A silicon-based surface code quantum computer

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Individual impurity atoms in silicon can make superb individual qubits, but it remains an immense challenge to build a multi-qubit processor: There is a basic conflict between nanometre separation desired for qubit-qubit interactions, and the much larger scales that would enable control and addressing in a manufacturable and fault tolerant architecture. Here we resolve this conflict by establishing the feasibility of surface code quantum computing using solid state spins, or 'data qubits', that are widely separated from one another. We employ a second set of 'probe' spins which are mechanically separate from the data qubits and move in-and-out of their proximity. The spin dipole-dipole interactions give rise to phase shifts; measuring a probe's total phase reveals the collective parity of the data qubits along the probe's path. We introduce a protocol to balance the systematic errors due to the spins being imperfectly located during device fabrication. Detailed simulations show that the surface code's threshold then corresponds to misalignments that are substantial on the scale of the array, indicating that it is very robust. We conclude that this simple 'orbital probe' architecture overcomes many of the difficulties facing solid state quantum computing, while minimising the complexity and offering qubit densities that are several orders of magnitude greater than other systems.

The code written for our numerical simulations is openly available online [1].

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Introduction

- Semiconductor technologies enabled the spectacular success of *classical* information processing devices.
- Can hope to make use of this technology for quantum systems.
- B. E. Kane, Nature (1998): use impurity atoms implanted in a pure silicon matrix as qubits
- Qubit-qubit interactions through direct contact interactions ➔ nanometer separation
- Need high precision of qubit locations and exquisitely small and precisely aligned electrode gates to modulate the interaction
- Since then: substantial theoretical and experimental progress
- *Main conflict*: want nanometer separation of qubits for interactions, but larger separation for control and addressing in a manufacturable and fault tolerant architecture.

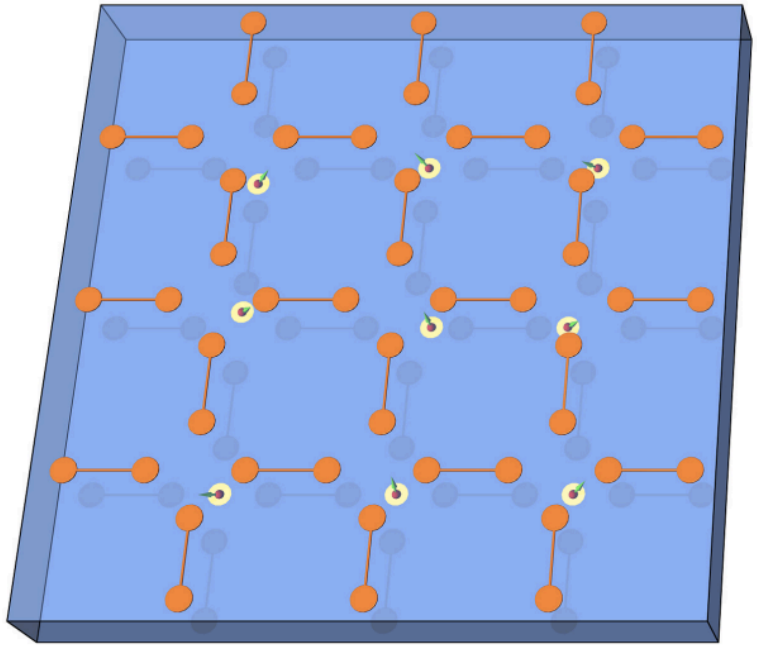
Surface codes

- Fault-tolerant, universal quantum computation with a 2D array of qubits
- Half of the qubits are *data qubits*, the other half are *syndrome qubits*
- For error correction, need parity measurements of 4 nearby data qubits
- Periodically perform entangling gates between data and syndrome qubits
- Error threshold of ~1% for i.i.d. errors

Long-distance entangling gates for spin qubits

Metallic floating gates
Trifunovic, Dial, Trif, Wooton, Abebe, Yacobi, Loss
PRX (2012)

