Tunneling dynamics of bosonic Josephson junctions assisted by a cavity field

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Short: The many-boson Schrödinger equation in a double well as a non-rigid pendulum with only 4 dynamical variables!?

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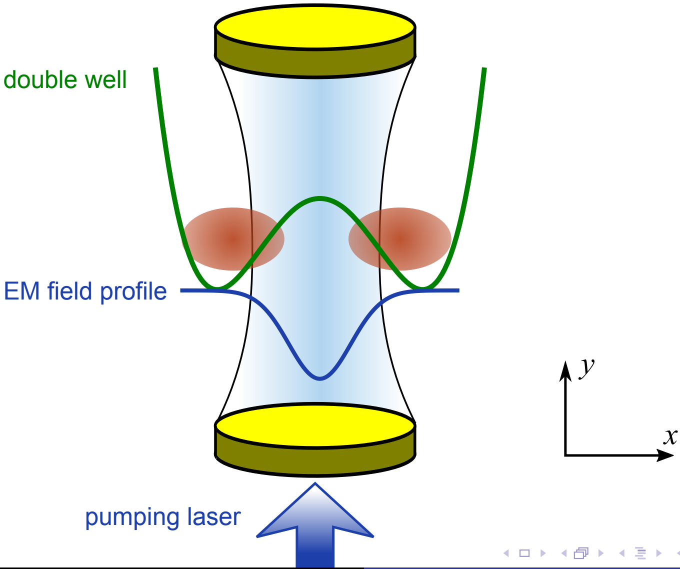
We study the interplay between the dynamics of a Bose-Einstein condensate in a double-well potential and that of an optical cavity mode. The cavity field is superimposed to the double-well potential and affects the atomic tunneling processes. The cavity field is driven by a laser red detuned from the bare cavity resonance; the dynamically changing spatial distribution of the atoms can shift the cavity in and out of resonance. At resonance the photon number is hugely enhanced and the atomic tunneling becomes amplified. The Josephson junction equations are revisited and the phase diagram is calculated. We find new solutions with finite imbalance and at the same time a lack of self-trapping solutions due to the emergence of a new separatrix resulting from enhanced tunneling.

Setup



Hamiltonian I

Total Hamiltonian:

$$\hat{H} = \hat{H}_L + \hat{H}_J + \hat{H}_{JL}$$

Photons in the cavity:

$$\hat{H}_L = -\hbar\Delta_C\, \hat{a}^\dagger \hat{a} - i\hbar\eta \left(\hat{a} - \hat{a}^\dagger\right)$$

Cavity-laser-detuning $\Delta_C = \omega_L - \omega_C$, laser intensity $\eta > 0$.
BJJ in Bose-Hubbard-approximation:

$$\hat{H}_J = \epsilon\, \hat{N}_A - J\left(\hat{b}_1^\dagger \hat{b}_2 + \hat{b}_2^\dagger \hat{b}_1\right) + \frac{U}{2}\left(\hat{b}_1^\dagger \hat{b}_1^\dagger \hat{b}_1 \hat{b}_1 + \hat{b}_2^\dagger \hat{b}_2^\dagger \hat{b}_2 \hat{b}_2\right)$$

Onsite interaction U and tunneling J.

Hamiltonian II

Photon-atom scattering (dispersive):

$$\hat{H}_{JL} = \hat{N}_L \left[W_0 \hat{N}_A + W_{12}(\hat{b}_1^\dagger \hat{b}_2 + \hat{b}_2^\dagger \hat{b}_1) \right]$$

Photon number $\hat{N}_L = \hat{a}^\dagger \hat{a}$, AC-Stark shift W_0, cavity-assisted tunneling W_{12}

Heisenberg equations of motion

$$
\begin{aligned}
i\hbar \frac{d}{dt}\hat{b}_1 &= W_0\, \hat{a}^\dagger \hat{a}\, \hat{b}_1 - (J - W_{12}\, \hat{a}^\dagger \hat{a})\hat{b}_2 + U\, \hat{n}_1 \hat{b}_1, \\
i\hbar \frac{d}{dt}\hat{b}_2 &= W_0\, \hat{a}^\dagger \hat{a}\, \hat{b}_2 - (J - W_{12}\, \hat{a}^\dagger \hat{a})\hat{b}_1 + U\, \hat{n}_2 \hat{b}_2, \\
i\hbar \frac{d}{dt}\hat{a} &= -[\hbar\Delta_C - W_0 \hat{N}_A - W_{12}(\hat{b}_1^\dagger \hat{b}_2 + \hat{b}_2^\dagger \hat{b}_1)]\hat{a} + i\hbar\eta,
\end{aligned}
$$

Mean-field approximation (I)

