# Signatures of Majorana Zero Modes in Spin-Resolved Current Correlations

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We consider a normal lead coupled to a Majorana bound state. We show that the spin-resolved current correlations exhibit unique features which distinguish Majorana bound states from other low-energy resonances. In particular, the spin-up and spin-down currents from a Majorana bound state are anti-correlated at low bias voltages, and become uncorrelated at higher voltages. This behavior is independent of the exact form of coupling to the lead, and of the direction of the spin polarization. In contrast, an ordinary low-energy Andreev bound state gives rise to a positive correlation between the spin-up and spin-down currents, and this spin-resolved current-current correlation approaches a non-zero constant at high bias voltages. We discuss experimental setups in which this effect can be measured.

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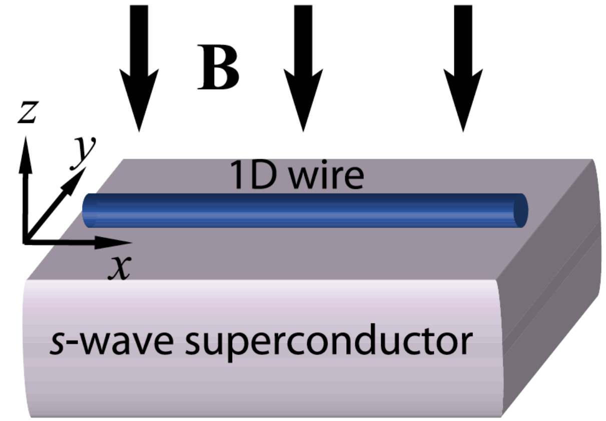
# Majorana fermions

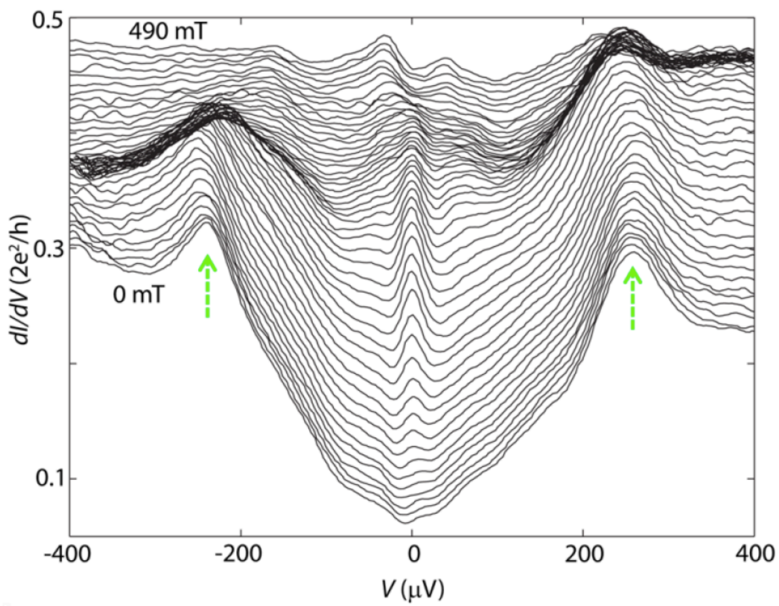
$$\gamma = \gamma^\dagger$$

- Non-local:
  immune to decoherence
- Non-abelian statistics:
  topological quantum computation




Setup to find
Majorana bound states (MBS)

