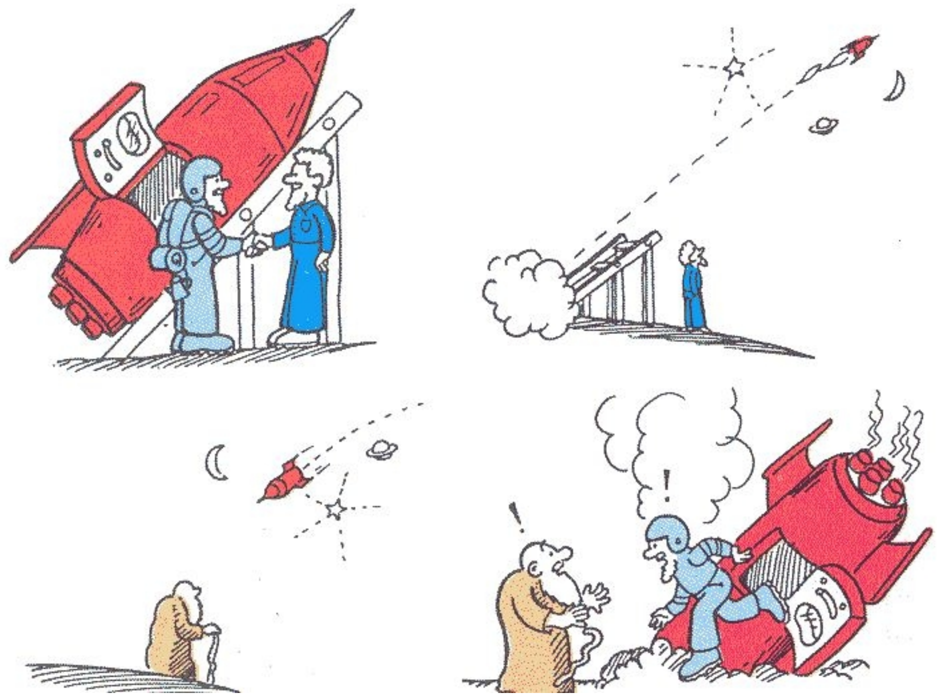
Twin paradox with macroscopic clocks in superconducting circuits

Phys. Rev. A 78, (2014)

Joel Lindkvist, Carlos Sabin, Ivette Fuentes, Andrzej Dragan, Ida-Maria Svensson, Per Delsing and Göran Johansson

