Universal spin-triplet superconducting correlations of Majorana fermions

Xin Liu, Jay D. Sau, and S. Das Sarma;arXiv:1501.07273

Marcel Serina

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Current context

- MFs in topological superconductors: Symmetry protected zero-energy subgap states, equal weight electron-hole superposition, $\gamma = \gamma^\dagger$
- Unique transport signatures: quantized 0-bias conductance, fractional Josephson effect
- Open challenge: to manipulate MFs by their spin properties

Resume of results

- MFs have only spin-triplet superconducting correlations which is model independent and guaranteed by the particle-hole symmetry for the TSCs in D class and particle-hole as well as time-reversal symmetries for the TSCs in DIII class.
- No matter whether the bulk superconducting gap is spin singlet or triplet, the MF-induced resonant Andreev reflection (AR) only injects spin-triplet Cooper pairs into the normal lead.
- New transport signature of MFs: TSC/spin-orbit coupl. semicond./TSC setup. SOC controlled oscillatory critical Josephson current.

Comments I

- Following is known from the zoology of TSCs in[1] and from random-matrix theory in[2]
- 4 classes of TSCs based on the symmetries of Hamiltonian are possible. Relevant are time-rev. (TR), particle-hole (PH) and spin-rotation SU(2). For TSC in BdG description PH is always present. DIII respects TR, D does not. Both do not respect SU(2), resulting $[\mathcal{H}_{BdG}; S_i] \neq 0$. S_i are SU(2) generators in BdG representation.
- Zero energy eigenstates are possible only in D and DIII. Those classes do not commute with S_i thus their eigenstates are not spin singlets. So MFs here cannot be spin singlets.

[1] Andreas P. Schnyder et al.,PhysRevB.78.195125 (2008)
[2] C.W.J.Beenakker et al.,PhysRevB.83.085413 (2011)

Comments II

- We can further investigate the SU(2) spin structure of D and DIII classes by calculating the commutators with SU(2) generators, construct the SU(2) representation and we find that eigenstates of these classes have always spin triplet structure.
- Based on these arguments, fact that MFs are always spin triplets is obvious.

Off-diagonal spectral function I

- From symmetry arguments, it can be shown that block off-diagonal part of the spectral function in the 4-component spinor basis $(\psi_\uparrow, \psi_\downarrow, \psi_\downarrow^\dagger, -\psi_\uparrow^\dagger)$, with d_0 and $\mathbf{d}$ being the spin-singlet and triplet pairing amplitudes respectively, can be written as

$$A^{\text{off}}(E) = \begin{pmatrix} 0 & d_0\sigma_0 + \mathbf{d}\cdot\boldsymbol{\sigma} \\ [d_0\sigma_0 + \mathbf{d}\cdot\boldsymbol{\sigma}]^\dagger & 0 \end{pmatrix}$$

- PH symmetry for 0-energy spectral function requires $\hat{C}A(E=0)\hat{C}^{-1} = A(E=0)$, where $\hat{C} = \tau_y\sigma_y K$ is the particle-hole operator and $\tau_{x,y,z}$ are the Pauli matrices in particle-hole space, $\sigma_{x,y,z}$ in spin space and K is complex conjugation.

Off-diagonal spectral function II

$$\hat{C} A^{\text{off}} \hat{C}^{-1} = \begin{pmatrix} 0 & (-d_0\sigma_0 + \mathbf{d}\cdot\sigma) \\ [(-d_0\sigma_0 + \mathbf{d}\cdot\sigma)]^\dagger & 0 \end{pmatrix}$$

- That gives us $d_0 = 0$, thus PH symmetry completely forbids spin-singlet pairing and allows spin-triplet only.
- D class has only one MF mode and everything is done. DIII class has two MF modes, which can appear as one fermion resulting in zero A^{off}. But TR symmetry guarantees that two MFs belong to different spin channel, what gives us triplet pairing again.

Scattering matrix of SC/NM junction

- General form of S-matrix on the interface of SC/normal metal according to[3]:

$$\hat{R}(E) = \begin{pmatrix} \hat{r}_{\mathrm{ee}}(E) & \hat{r}_{\mathrm{eh}}(E) \\ \hat{r}_{\mathrm{he}}(E) & \hat{r}_{\mathrm{hh}}(E) \end{pmatrix}$$

$$\hat{r}_{\mathrm{eh}}(E) = a_0\sigma_0 + a_j\sigma_j = -\sigma_y \hat{r}^*_{\mathrm{he}}(-E)\sigma_y$$

$$\hat{r}_{\mathrm{ee}}(E) = b_0\sigma_0 + b_j\sigma_j = \sigma_y \hat{r}^*_{\mathrm{hh}}(-E)\sigma_y$$