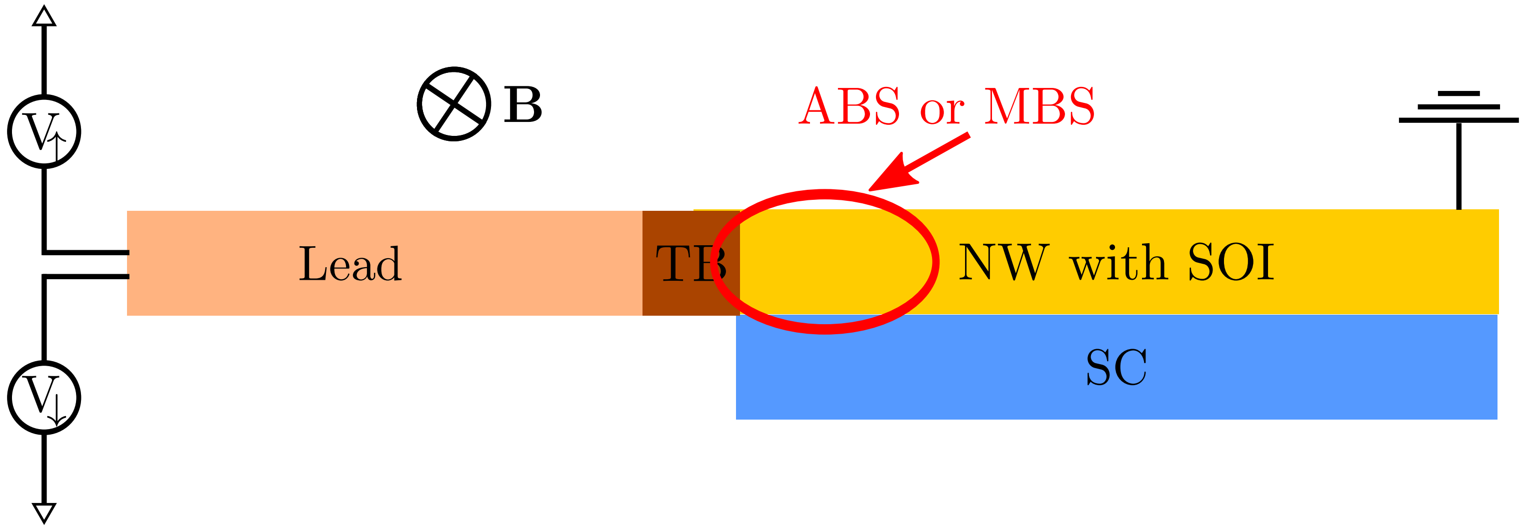
Lutchyn et al., PRL **105**, 077001 (2010)
Oreg et al., PRL **105**, 177002 (2010)
Alicea, PR B **81**, 125318 (2010)




Experimental results were not conclusive:
Disorder, Kondo, smooth confinement,
Andreev bound states (ABS)

Mourik *et al.*, Science **336**, 1003 (2012)
Deng *et al.*, Nano Lett. **12**, 6414 (2012)
Das *et al.*, Nat. Phys. **8**, 887 (2012)
Churchill *et al.*, PRB **87**, 241401 (2013)
Finck *et al.*, PRL **110**, 126406 (2013)

# Setup




Measure spin-resolved current correlations:

$$P_{ss'} = \int_{-\infty}^{\infty} dt \langle \delta \hat{I}_s(0) \delta \hat{I}_{s'}(t) \rangle$$

$$\delta \hat{I}_s = \hat{I}_s - \langle \hat{I}_s \rangle$$

$$s = \uparrow, \downarrow$$

# Simple model - MBS

$$H = H_L + H_T$$

$$H_L = \sum_{k,s} \epsilon_k \psi^\dagger_{ks} \psi_{ks}$$

$$H_T = i\gamma \cdot \sum_{k,s} \left( t_s \psi_{ks} + t_s^* \psi^\dagger_{ks} \right)$$

lead modes

Majorana fermion

Coupling of leads to MBS

Scattering matrix yields correlations

$$P_{\uparrow\downarrow} = -\frac{2e^2}{h} \Gamma_\uparrow \Gamma_\downarrow \frac{eV}{(eV)^2 + \Gamma^2}$$

$$P_{\uparrow\downarrow} \leq 0$$
$$P_{\uparrow\downarrow} \propto 1/V$$

$$\Gamma_s = 2\pi\nu_0 |t_s|^2$$
$$\Gamma = 2\pi\nu_0 (|t_\uparrow|^2 + |t_\downarrow|^2)$$

# Simple model - ABS

$$H = H_L + \tilde{H}_T$$

$$H_L = \sum_{k,s} \epsilon_k \psi^\dagger_{ks} \psi_{ks}$$

lead modes

$$\tilde{H}_T = a^\dagger \sum_k \left( \tilde{t}_\uparrow \psi_{k\uparrow} + \tilde{t}_\downarrow \psi^\dagger_{k\downarrow} \right) + \mathrm{H.c.}$$