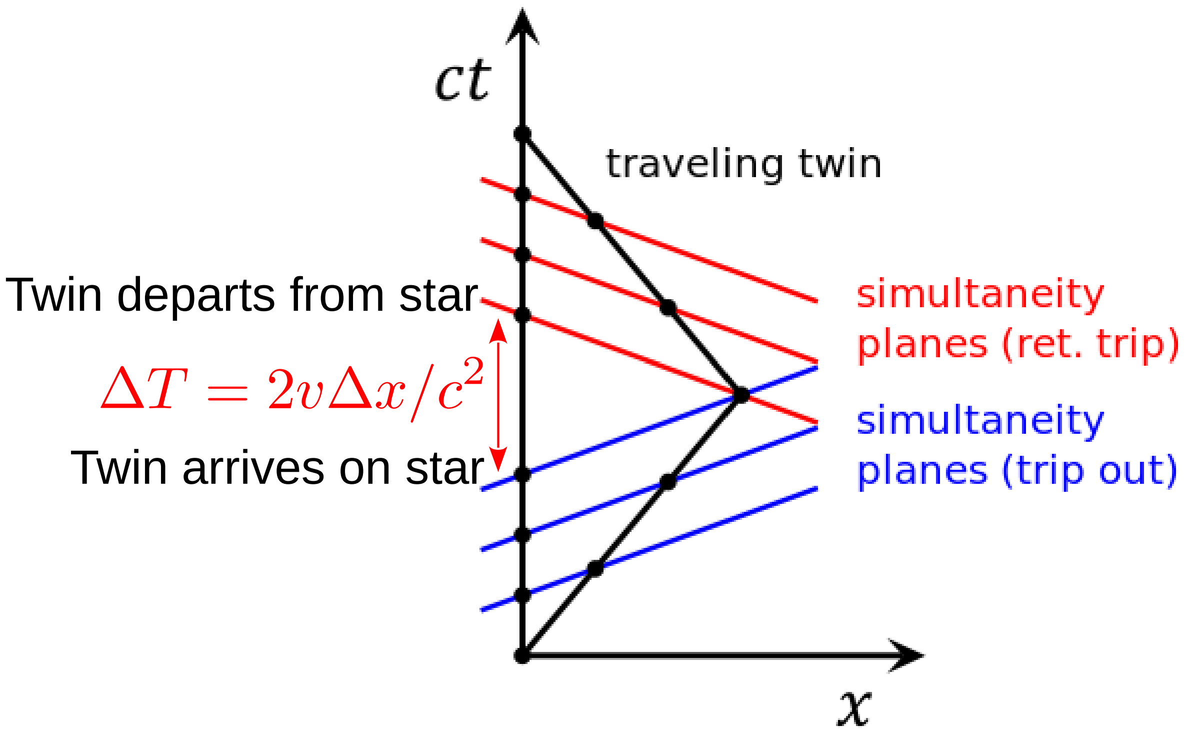


Journal club presented by Simon Nigg

University of Basel, January 13th 2015

The twin paradox (reminder)

inertial reference frame of earth-bound twin



Simultaneity in moving (inertial) frame:

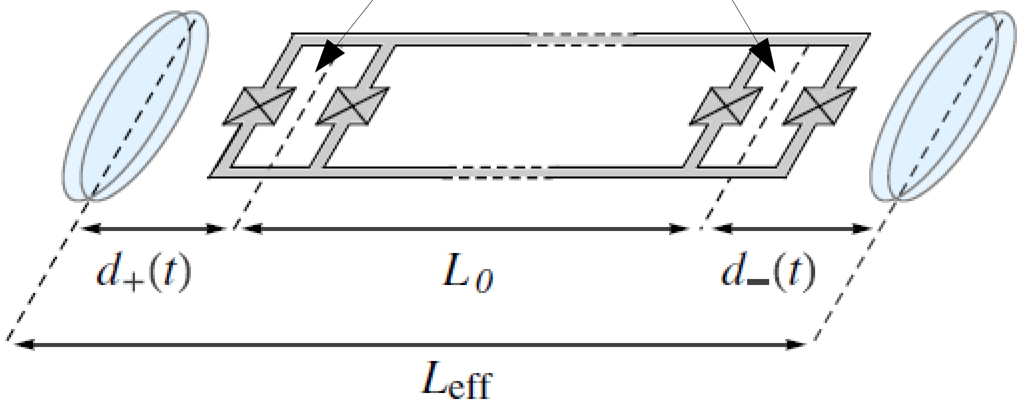
$$t' = \frac{t \pm v\frac{x}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} = \text{const.} \Leftrightarrow ct \pm vx/c = \text{const.}$$

Simulating a moving clock

Tunable superconducting **transmission line resonator**

Clock = coherent state in fundamental mode

Flux tunable SQUIDS = variable inductors



Friis et al., PRL 110, 113602 (2013)