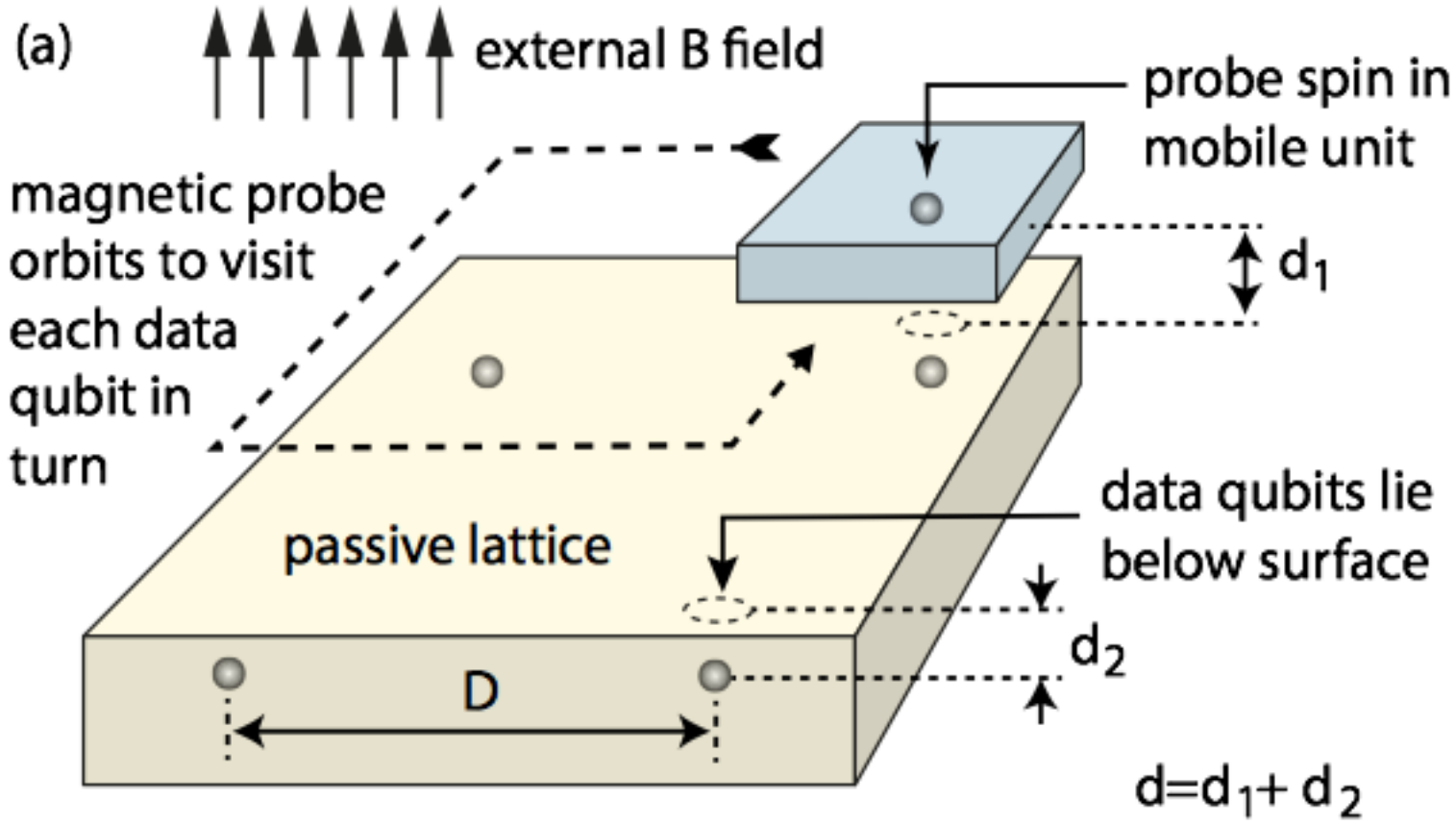
Ferromagnets
Trifunovic, Pedrocchi, Loss
PRX (2013)