Creation operator ABS

Coupling of leads to ABS

Scattering matrix yields correlations

$$P_{\uparrow\downarrow} = \frac{2e^2}{h} \frac{\tilde{\Gamma}_\uparrow \tilde{\Gamma}_\downarrow}{\tilde{\Gamma}} \left\{ \left[ \frac{(\tilde{\Gamma}_\uparrow - \tilde{\Gamma}_\downarrow)^2}{\tilde{\Gamma}^2} + \cos^2\theta \right] \arctan \frac{2eV}{\tilde{\Gamma}} + \left[ \frac{(\tilde{\Gamma}_\uparrow - \tilde{\Gamma}_\downarrow)^2}{\tilde{\Gamma}^2} - \cos^2\theta \right] \frac{2eV/\tilde{\Gamma}}{1 + (2eV/\tilde{\Gamma})^2} \right\}$$

$$P_{\uparrow\downarrow} \geq 0$$
$$P_{\uparrow\downarrow} \to \mathrm{const.}$$

$$\tilde{\Gamma}_s = 2\pi\nu_0 |\tilde{t}_s|^2$$
$$\tilde{\Gamma} = 2\pi\nu_0 (|\tilde{t}_\uparrow|^2 + |\tilde{t}_\downarrow|^2)$$

# Microscopic model

$$H = H_L + H_{\rm nw} + H_T$$

$$H_L = \sum_{k,s} \epsilon_k \psi^\dagger_{ks}\psi_{ks}$$

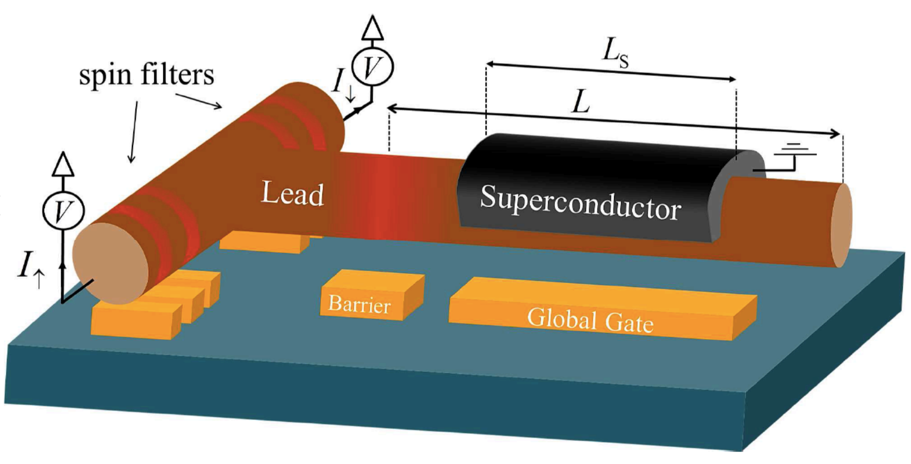
SOI Zeeman

$$\tilde{H}_{\rm nw} = \int_{-L/2}^{L/2} dx \Phi^\dagger(x) \left[ \left( \frac{-\partial_x^2}{2m_e} - \mu \right) \tau^z + i\alpha_R \tau^z \sigma^z \partial_x + \boldsymbol{B}\cdot\boldsymbol{\sigma} + \Delta(x)\tau^x \right] \Phi(x)$$

$$H_T = -\sum_{k,p,s} t_{kp} \phi^\dagger_{ps}\psi_{ks} + {\rm H.c.}$$

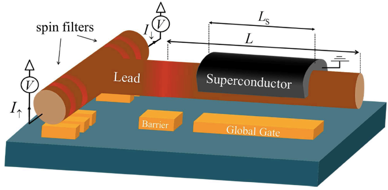
$$\Delta(x) = \Delta_0 \Theta(L_s/2 - |x|)$$