Shaking cavity of constant length: $d_+(t) + d_-(t) = \mathrm{const.}$

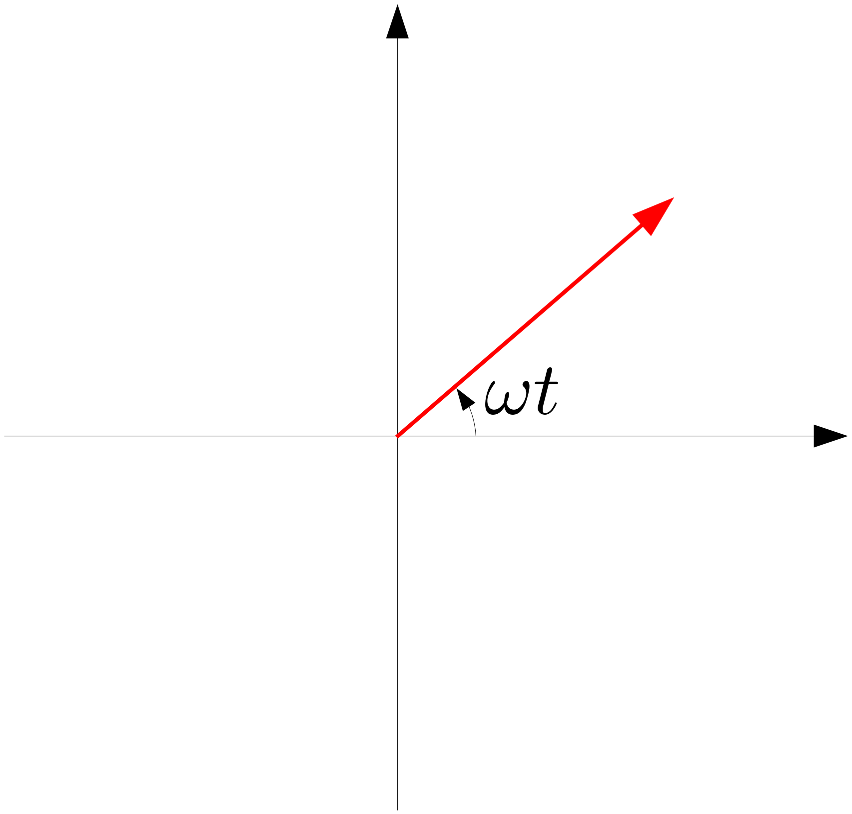
cavity loss rate

$d_\pm \ll L_0 \Rightarrow$ repeat many times! $N_{\mathrm{rep}} \sim \kappa^{-1}$

Clock hand = coherent state amplitude

$$|\psi(0)\rangle = |\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle \qquad H = \omega a^\dagger a$$

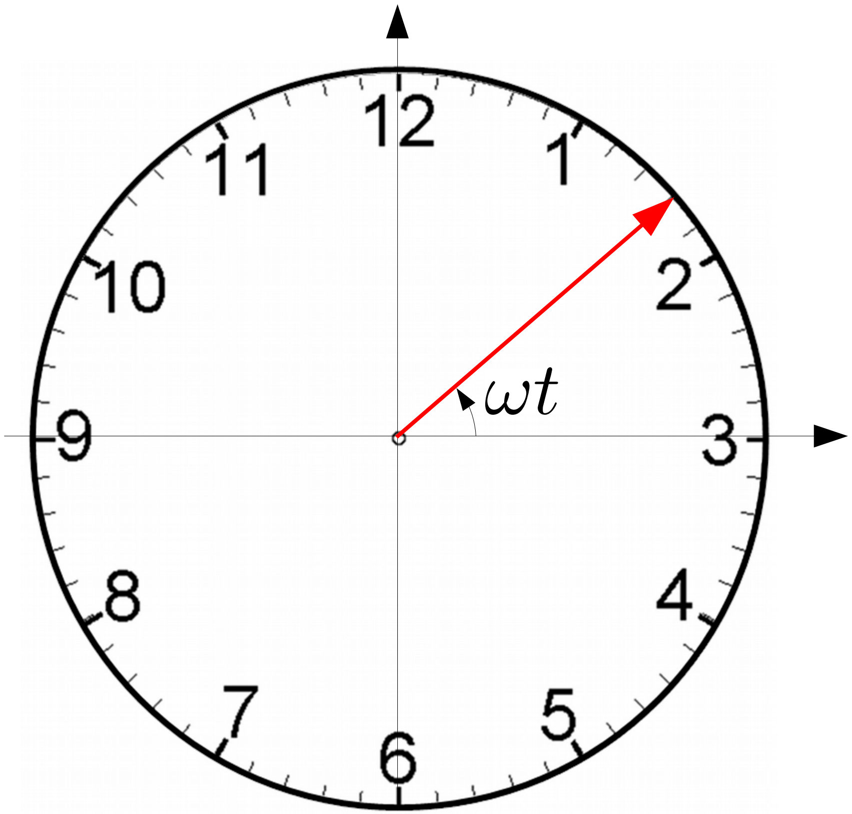
$$\Rightarrow |\psi(t)\rangle = |\alpha e^{-i\omega t}\rangle$$