A long story cut short:

$$\hat{\Psi}^\dagger = \sum_{k=1}^{\infty} \hat{b}_k^\dagger(t) w(x) \underbrace{\longrightarrow}_{Bose-Hubbard} \hat{\Psi}^\dagger = \sum_{k=1}^{2} \hat{b}_k^\dagger(t) w(x)$$

$$\underbrace{\longrightarrow}_{Mean-field} \hat{\Psi}^\dagger = \hat{b}^\dagger(t) FCS(z, \theta, \xi, \Phi)$$

Assumption: full coherent state (FCS)

$$|FCS\rangle = |\beta_1\rangle_A \otimes |\beta_2\rangle_A \otimes |\alpha\rangle_L \quad (1)$$

Operators in Heisenberg equations replaced by their eigenvalues.
$\hat{b}_j|\beta_j\rangle_A = \beta_j|\beta_j\rangle_A$, $\beta_j = \sqrt{N_j(t)}\, e^{i\theta_j(t)}$
$N_j(t)$ average number of atoms in jth with phase $\theta_j(t)$
$\hat{a}|\alpha\rangle_L = \alpha|\alpha\rangle_L$, where $\alpha = \xi(t)\, e^{i\phi(t)}$, $N(t) = \xi(t)^2$ the average number of cavity photons with phase $\phi(t)$

Mean-field approximation (II)

Population imbalance: $z(t) = (N_1(t) - N_2(t))/N_A$; Relative phase $\theta(t) = \theta_2(t) - \theta_1(t)$; Cavity population $\phi(t)$; Cavity phase $\phi(t)$.

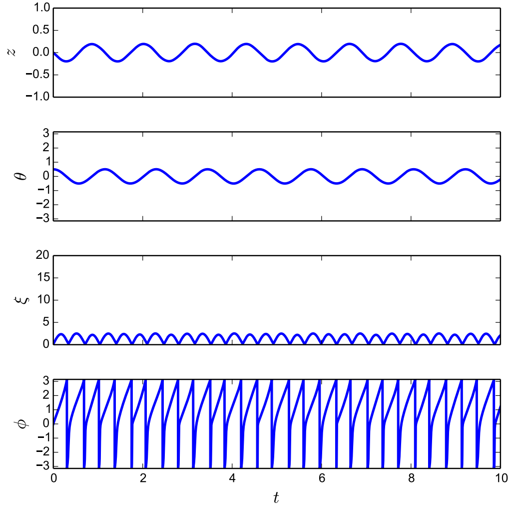
$$
\begin{aligned}
\dot{z} &= -2\nu\sqrt{1-z^2}\sin\theta; \qquad \nu = (J - W_{12}\xi^2)/\hbar \\
\dot{\theta} &= \left(\tilde{g} + \frac{2\,\nu}{\sqrt{1-z^2}}\cos\theta\right) z; \qquad \tilde{g} = UN_A/\hbar \\
\dot{\xi} &= \eta\,\cos\phi \\
\dot{\phi} &= \delta_C - \frac{\eta}{\xi}\sin\phi; \qquad \delta_C = \Delta_C - N_A(W_0 + W_{12}\sqrt{1-z^2}cos\theta)/\hbar
\end{aligned}
$$

Fixed point analysis: zero imbalance $z = 0$ (I)

$\dot{\theta} = 0$ (time-independent phase)
$\dot{z} = 0$ implies $\theta = 0$, or $\theta = \pi$
$\dot{\xi} = 0$ implies $\phi = \pm\pi/2$
From $\dot{\phi} = 0$ (time-independent cavity phase) yields cavity population ξ
Notation: $\mathbf{X} = (z, \theta, \xi, \phi)$

$$
\begin{aligned}
\mathbf{X}_1 &= \left(0, 0, \frac{\hbar\eta}{\hbar\Delta_C - N_A(W_0 + W_{12})}, \frac{\pi}{2}\right), \\
\mathbf{X}_2 &= \left(0, 0, \frac{\hbar\eta}{N_A(W_0 + W_{12}) - \hbar\Delta_C}, -\frac{\pi}{2}\right), \\
\mathbf{X}_3 &= \left(0, \pi, \frac{\hbar\eta}{\hbar\Delta_C - N_A(W_0 - W_{12})}, \frac{\pi}{2}\right), \\
\mathbf{X}_4 &= \left(0, \pi, \frac{\hbar\eta}{N_A(W_0 - W_{12}) - \hbar\Delta_C}, -\frac{\pi}{2}\right).
\end{aligned}
$$

Fixed point analysis: zero imbalance (II)



$\mathbf{X}(t = 0) = (0, 0.5, 0, 0)$

$\hbar\Delta_C = -100J$
$W_0 N_A = -90J$
$W_{12} N_A = -30J$
$U N_A = 12J$
$N_A = 1000$
$\hbar\eta = 20J$

Fixed point analysis: finite imbalance equilibrium, $z \neq 0$ (I)