- $\hat{r}_{(\mathrm{ee,hh})}$ and $\hat{r}_{\mathrm{eh,he}}$ are the normal reflection and AR matrices respectively, and $b_{0,x,y,z}$ and $a_{0,x,y,z}$ are the normal reflection and AR coefficients respectively

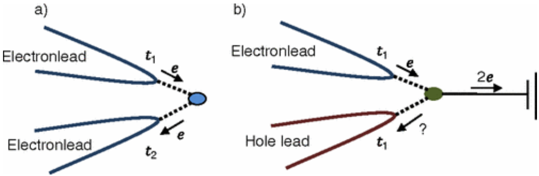
[3] C.W.J.Beenakker et al.,PhysRevB.83.085413 (2011)

Symmetry properties of S-matrix

- a_0 provides spin-singlet channel, $a_{x,y,z}$ spin-triplet channels. As MFs are only spin-triplets we expect that MF induced resonant AR can occur only in spin-triplet channels. Authors show this first for $E = 0$ with S-matrix theory for DII class and numerically calculate the S-matrix further from $E = 0$ inside the gap for both classes.
- For D class TSC it is known[4] that incoming electron and reflected hole in resonant AR have the same spin, thus providing spin-triplet channel.

[4] J.J.He et al.,PhysRevLett.112.037001 (2014)

Majorana-induced resonant Andreev reflection (MIRAR)



(a) Conventional resonant tunneling: two leads are coupled to a resonant level with coupling amplitudes t_1 and t_2, respectively. Resonant tunneling with unity transmission probability can happen only if $t_1 = t_2$. (b) MIRAR: a single-lead coupled to a Majorana level plays the role of both an electron lead and a hole lead. The coupling amplitudes of the leads to the Majorana level are ensured to be the same. The Majorana mode is attached to a superconductor which is grounded.

DIII class TSC/NM interface

- For a DIII class TSC/NM interface, the condition $\hat{T}\hat{R}^{\dagger}\hat{T}^{-1} = \hat{R}$, due to time reversal symmetry, constrains $b_{x,y,z} = 0$. Here $\hat{T} = i\sigma_y K$. The unitarity of $\hat{R}$ further requires *either* $(|b_0|^2 + |a_0|^2 = 1 \text{ and } a_x = a_y = a_z = 0)$ *or* $(|a_x|^2 + |a_y|^2 + |a_z|^2 = 1 \text{ and } b_0 = a_0 = 0)$.
- The former corresponds to the trivial spin-singlet AR with $Q = \mathrm{Pf}\left(i\sigma_y \hat{R}\right) = 1$. More importantly, the latter indicates the perfect spin-triplet AR and results in the nontrivial topological invariant $Q = -1$.

Model Hamiltonians I

$$
\begin{aligned}
H_{\mathrm{TS}}^{\mathrm{D}} &= (-2t\cos(k) - \mu_{\mathrm{s}})\tau_z \otimes \sigma_0 - M\tau_0 \otimes \sigma_z \\
&+ 2t_{\mathrm{so}}\sin(k)\tau_z \otimes \sigma_y + \Delta\tau_x \otimes \sigma_0
\end{aligned}
$$

$$
\begin{aligned}
H_{\mathrm{TS}}^{\mathrm{DIII}} &= (-2t\cos(k) - \mu_{\mathrm{s}})\tau_z \otimes \sigma_0 \\
&\quad +2t_{\mathrm{so}}\sin(k)\tau_z \otimes \sigma_z + \Delta(k)\tau_x \otimes \sigma_0
\end{aligned}
$$

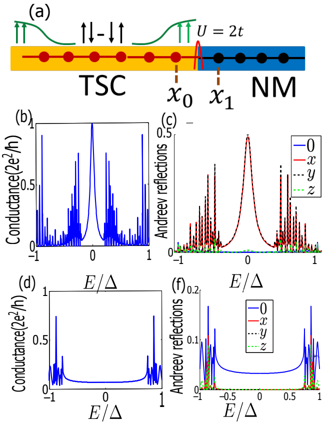
t is the spin independent hopping, t_{so} is the SOC hopping and μ_{s} is the chemical potential of the SOC wire with M the Zeeman coupling strength and Δ the proximity induced superconducting gap. In DIII $\Delta(k) = (\Delta_0 - \Delta_1\cos(k))$