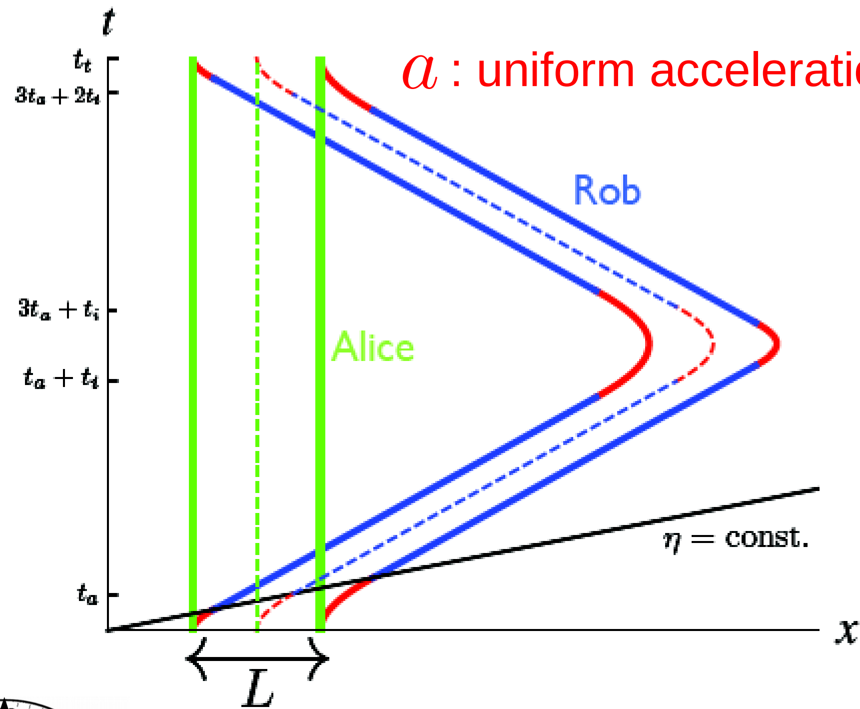
Clock hand = coherent state amplitude

$$|\psi(0)\rangle = |\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle \qquad H = \omega a^\dagger a$$

$$\Rightarrow |\psi(t)\rangle = |\alpha e^{-i\omega t}\rangle$$



Some technical elements



a : uniform acceleration

Rob's modes before the trip: a_n

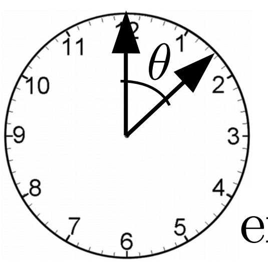
Rob's modes after the trip: b_n

Bogoliubov transformation to relate frames:

$$b_m = \sum_n \left(A^*_{mn} a_n - B_{mn} a^\dagger_n \right)$$

A_{mn} : mode-mixing $m \neq n$

B_{mn} : particle creation (DCE)



(In a frame co-moving with Alice.)

$$|i\alpha\rangle_1 = \exp[i\alpha a^\dagger_1 + i\alpha a_1]\,|\emptyset\rangle \rightarrow \exp[i\alpha b^\dagger_1 + i\alpha b_1]\,|\emptyset\rangle$$

$$\exp[i\alpha(A_{11} - B_{11})a^\dagger_1 + i\alpha(A^*_{11} - B^*_{11})a_1]\,|0\rangle_1\,|\Omega\rangle_{2\ldots\infty} = |i\tilde{\alpha}e^{i\theta}\rangle_1\,|\Omega\rangle_{2\ldots\infty}$$

Rob's clock dephasing: $$\tan\theta = \frac{-\mathrm{Im}\,(A_{11} - B_{11})}{\mathrm{Re}\,(A_{11} - B_{11})}$$