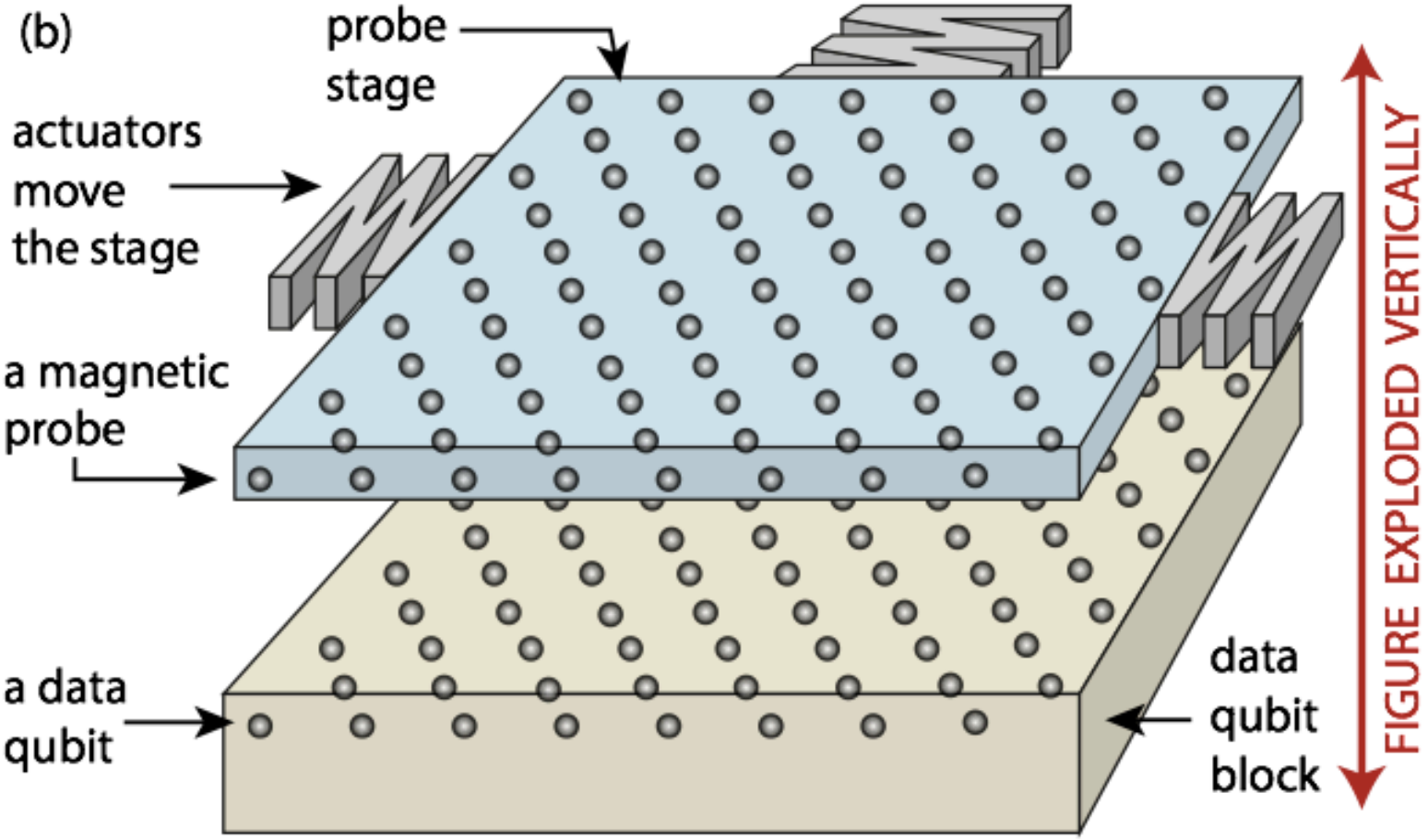
Their proposal

- Syndrome qubits are not static but periodically move close to the 4 adjacent data qubits
- Use long range dipole fields rather than contact interactions
  ➔ Allows to select the scale of the device according to technological abilities
- No need for direct gating between physical qubits; no need for extreme precision in the location of impurities
- Requires active control of only the electron spins, rather than the nuclear spins