Model Hamiltonians II

- $H_{\mathrm{NM}} = (-2t\cos(k) - \mu_{\mathrm{n}})\tau_z \otimes \sigma_0$ is the Hamiltonian of normal metal. Authors calculate S-matrix at $x = x_1$ (site in the NM closest to the TSC) using Fisher-Lee relation $\hat{R}_{ij}(E, r) = -\delta_{ij} + i\hbar\sqrt{v_i v_j} G^R_{ij}(E, r)$, where G^{R} is the retarded Green's function with $i, j = 1, \ldots, 4$ and $v_{i(j)}$ is the velocity of the particle at energy E in $i(j)$ channels.
- Aim is to see the difference in AR amplitude in the trivial and topological state as well as compare the results for both classes

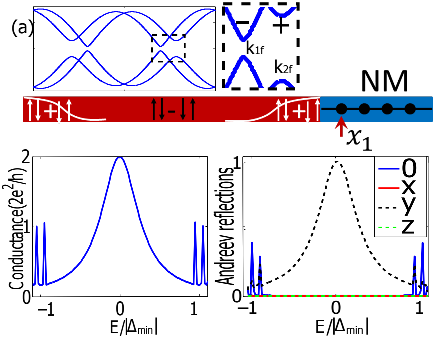
S-matrix analysis for D class Hamiltonian

- (a) Time-reversal broken TSC/NM junction. The red peak indicates the location of the barrier with the potential $U = 2t$. (b) and (c) The superconductor is in the topological nontrivial regime. (d) and (f) The superconductor is in the topological trivial regime.



S-matrix analysis for DIII class Hamiltonian

- TR invariant TSC/NM junction. The bulk dispersion of the TSC shows two SC gaps. The inset shows the sign of the minimal gap $\Delta_{\rm min}$ is negative. The conductance of the TSC/NM junction as a function of energy. AR probabilities for spin-singlet and spin-triplet channels.



SOC controlled critical Josephson current I

- Authors have shown, that MIRAR always injects spin-triplet Cooper pairs. Recet studies show[5], that SOC can rotate the **d** vector of the spin-triplet Cooper pairs and leave spin-singlet ones unaffected.
- SOC can then distinguish singlet and triplet Cooper pairs. Authors assume TSC/SOC-semicond./TSC Josephson junction and show that rotation of **d** vector results in oscillation of the critical Josephson current. TSC is described by $H_{\mathrm{TS}}^{\mathrm{D}}$.
- $H_{\mathrm{semi}} = (-2t\cos(ka) - \mu_{\mathrm{s}})\tau_z \otimes \sigma_0 + 2t'_{\mathrm{so}}\sin(ka)\tau_z \otimes \sigma_y$, what induces precession of spin in $x - z$ plane, so $|\uparrow\uparrow\rangle$ changes to $|\nearrow\nearrow\rangle$ and $\langle\uparrow\uparrow|\nearrow\nearrow\rangle = (1+\cos\theta)/2$.

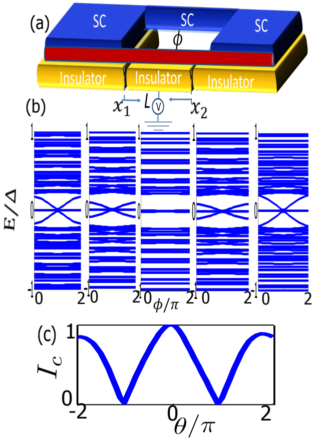
[5]Xin Liu et al.,PhysRevLett.113.227002 (2014)

SOC controlled critical Josephson current II

- $\theta = N\frac{t'_{\rm so}}{t}$, N is the number of sites in the normal SOC region and this holds in the limit $t'_{so} \ll t$
- $I_{\rm s} = \frac{2e}{h}\sum_n \partial E_n/\partial\phi = I_c \sin(\phi/2)$ where $I_{\rm c}$ is the critical current, with the summation over all the negative Andreev levels.
- Similar results have been predicted for SFS junctions, where MFs do not play any role and where $0-\pi$ transition is observed for $I_{\rm c} \to 0$. In contrast, here we do not observe such transition. Authors claim, that their prediction of the SOC controlled critical current is a transport signature of MFs.

SOC controlled critical Josephson current III

- (a) TSC/SOC/TSC junction. (b) Andreev levels for different SOC strengths which correspond to the precession angles $\theta = 0, 3\pi/4, \pi, 5\pi/4, 2\pi$ respectively. (c) The oscillatory critical current as a function of θ. The current is normalized by the critical current at $t'_{so} = 0$.



Summary

- MFs as a zero energy bound states of TSCs always have spin-triplet correlations regardless of the bulk superconducting gap.
- This property can be used in TSC/SOC-semicond./TSC setup to obtain the SOC-dependent critical Josephson current as another transport signature of MFs.