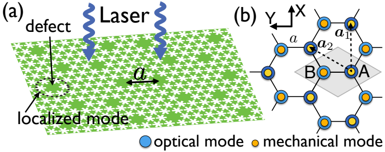
Journal Club, 4 November 2014

Optomechanical Dirac Physics

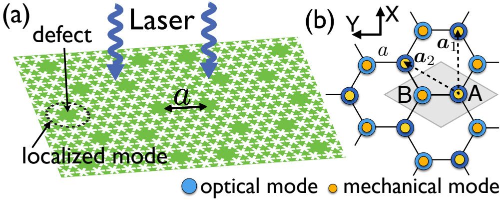
M. Schmidt, V. Peano, and F. Marquardt



arXiv:1410.8483 [cond-mat.mes-hall]

Proposal

- Implement and study Dirac physics in “optomechanical metamaterials”
- Platform: optomechanical crystal arrays



- Possibility to engineer non-trivial band structures of photons and phonons
- Advantages: tunability of the of the band structure, possibility to observe related effects by monitoring the emitted light

Optomechanical crystal

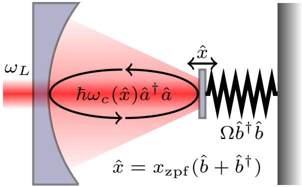
Photonic crystal

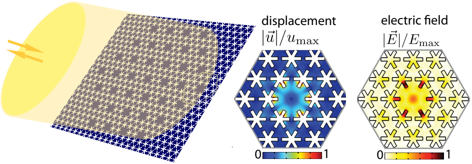
- Thin patterned dielectric slab (Si) with localized optical modes at defects ($\omega_c \sim 100$ THz).

Optomechanical crystal

- Photonic crystal with co-localized optical and vibrational modes at defects ($\Omega \sim 1$ GHz).
- On-site radiation-pressure interaction.

$$\hat{H}_{\mathrm{RP}} = g_0 \hat{a}^\dagger \hat{a} \left(\hat{b} + \hat{b}^\dagger \right)$$





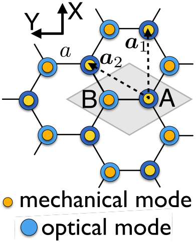
Optomechanical array with honeycomb geometry

Model: 2D honeycomb lattice of identical optomechanical cells

Unit cell

Two pairs of mechanical ($\hat{b}_j$) and optical ($\hat{a}_j$) modes.
Optical modes driven into a coherent state:

$$\hat{a}_j \quad \rightarrow \quad \alpha_j + \hat{a}_j .$$



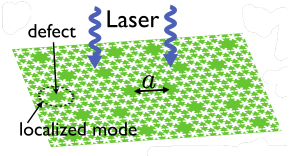
One-site linearized Hamiltonian

$$\hat{H}_j = \Omega \hat{b}_j^\dagger \hat{b}_j - \Delta_j \hat{a}_j^\dagger \hat{a}_j - g_j \left(\hat{b}_j^\dagger \hat{a}_j + \hat{a}_j^\dagger \hat{b}_j \right)$$

Detuning: $\Delta = \omega_c - \omega_L$, Coupling: $g_j = g_0 \alpha_j$

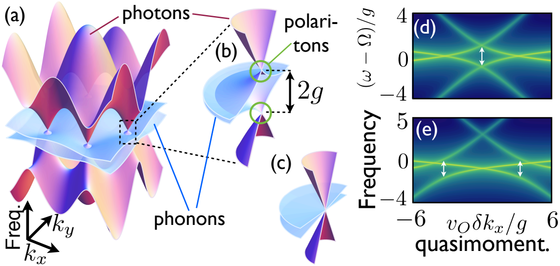
Tunneling between neighboring sites

$$\hat{H}_{\text{hop}} = - \sum_{\langle ij \rangle} \left(J_{ij} \hat{a}_i^\dagger \hat{a}_j + K_{ij} \hat{b}_i^\dagger \hat{b}_j \right)$$



Hamiltonian: $\hat{H} = \sum_j \hat{H}_j + \hat{H}_{\text{hop}}$ $\quad (j = [m, n, \sigma = \pm 1])$

Band structure (translation-invariant system)



Emergence of photon-phonon polariton Dirac cones

- 4 polariton bands, constructed out the original 2 photon and 2 phonon bands

With noise and dissipation: $\hat{H} = \sum_j \left(\hat{H}_j + \hat{H}_{\kappa,j} + \hat{H}_{\Gamma,j} \right) + \hat{H}_{\text{hop}}$