Orbital probe parity measurement



A scalable device



Physical realization

- Qubits: electrons bound to isolated donor impurities in a semiconductor
- Measurement of the syndrome qubit: spin-to-charge mapping, or optically if the spin-bearing entity has conditional optical transitions (e.g. for NV-centers in diamond)
- Length scales: can be scaled up and down due to long-range nature of dipolar interaction; smaller systems increases engineering challenge but enhance operating speed
- Assume: 40nm minimal separation between probe and data qubit; 400nm separation between data qubits (direct interaction between them negligible)

Dipolar interaction

$$H_{2S} = \mu_B B(g_1\sigma_1^z + g_2\sigma_2^z) + \frac{J}{r^3}\left(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 - 3(\hat{\boldsymbol{r}}\cdot\boldsymbol{\sigma}_1)(\hat{\boldsymbol{r}}\cdot\boldsymbol{\sigma}_2)\right)$$

Assume that Zeeman energies differ strongly

$$\Delta = \mu_B B(g_1 - g_2) \gg J/r^3$$

E.g., use different spin species: data qubits are phosphorus atom in silicon, while probes are a different donor species or NV centers in diamond.

This prevents the spins from ‘flip-flopping’

Syndrome measurement

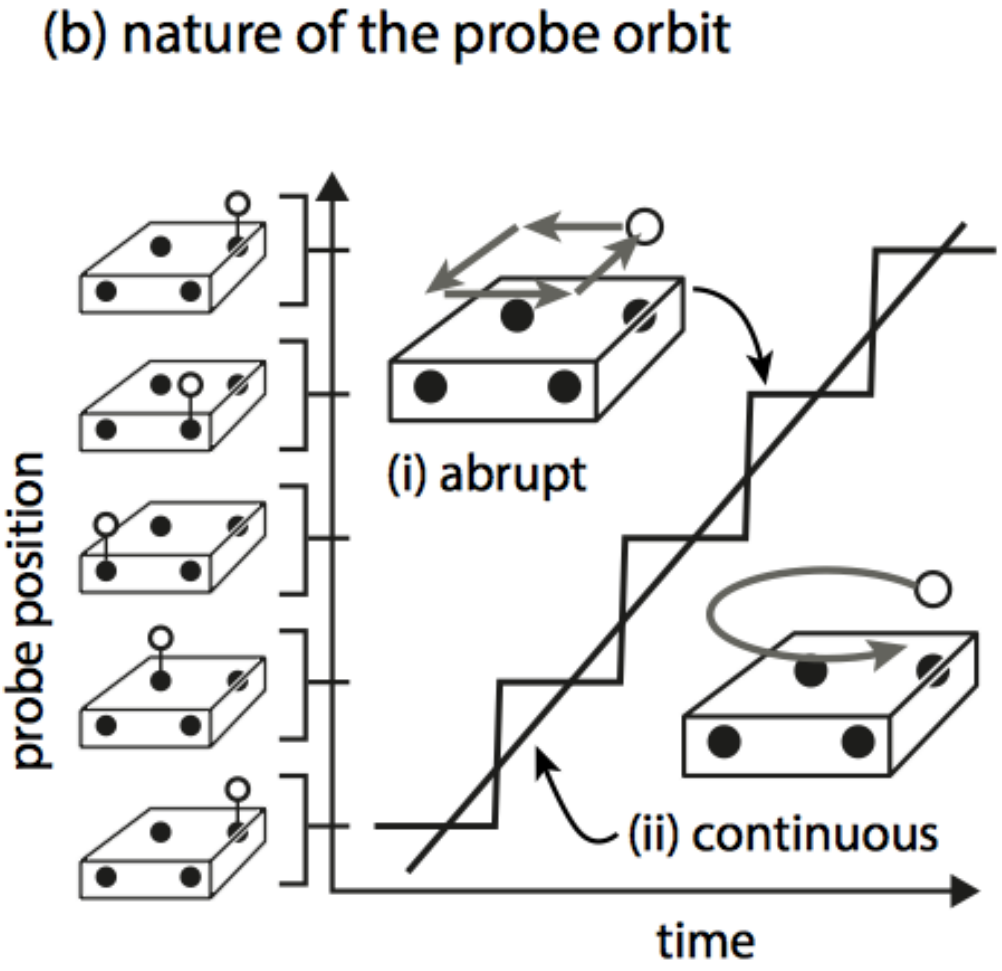
In a reference frame that subsumes that Zeeman evolution of the spins, the dipolar interaction takes the form

$$S(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \exp(i\theta) & 0 & 0 \\ 0 & 0 & \exp(i\theta) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Numerical simulation of the spin-spin dynamics shows that this expression is accurate up to errors of order 10^{-4} (negligible compared to other error sources).