A_{mn}, B_{mn} evaluated perturbatively in $h = aL/c^2$

Perturbative results ($\mathcal{O}(h^2)$) $h = aL/c^2$

$$
\begin{aligned}
A_{11} \; = \; & \left(1+6\alpha_{11}^{(2)}h^2\right)e^{i(4\theta_a+2\theta_i)} \\
& +h^2\sum_{k=2}^{\infty}\left(\alpha_{k1}^{(1)}\right)^2 \times \\
& \Big[2e^{(k+3)i\theta_a+2i\theta_i}+2e^{2(k+1)i\theta_a+(k+1)i\theta_i} \\
& -2e^{(3k+1)i\theta_a+(k+1)i\theta_i}-2e^{(3k+1)i\theta_a+2ki\theta_i} \\
& +2e^{(k+3)i\theta_a+(k+1)i\theta_i}-2e^{4i\theta_a+(k+1)i\theta_i} \\
& +e^{2(k+1)i\theta_a+2i\theta_i}+e^{4ki\theta_a+2ki\theta_i}+e^{2(k+1)i\theta_a+2ki\theta_i}\Big] \\
& -h^2\sum_{k=2}^{\infty}\left(\beta_{k1}^{(1)}\right)^2 \times \\
& \Big[2e^{(-k+3)i\theta_a+2i\theta_i}+2e^{2(-k+1)i\theta_a+(-k+1)i\theta_i} \\
& -2e^{(-3k+1)i\theta_a+(-k+1)i\theta_i}-2e^{(-3k+1)i\theta_a-2ki\theta_i} \\
& +2e^{(-k+3)i\theta_a+(-k+1)i\theta_i}-2e^{4i\theta_a+(-k+1)i\theta_i} \\
& +e^{2(-k+1)i\theta_a+2i\theta_i}+e^{-4ki\theta_a-2ki\theta_i}+e^{2(-k+1)i\theta_a-2ki\theta_i}\Big]
\end{aligned}
$$

$$\theta_a = \frac{\pi\,\mathrm{arcsinh}\,(at_a/c)}{2\,\mathrm{arctanh}\,(h/2)},$$

$$\theta_i = \pi c t_i/(\gamma L),$$

$$\gamma \; = \; \sqrt{(at_a/c)^2+1}$$

Solution of wave eq. in inertial frame:

$$u_n(t,x) = \frac{1}{\sqrt{\pi n}}\sin\left(\omega_n(x-x_l)\right)e^{-i\omega_n t}$$

Rindler coordinates:

$$
\begin{aligned}
x \; &= \; \frac{c^2}{a}e^{a\xi/c^2}\cosh\left(a\eta/c\right), \\
t \; &= \; \frac{c}{a}e^{a\xi/c^2}\sinh\left(a\eta/c\right).
\end{aligned}
$$

Solution of wave eq. in accel. frame:

$$v_m(\eta,\xi) = \frac{1}{\sqrt{\pi m}}\sin\left(\Omega_m(\xi-\xi_l)\right)e^{-i\Omega_m\eta}$$

Klein-Gordon inner products: $\alpha_{mn} \; = \; (v_m, u_n)$ $\beta_{mn} \; = \; -(v_m, u_n^*)$

Perturbative results ($\mathcal{O}(h^2)$) $h = aL/c^2$

$$
\begin{aligned}
B_{11} \;=\;& 2ih^2\beta_{11}^{(2)}\left[\sin\left(4\theta_a+2\theta_i\right)-\sin\left(2\theta_a+2\theta_i\right)+\sin\left(2\theta_a\right)\right] \\
& +2ih^2\sum_{k=2}^{\infty}\left(\alpha_{k1}^{(1)}\beta_{k1}^{(1)}\right)\times \\
& [\sin\left(\left(4\theta_a+2\theta_i\right)k\right)-2\sin\left(\left(3\theta_a+2\theta_i\right)k\right)\cos\left(\theta_a\right) \\
& -2\sin\left(\left(3\theta_a+\theta_i\right)k\right)\cos\left(\theta_a+\theta_i\right)+\sin\left(\left(2\theta_a+2\theta_i\right)k\right) \\
& +2\sin\left(\left(2\theta_a+\theta_i\right)k\right)\cos\left(\theta_i\right)+\sin\left(2\theta_a k\right) \\
& +2\sin\left(\left(\theta_a+\theta_i\right)k\right)\cos\left(3\theta_a+\theta_i\right) \\
& +2\sin\left(\theta_a k\right)\cos\left(3\theta_a+2\theta_i\right)-2\sin\left(\theta_i k\right)\cos\left(2\theta_a+\theta_i\right)]
\end{aligned}
$$

Perturbative results ($\mathcal{O}(h^2)$)

Proper time dilations of accelerating clock:
(neglecting particle creation and mode mixing)**:**

acceleration

$$h = aL/c^2$$

t_a (acceleration time)

L (cavity rest length)

Finite size clock:

$$\tau^a_{\mathrm{cav}} = \frac{\theta_a}{\omega_1} = \frac{L}{c}\frac{\mathrm{arcsinh}(at_a/c)}{2\mathrm{arctanh}(h/2)}$$