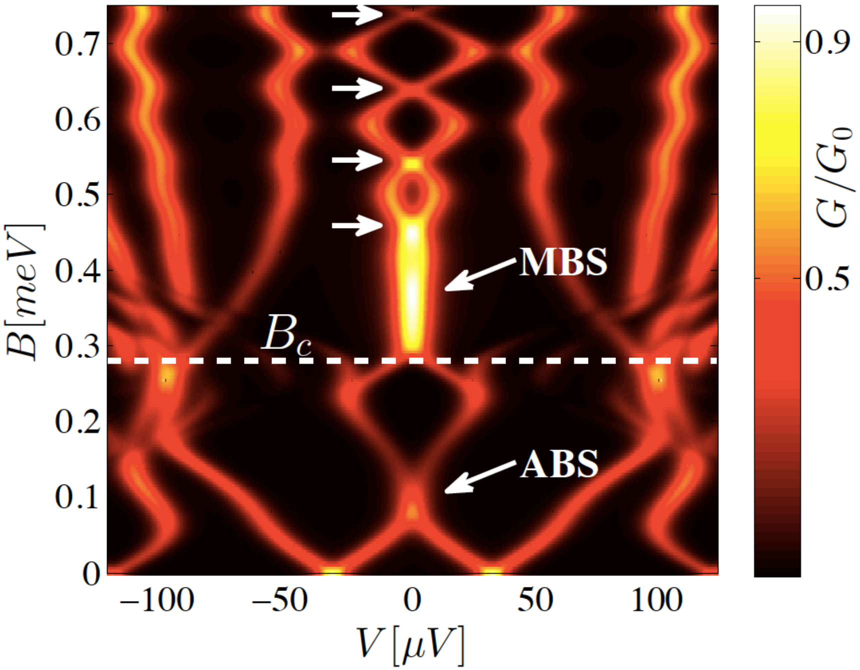
- Discretize to obtain tight-binding Hamiltonian of 1D lattice
- Parameters consistent with Delft experiment




$$\Phi^\dagger = (\phi^\dagger_\uparrow, \phi^\dagger_\downarrow, \phi_\downarrow, -\phi_\uparrow)$$