Performing S(π/2) between the syndrome and each data qubit allows for four-qubit parity measurements.

Two ways to move the probe



Advantages of (ii):

- Easier to realize
- Less sensitive to dislocations (see later)

Disadvantages of (ii):

- Slower

Dislocation errors

Spins occupying positions that deviate from their ideal location lead to *systematic* errors. Such dislocations replace the ideal parity projector

$$\hat{P}_{\text{even}} = |0000\rangle\langle 0000| + |1100\rangle\langle 1100| + \ldots + |1111\rangle\langle 1111|$$

by

$$\begin{aligned}\hat{P}'_{\text{even}} = A\big(&|0000\rangle\langle 0000| \;+\; |1111\rangle\langle 1111|\big) + \\ B\big(&|0011\rangle\langle 0011| \;+\; |1100\rangle\langle 1100|\big) + \\ C\big(&|0110\rangle\langle 0110| \;+\; |1001\rangle\langle 1001|\big) + \\ D\big(&|0101\rangle\langle 0101| \;+\; |1010\rangle\langle 1010|\big) + \hat{W}\end{aligned}$$

W: projectors onto odd states

Dislocation errors

- The effects of systematic errors accumulate over time.
- In order to combat this effect, one can apply bit flips to randomly chose pairs of qubits before and after syndrome measurement. This permutes the weights A, B, C, D and allows to “smooth out” the irregularities.
- The bit flips can also be applied to the probe spin instead of the data qubits, which may be preferable in practice.

Error model

Infidelity in key operations	Simulation parameter employed	Experimental	
		Best reported	Ref
Probe preparation	1%	1%	[12]
Probe measures 0	9%	4.6%	[a]
Probe measures 1	1%	1.2%	[a]
Probe rotations	0.1%	0.07%	[14]
Probe 'jitter'	4%	–	–
Data qubit error	0.2%	0.05%+0.07%	[15][14]