Point-like clock:

$$\tau^a_{\mathrm{point}} = \frac{c}{a}\mathrm{arcsinh}(at_a/c)$$

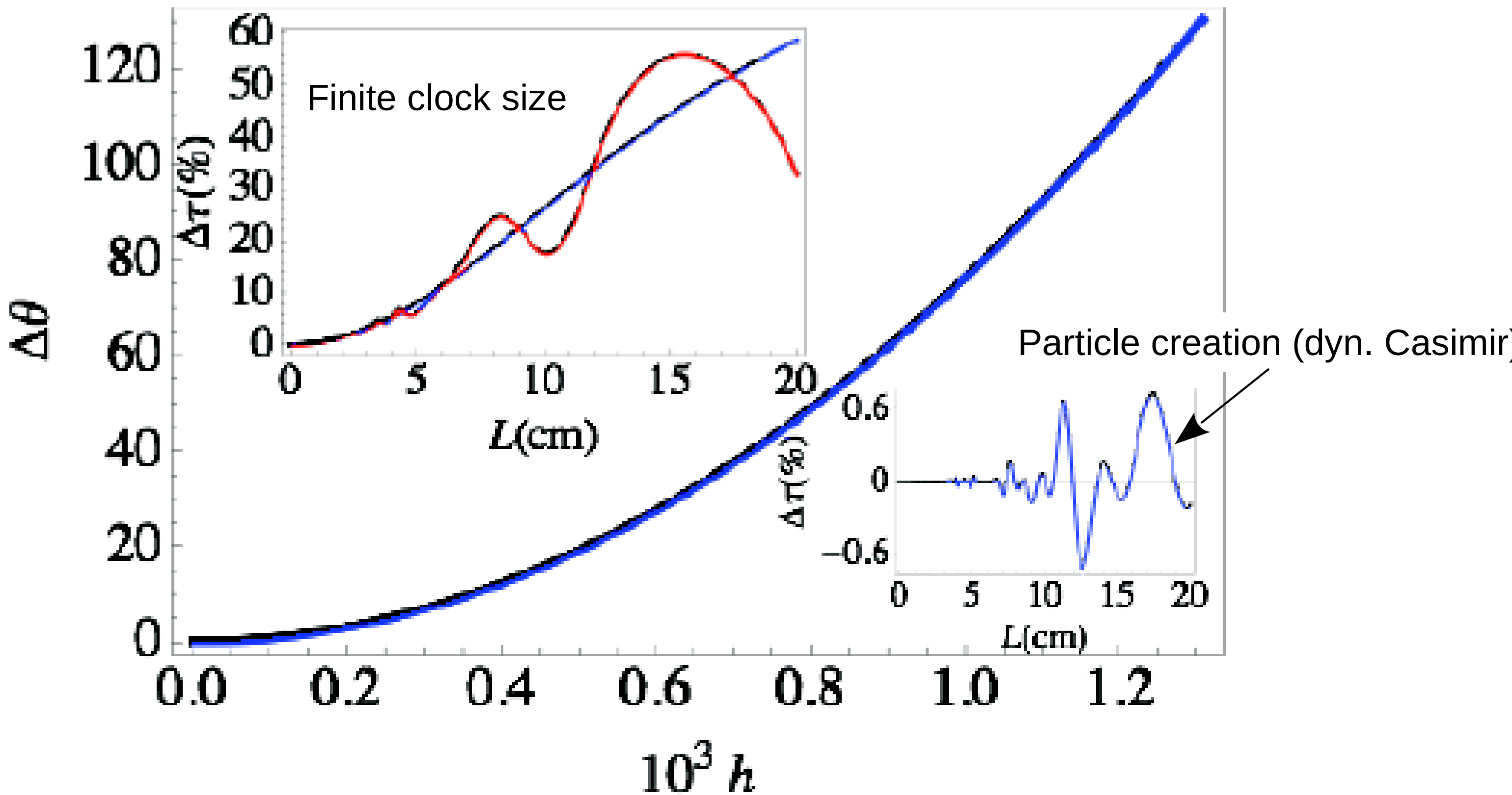
$$\frac{\tau^a_{\mathrm{cav}}}{\tau^a_{\mathrm{point}}} = \frac{(h/2)}{\mathrm{arctanh}(h/2)} = 1 \underbrace{- \frac{h^2}{12} + \mathcal{O}(h^4)}$$

finite clock size effect

Extended clock is slowed down more than point-like clock!