# Differential conductance & LDOS

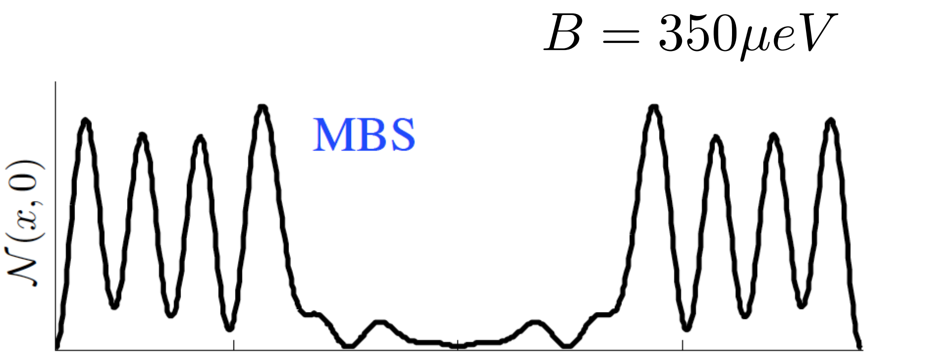


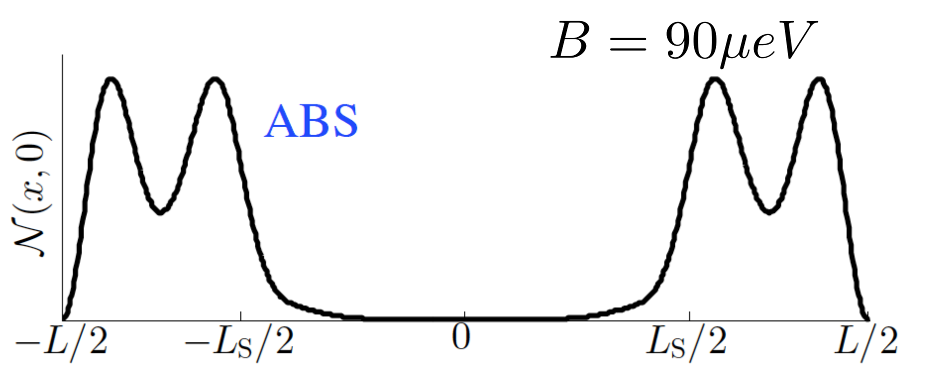




Oscillations due to MBS hybridization

$$T = 30mK$$

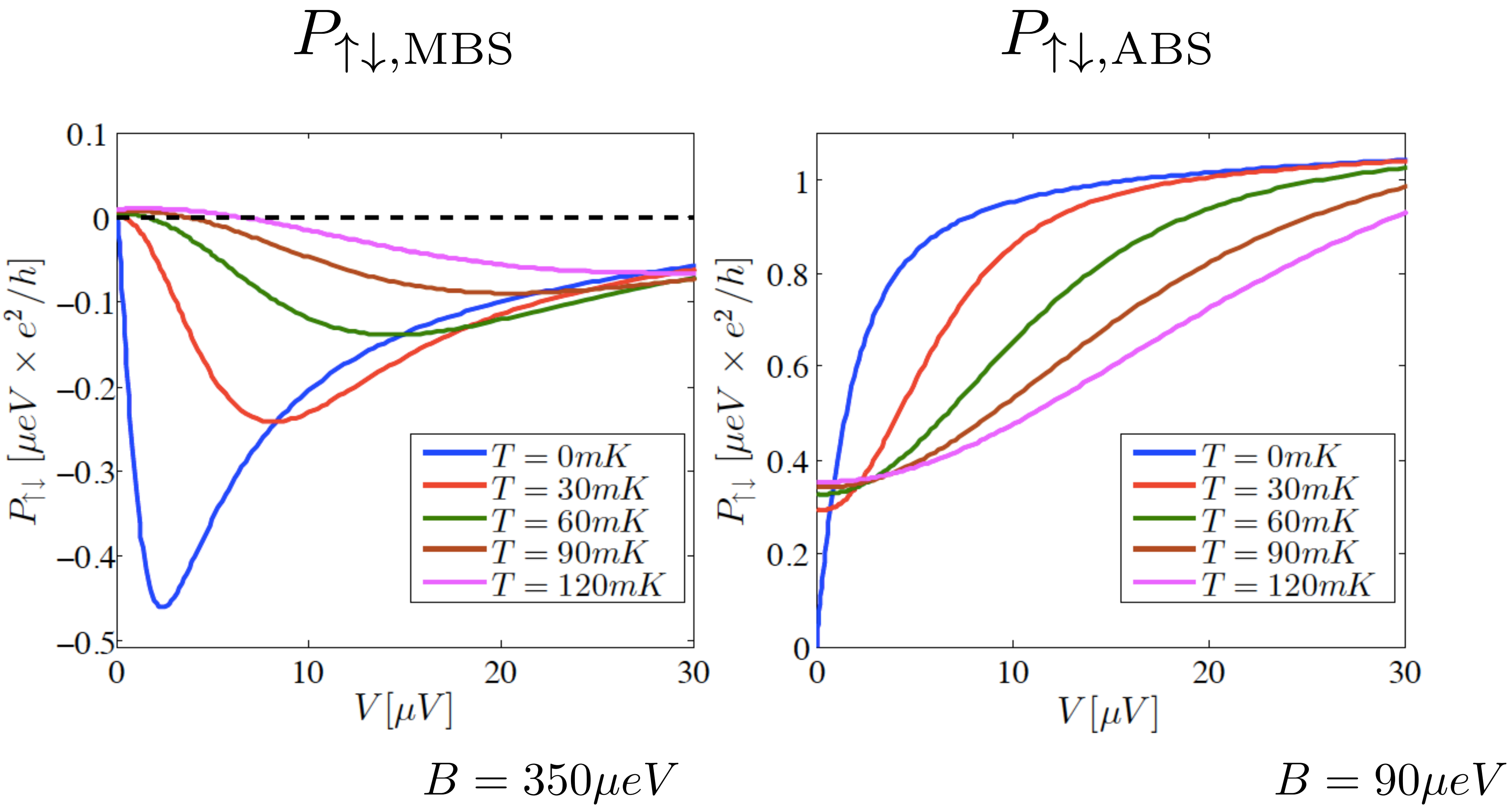
$$B_c = \sqrt{\mu^2 + \Delta_0^2}$$