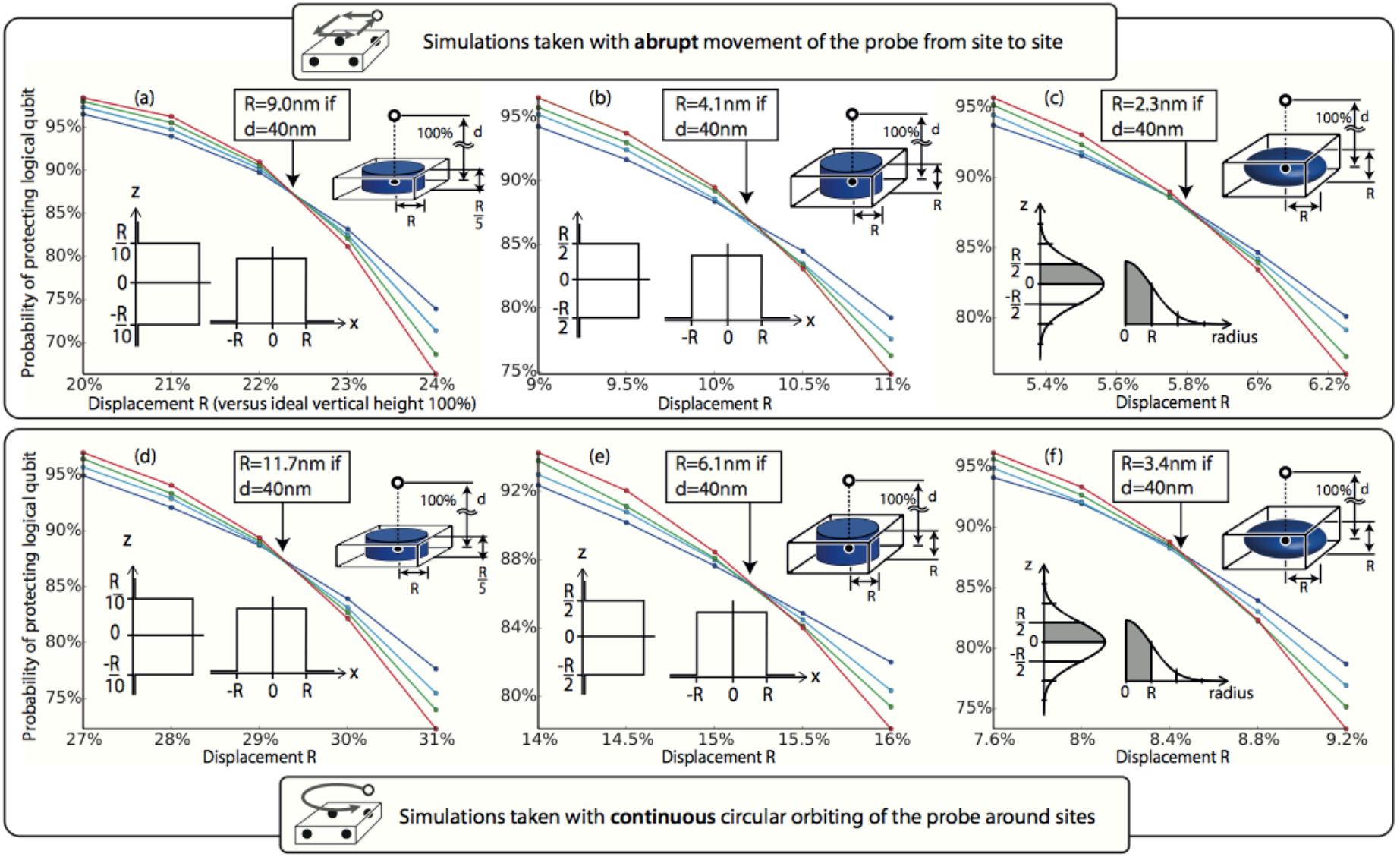
[a] Private communication with the author of [13]

'Jitter': random variation in dipolar coupling due to e.g. vibrations (i.e. deviations from $\theta=\pi/2$)