Higher order results (graphical)

Additional phase shift of accelerated clock



$t_a = 1\,\mathrm{ns}$ (acceleration time)

$t_i = 0$ (inertial travel time)

$L = 1.1\,\mathrm{cm}$ (cavity rest length)

$d_{\pm} = 1.7\,\mathrm{mm}$ (cavity displacement)

$v_{\mathrm{max}} = 0.014c$

$N_{\mathrm{rep}} = 500 \Rightarrow T_{\mathrm{travel}} = 2\,\mu\mathrm{s}$

Summary and conclusions

Summary:

- Simulate twin paradox with superconducting resonators sandwiched between two SQUIDS.
- Time dependent flux biases to simulate motion of cavity with constant length.
- Prepare coherent state in cavity and use its phase as clock hand.

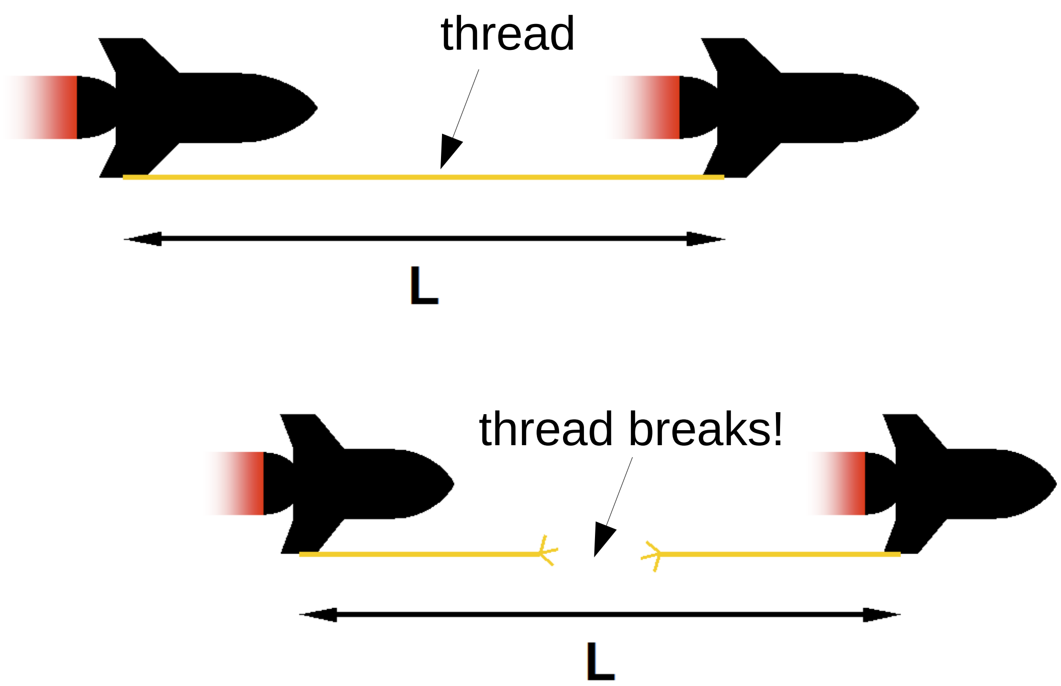
Results:

- Twin paradox can be demonstrated in a ground-based experiment at velocities approaching 1.4% of c.
- Phase shift of up to 130 deg.
- At high accelerations the extension of the clock becomes relevant: time dilation increases with clock's spatial dimension.
- Interplay between relativity and quantum field theory effects (dynamical Casimir effect).

Outlook:

- How about using squeezed cavity states instead of coherent states ?
- Further investigate overlap between quantum theory and relativity ?

Bell's* spaceship paradox



- In the rest frame, both spaceships accelerate the same → distance between them remains the same
- Thread length is contracted → thread breaks!

- In accelerated trailing spaceship accelerates less then leading spaceship → distance between spaceships increases
- Proper length of string remains constant → thread breaks!

(*) Originally designed by Dewan and Beran (1959)