac{k^2}{2m} - \mu$$

Position dependent superconducting order parameter

$$\mathbf{\Delta} = |\Delta| (\cos\varphi(r), \sin\varphi(r), 0)$$

Consider magnetic texture of the form:

$$\hat{\mathbf{S}}_j = (\cos 2k_h a j \sin\theta, \sin 2k_h a j \sin\theta, \cos\theta)$$

$$\bar{\Psi}(\mathbf{r}) = U(r)\Psi(\mathbf{r}) \text{ where } U(r) = e^{i\tau_z \frac{\varphi(r)}{2}}$$

Transformed BdG equation takes the form

$$\left[E - \tau_z \left(\frac{(\mathbf{k} - \frac{\nabla\varphi}{2}\tau_z)^2}{2m} - \mu \right) - |\Delta|\tau_x\right]\bar{\Psi}(\mathbf{r})$$
$$= - J \sum_j \mathbf{S}_j \cdot \sigma\delta(\mathbf{r}_j)\bar{\Psi}(\mathbf{r}).$$

$$|\nabla\varphi| \lesssim 2\pi/\xi \ll k_F$$

Single magnetic moment:

Without supercurrent

$$E = \pm\varepsilon = \pm|\Delta|\frac{1-\alpha^2}{1+\alpha^2} \qquad \alpha = \pi\nu_0 SJ$$

With supercurrent

$$\pm\varepsilon \rightarrow \pm\varepsilon \mp \frac{|\Delta|}{6}\left(\frac{\tilde{\varepsilon}_\varphi}{|\Delta|}\right)^2 \qquad \tilde{\varepsilon}_\varphi = \frac{v_F|\nabla\varphi|}{2} \qquad \tilde{\varepsilon}_\varphi/|\Delta| \ll 1$$

Chain of magnetic impurities: $H\Psi' = E\Psi'$ $H = \begin{pmatrix} h_{ij} & \Delta_{ij} \\ (\Delta_{ij})^\dagger & -h^*_{ij} \end{pmatrix}$

$h_{ij} = \tilde{\varepsilon}_0$ and $\Delta_{ij} = 0$ when $i = j$

$$h_{ij} = |\Delta| \frac{e^{-\frac{r_{ij}}{\xi}}}{k_F r_{ij}} \left(i \frac{\varepsilon_\varphi}{|\Delta|} \operatorname{sign}(i-j) \cos k_F r_{ij} - \sin k_F r_{ij} \right) \langle \uparrow i | \uparrow j \rangle$$

$$\Delta_{ij} = |\Delta| \frac{e^{-\frac{r_{ij}}{\xi}}}{k_F r_{ij}} \cos k_F r_{ij} \langle \uparrow i | \downarrow j \qquad i \neq j$$

$$\varepsilon_\varphi = \frac{v_F |\nabla\varphi|}{2} \cos\beta$$

Helical magnetic order

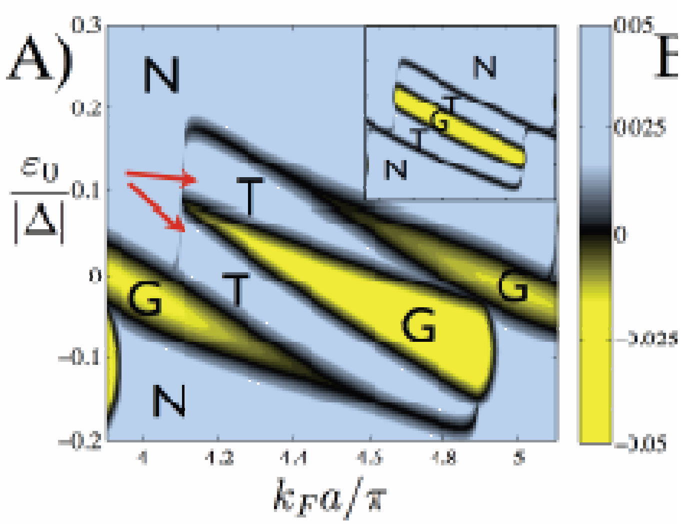
$$\langle \uparrow i | \uparrow j \rangle = \cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2} e^{-2ik_h x_{ij}}$$

$$\langle \uparrow i | \downarrow j \rangle = i \sin k_h x_{ij} \sin\theta$$

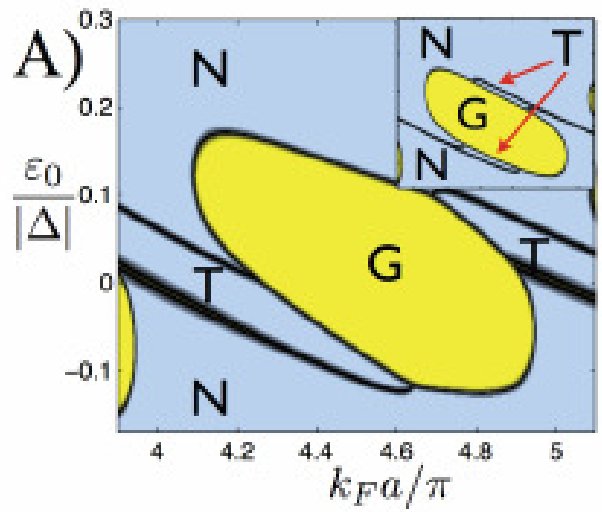
The spectrum is calculated.

Helical structure is nonplanar $\theta \neq \frac{\pi}{2}$ $\xi_0/a \ll 1$

Different phases are separated by the condition: $\min E(k) = 0$



$\theta = 2\pi/5, k_h a = \pi/10, \varepsilon_\varphi = |\Delta|/3, \xi = 50a$



$\theta = \tau/5, k_h a = \pi/3, \varepsilon_\varphi = |\Delta|/3, \xi = 50a$

In addition to adding gapless regions, finite supercurrent also pushes some gapless regions to the topologically nontrivial gapped phase indicated by the red arrows.

for some helical configurations it is possible to dramatically increase the topologically nontrivial region in the phase diagram.

Main results:

1. For nonplanar magnetic structures:
Supercurrent tunes the gapless phase to the gapped topological phase.
In some cases the topological phase increases in the phase diagram.

2. For planar magnetic helix:
Without the supercurrent system is gapped.
Supercurrent drives the system towards the gapless phase.