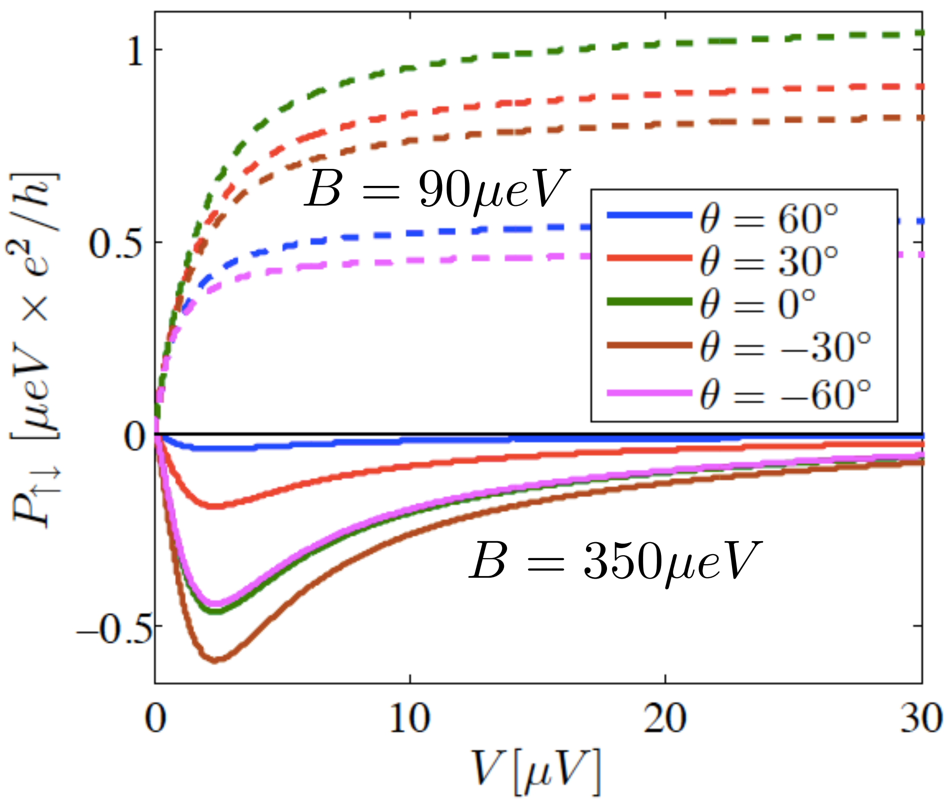

Local density of states:
STM measurement not conclusive

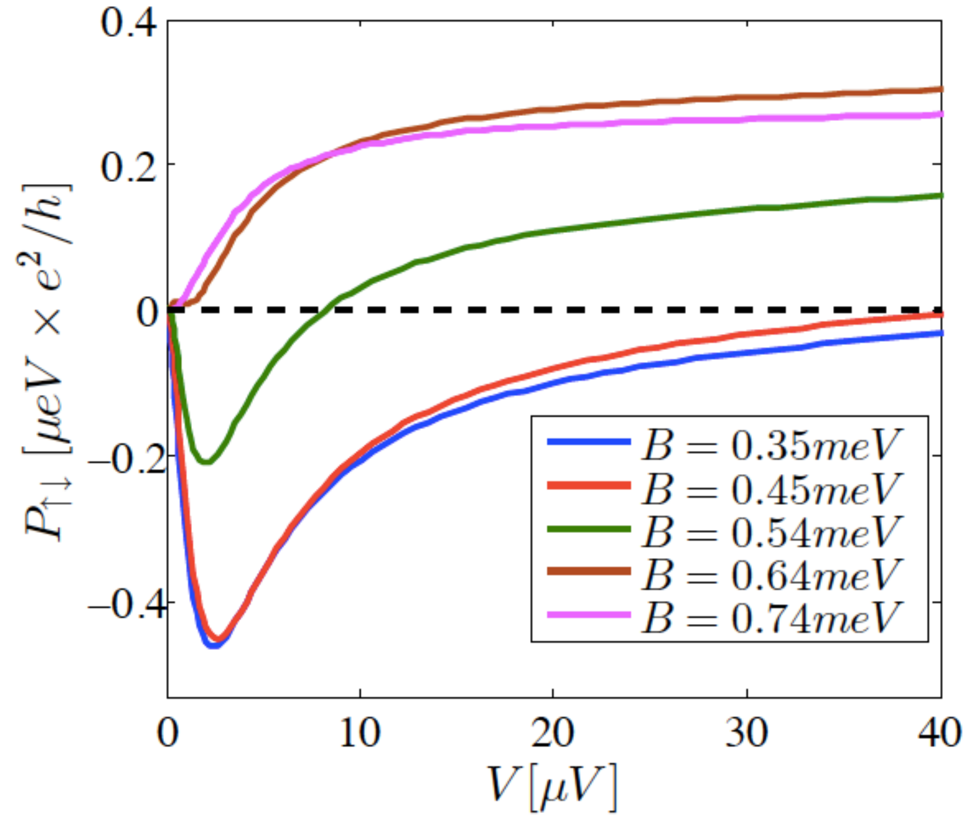
# Current correlations




Reproduces behavior predicted by simple model

# Current correlations



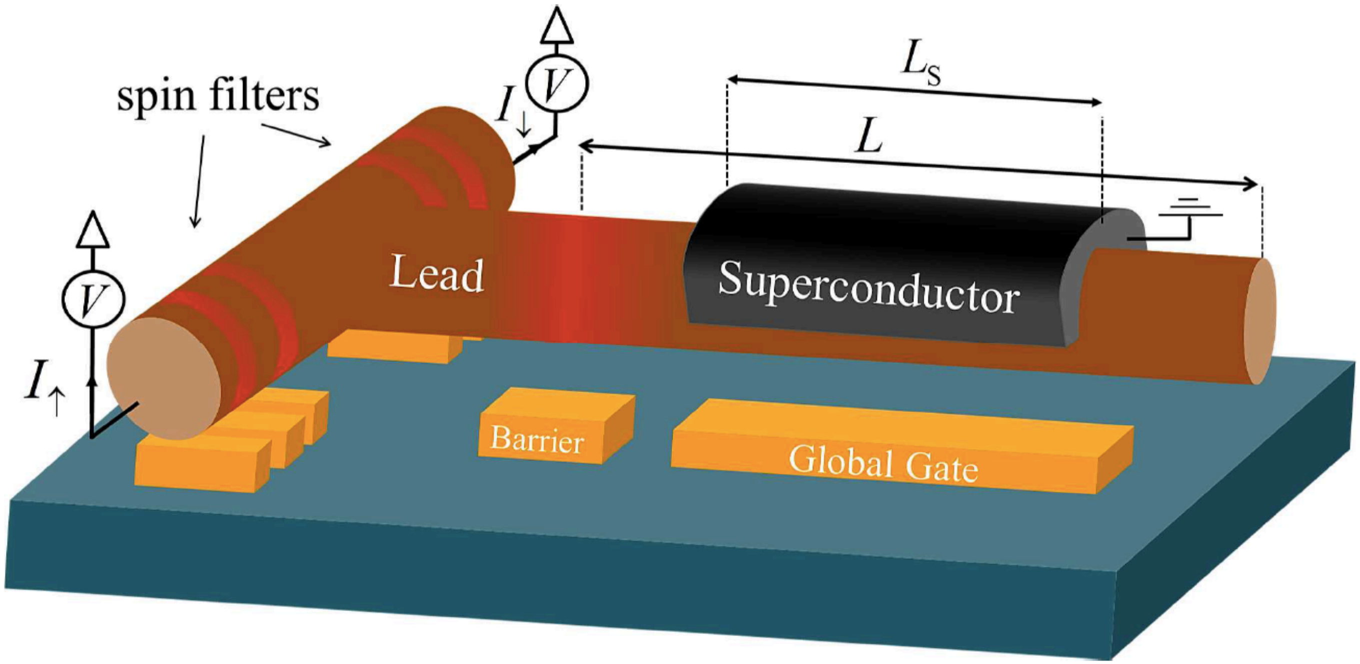




$$\boldsymbol{B} = B(\sin\theta, 0, \cos\theta)$$

Crossover MBS to ABS

$$T = 0$$

# Experimental realization




- T-junction of NWs
- Gate-defined QDs act as spin-filters
- Tune spin filters to opposite resonance to measure $P_{\uparrow\downarrow}$