- Band structure is visible in the emitted far-field radiation (scattered laser-drive photons), even at room temperature.

$$S(\mathbf{k}, \omega) = \sum_\sigma \int dt \, e^{i\omega t} \left\langle \hat{a}^\dagger_{\mathbf{k}\sigma}(t) \hat{a}_{\mathbf{k}\sigma} \right\rangle$$

Without interaction $(g = 0)$
Excitations on sublattice A or B $\quad\rightarrow\quad$ binary d.o.f $\sigma_z = \sigma = \pm 1$

$$\hat{H}_i = \nu_i \, \hat{\boldsymbol{\sigma}} \cdot \delta\mathbf{k} = \nu_i \left(\hat{\sigma}_x \delta k_x + \hat{\sigma}_y \delta k_y\right) \qquad \big(i = O, M\big)$$

$\delta\mathbf{k} = \mathbf{k} - \mathbf{K}$, $\nu_O = 3aJ/2$, $\nu_M = 3aK/2$

Interacting case $(g \neq 0)$
Particle type $\quad\rightarrow\quad$ second binary d.o.f $\tau_z = \tau = \pm 1$

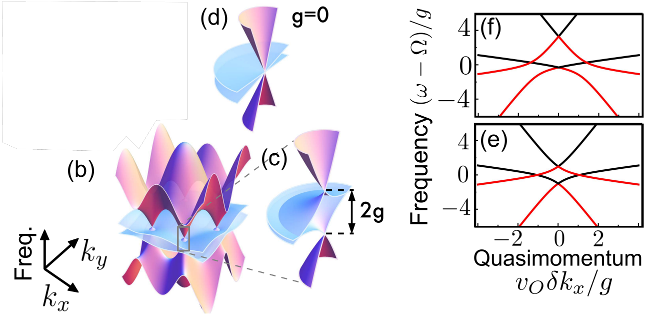
$$\hat{H}_D = \frac{\delta\omega}{2}\hat{\tau}_z + \left(\bar{\nu} + \frac{\delta\nu}{2}\hat{\tau}_z\right)\hat{\boldsymbol{\sigma}} \cdot \delta\mathbf{k} - g\hat{\tau}_x + \bar{\omega}$$

$\delta\omega = -\Delta - \Omega$, $\bar{\nu} = (\nu_0 + \nu_M)/2$, $\delta\nu = \nu_0 - \nu_M$, $\bar{\omega} = (\Omega - \Delta)/2$.

Dispersive spectrum

$$\omega_{\tau,\sigma}(\mathbf{k}) = \bar{\omega} - \sigma\bar{\nu}|\delta\mathbf{k}| + \tau\sqrt{g^2 + \frac{(\delta\omega - \sigma\delta\nu|\delta\mathbf{k}|)^2}{4}}$$

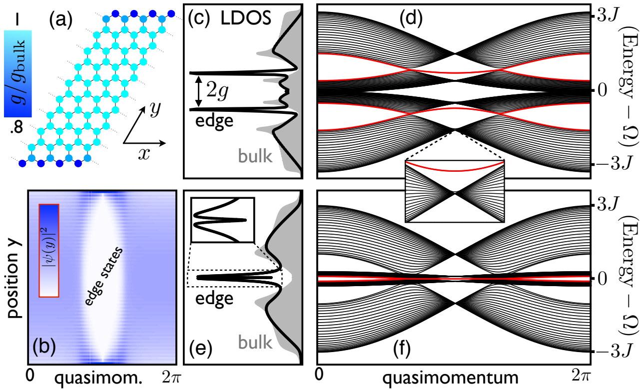
The velocity is momentum-dependent and varies on the scale g/Ja.



- Two cones split by $\sqrt{\delta\omega^2 + 4g^2}$
- Sweeping the laser detuning ($\delta\omega$), upper cones evolves from purely optical (ν_0), over polaronic ($\bar{\nu}$), to purely mechanical (ν_M).

Edge states

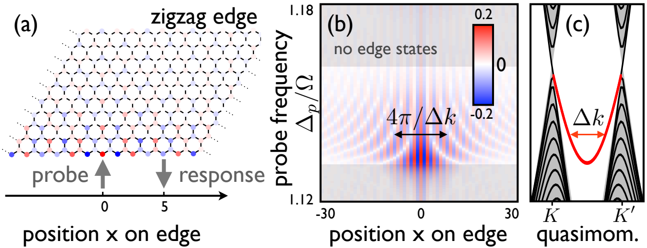
The physics of edge-state is modified by inhomogeneous optomechanical couplings ($g_{\mathrm{edge}} < g_{\mathrm{bulk}}$).