$\dot{\theta} = 0$ (time-independent phase); $\dot{z} = 0$ implies $\theta = 0$, or $\theta = \pi$
$\dot{\xi} = 0$ implies $\phi = \pm\pi/2$; $\dot{\phi} = 0$ yields cavity population ξ

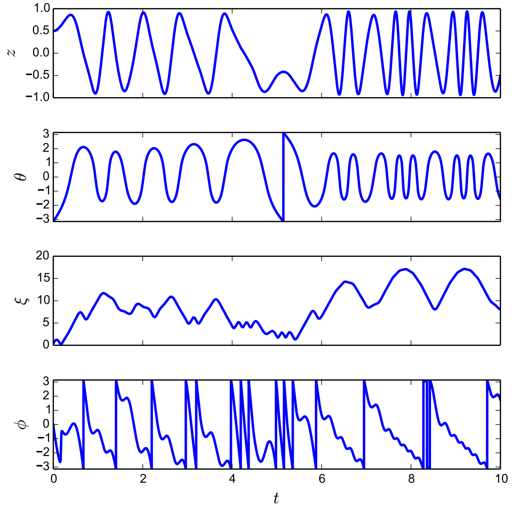
Notation: $\mathbf{X} = (z, \theta, \xi, \phi)$ and $\bar{z} = \pm\sqrt{1 - \left(\frac{2(J - W_{12}\,\bar{\xi}^2)}{UN_A}\right)^2}$.

$$\mathbf{X}_5 = \left(\bar{z}, 0, \frac{\hbar\eta}{\hbar\,\Delta_C - N_A\left(W_0 + W_{12}\sqrt{1-\bar{z}^2}\right)}, \frac{\pi}{2}\right),$$

$$\mathbf{X}_6 = \left(\bar{z}, 0, \frac{\hbar\eta}{N_A\left(W_0 + W_{12}\sqrt{1-\bar{z}^2}\right) - \hbar\,\Delta_C}, -\frac{\pi}{2}\right),$$

$$\mathbf{X}_7 = \left(\bar{z}, \pi, \frac{\hbar\eta}{\hbar\,\Delta_C - N_A\left(W_0 - W_{12}\sqrt{1-\bar{z}^2}\right)}, \frac{\pi}{2}\right),$$

$$\mathbf{X}_8 = \left(\bar{z}, \pi, \frac{\hbar\eta}{N_A\left(W_0 - W_{12}\sqrt{1-\bar{z}^2}\right) - \hbar\,\Delta_C}, -\frac{\pi}{2}\right).$$



$\mathbf{X}(t = 0) = (0.5, -\pi, 0, 0)$

$\hbar\Delta_C = -100J$

$W_0 N_A = -90J$

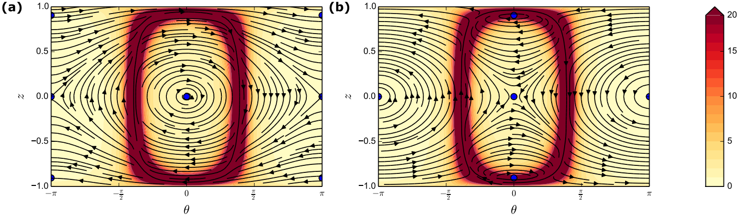
$W_{12} N_A = -30J$

$UN_A = 12J$

$N_A = 1000$

$\hbar\eta = 20J$

Phase space from adiabatic elimination of cavity dynamics



Assume detuning is much larger than tunneling: $\hbar\delta_C \gg \nu$
Average number of cavity photons: $\bar{\xi} = \frac{\eta}{|\delta_C|}$ Effective ODEs:

$$\begin{aligned} \dot{z} &= -2\bar{\nu}\sqrt{1-z^2}\sin\theta, \\ \dot{\theta} &= \left(\tilde{g} + \frac{2\,\bar{\nu}}{\sqrt{1-z^2}}\cos\theta\right) z. \end{aligned}$$

Here, $\bar{\nu} = (J - W_{12}\bar{\xi}^2) = (J - W_{12}\eta^2/\delta_C^2)$, the colors represent $\bar{\xi}^2$

- Addition of cavity field to the BJJ adds another time-scale to the dynamics, similar to adding a second mass
- Cavity resonance changes self-trapping to "anti-self-trapped" dynamics
- Mean-field overstrained? Full $N = 1000$ dynamics really covered by only 4 (!) parameters?! From the abstract of PRA **89**, 023602 (Jul 4, 2012 – Feb 5, 2014), which analyzes the BJJ with a general many-body treatment:
  "Even for arbitrarily large particle numbers and arbitrarily weak interaction strength the dynamics is many-body in nature and the fragmentation universal. There is no weakly interacting limit where the Gross-Pitaevskii mean field is valid for long times."

Acknowledgement

Thank you for your attention!

Further questions?