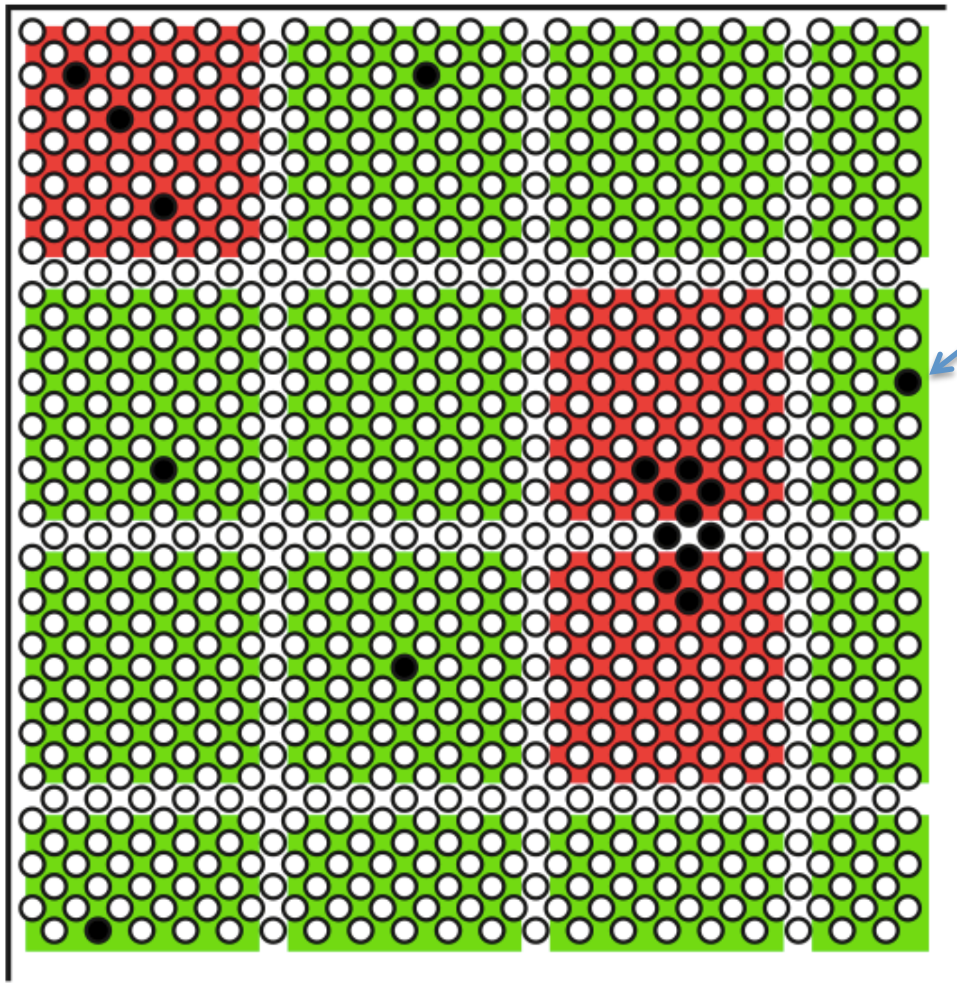
Numerical simulations

- Error rates are assumed to be fix. Find a threshold for how much data qubit locations can deviate from their ideal locations.
- Use a standard error correction algorithm (minimum-weight perfect matching).
- 50,000 numerical experiments per data point
- Simulate systems consisting of N = 221, 313, 421, 545 physical qubits

Results


Simulations taken with **abrupt** movement of the probe from site to site
(a)
R=9.0nm if d=40nm
(b)
R=4.1nm if d=40nm
(c)
R=2.3nm if d=40nm
Probability of protecting logical qubit
Displacement R (versus ideal vertical height 100%)
Displacement R
(d)
R=11.7nm if d=40nm
(e)
R=6.1nm if d=40nm
(f)
R=3.4nm if d=40nm
radius
Simulations taken with **continuous** circular orbiting of the probe around sites

Quantum computation: lattice surgery approach



Damaged/missing qubit

Timescales

- For a probe - data qubit separation of 40nm, a total interaction time of 1.2ms is required to acquire the appropriate phase.
- ➔ operations in the 1kHz regime for abrupt motion; slowdown of ~10 for continuous circular motion
- A hybrid approach may be optimal

Mechanics

- Prototypical system that enables such motions with sub-nanometer positional accuracy: tip of an atomic force microscope cantilever
- Could in principle use an array of tips on a single cantilever. However, scalability is challenging due to practical constraints such as height uniformity of the probe tips.
- More viable option: micro electromechanical systems (MEMS):
  - Typically manufactured from silicon-on-insulator wafers
  - Uniformity of oxide layer ➔ High homogeneity of the probe

Material systems: Silicon

- Silicon is the predominant material for the realization of MEMS devices
- Silicon can be isotopically purified to a high degree
- ➔ Electron spins of donor impurities (e.g. phosphorus) show extraordinarily long coherence times (up to 2s)
- ➔ Data qubit error rates <0.1% over one 1.2ms period
- Initialization, manipulation and read-out of the electron spin of single phosphorus impurities have been achieved with high fidelity

Material system: diamond nitrogen-vacancy centers

- Optically addressable spin states, which can be manipulated even at room temperature
- Coherence times are long enough (T_2 = 600ms at 77K)
- Yield of active NV centers per implanted nitrogen atom is <30%: improvements are necessary

Conclusions

- The authors have described a new scheme for implementing surface code quantum computing, base on an array of donor spins in silicon.
- Parity measurements can be achieved using continuous phase acquisition onto another ‘probe’ qubit, removing the challenging requirement for direct gating between physical qubits.
- Simulations show that this approach is extremely robust against deviations in the location of the qubits.
- Since the scheme is based on long range dipole interactions, the system is essentially scale independent.