- Edge modes are dispersive
- Vanishing DOS at the Dirac points is smeared out slightly by dissipation

Edge state transport

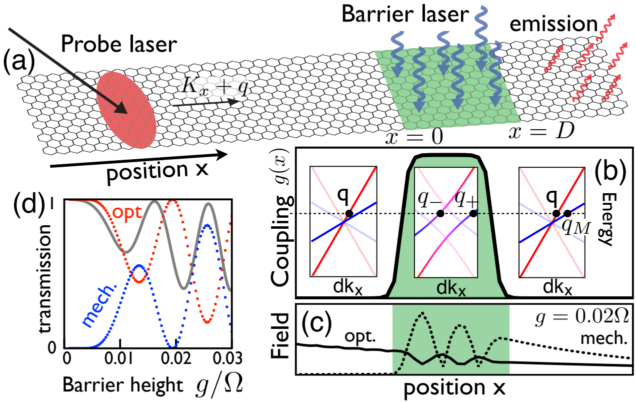
- Zigzag edge forms a polariton waveguide for locally-injected excitations (e.g. with a tapered optical fiber).
- Group velocity is tunable in-situ via the laser amplitude.



Optomechanical Klein tunneling

Unimpeded transmission of relativistic particles through arbitrary long and high potential barriers.

Effective potential landscape with a non-uniform driving laser profile.



Optomechanical Klein tunneling

Unimpeded transmission of relativistic particles through arbitrary long and high potential barriers.

Effective potential landscape with a non-uniform driving laser profile.

Photon moving toward the barrier:

$$|\psi_{\rm in}\rangle = e^{iq_O x}|\sigma_x = 1, \tau = 1\rangle$$
$$\rightarrow \quad |\psi_{\rm out}\rangle = t_0 e^{iq_O x}|1,1\rangle + t_M \sqrt{\nu_0/\nu_M} e^{iq_M x}|1,-1\rangle$$

$q_M = \nu_0 q_0/\nu_m$, $|t_M|^2 + |t_0|^2 = 1$

Probability to convert a photon into a phonon:

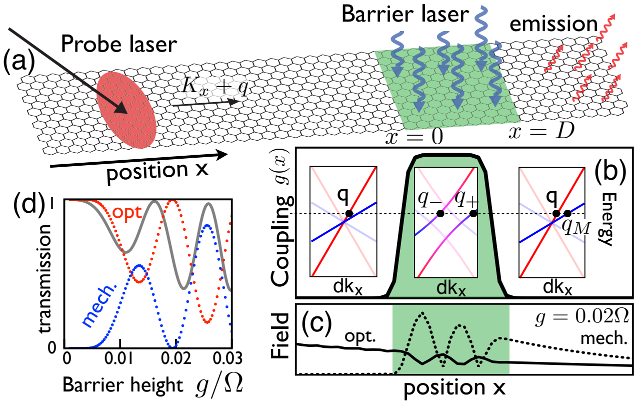
$$|t_M|^2 = \frac{\sin^2[(q_+ - q_-)D/2]}{1 + v_O^3 q_O^2/(4 v_M g^2)}$$

$q_\pm$: momenta of the right moving polaritons

Optomechanical Klein tunneling

Unimpeded transmission of relativistic particles through arbitrary long and high potential barriers.

Effective potential landscape with a non-uniform driving laser profile.



LDOS and transmission amplitudes

Photonic retarded Green's function

$$\tilde{G}_{OO}(\omega, \mathbf{j}, \mathbf{l}) = -i \int_{-\infty}^{\infty} dt e^{i\omega t} \Theta(t) \langle [\hat{a}_{\mathbf{j}}(t), \hat{a}_{\mathbf{l}}^{\dagger}(0)] \rangle.$$

LDOS

$$\rho(\omega, \mathbf{j}) = -2 \mathrm{Im} \tilde{G}_O(\omega, \mathbf{j}, \mathbf{j})$$

Transmission amplitude

$$\left\langle \hat{a}_{\mathbf{l}}^{(\mathrm{out})}(t) \right\rangle = t_O(\omega, \mathbf{l}, \mathbf{j}) \left\langle \hat{a}_{\mathbf{j}}^{(\mathrm{in})}(t) \right\rangle, \qquad \left\langle \hat{a}_{\mathbf{j}}^{(\mathrm{in})}(t) \right\rangle = f e^{-i\omega t}$$

$$t_O(\omega, \mathbf{l}, \mathbf{j}) = \delta_{\mathbf{lj}} - i\kappa \tilde{G}_{OO}(\omega, \mathbf{l}, \mathbf{j})$$