# Conclusion

- Spin resolved current correlation
  with unique signatures of MBS
- Explicitly distinguish signatures of ABS and MBS
- Other effects are expected to yield qualitatively similar results

# App: Scattering matrix

$$\begin{pmatrix} r^{ee} & r^{eh} \\ r^{he} & r^{hh} \end{pmatrix} = 1 - 2\pi i W^\dagger (E + i\pi W W^\dagger)^{-1} W$$

MBS

$$W = \sqrt{\nu_0}(t_\uparrow, t_\downarrow, t_\uparrow^*, t_\downarrow^*)$$

$$r_{ss'}^{ee} = \delta_{ss'} + \frac{2\pi\nu_0 t_s^* t_{s'}}{iE - \Gamma}$$

$$r_{ss'}^{he} = \frac{2\pi\nu_0 t_s t_{s'}}{iE - \Gamma}$$

$$r^{hh}(E) = r^{ee}(-E)^*$$

$$r^{eh}(E) = r^{he}(-E)^*$$

ABS

$$W = \sqrt{\nu_0}\begin{pmatrix} \tilde{t}_\uparrow & 0 & 0 & \tilde{t}_\downarrow^* \\ 0 & \tilde{t}_\downarrow & \tilde{t}_\uparrow^* & 0 \end{pmatrix}$$

$$r^{ee} = \frac{iE}{iE - \tilde{\Gamma}/2} + \frac{(\tilde{\Gamma}_\uparrow - \tilde{\Gamma}_\downarrow)/2}{iE - \tilde{\Gamma}/2}\sigma^z$$

$$r^{he} = \frac{2\pi\nu_0 \tilde{t}_\uparrow \tilde{t}_\downarrow}{iE - \tilde{\Gamma}/2}\sigma^x$$

$$\Gamma = 2\pi\nu_0(|t_\uparrow|^2 + |t_\downarrow|^2)$$

$$\Gamma_s = 2\pi\nu_0 |t_s|^2$$

# App: Current and current correlation

$$\langle \hat{I}_s \rangle = \frac{e}{h} \sum_{\substack{s' \in \uparrow,\downarrow \\ \alpha,\beta \in e,h}} \mathrm{sgn}(\alpha) \int dE A^{\beta\beta}_{s's'}(s,\alpha;E) f_\beta(E)$$

$$P_{ss'} = \frac{e^2}{h} \sum_{\substack{\sigma\sigma' \in \uparrow,\downarrow \\ \alpha,\beta,\gamma\delta \in e,h}} \mathrm{sgn}(\alpha)\mathrm{sgn}(\beta) \int dE A^{\gamma\delta}_{\sigma\sigma'}(s,\alpha;E) A^{\delta\gamma}_{\sigma'\sigma}(s',\beta;E) f_\gamma(E)[1-f_\delta(E)]$$

$$A^{\gamma\delta}_{\sigma\sigma'}(s,\alpha;E) = \delta_{s\sigma}\delta_{s\sigma'}\delta_{\alpha\gamma}\delta_{\alpha\delta} - [r^{\alpha\gamma}_{s\sigma}]^* r^{\alpha\delta}_{s\sigma'}$$

$$f_e(E) = 1 - f_h(-E)$$

# Model parameters

$$\Delta_0 = 250\mu eV$$

$$E_{\text{so}} = m_e \alpha_R^2 / 2 = 50\mu eV$$

$$L = 2.5\mu m,\ L_S = 1.4\mu m$$

$$a = L/N$$

$$E_{\text{so}} = u^2/t$$

$$l_{\text{so}} = ta/u$$

$$4t = 40\Delta_0$$