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Majorana Fermion Rides on a Magnetic Domain Wall

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We propose using a mobile magnetic domain wall as a host of zero-energy Majorana fermions in a spin-orbit coupled nanowire sandwiched by two ferromagnetic strips and deposited on an *s*-wave superconductor. The ability to control domain walls by thermal means allows to braid Majorana fermions nonintrusively, which obey non-Abelian statistics. The analytical solutions of Majorana fermions inside domain walls are obtained in the strong spin-orbit regime.

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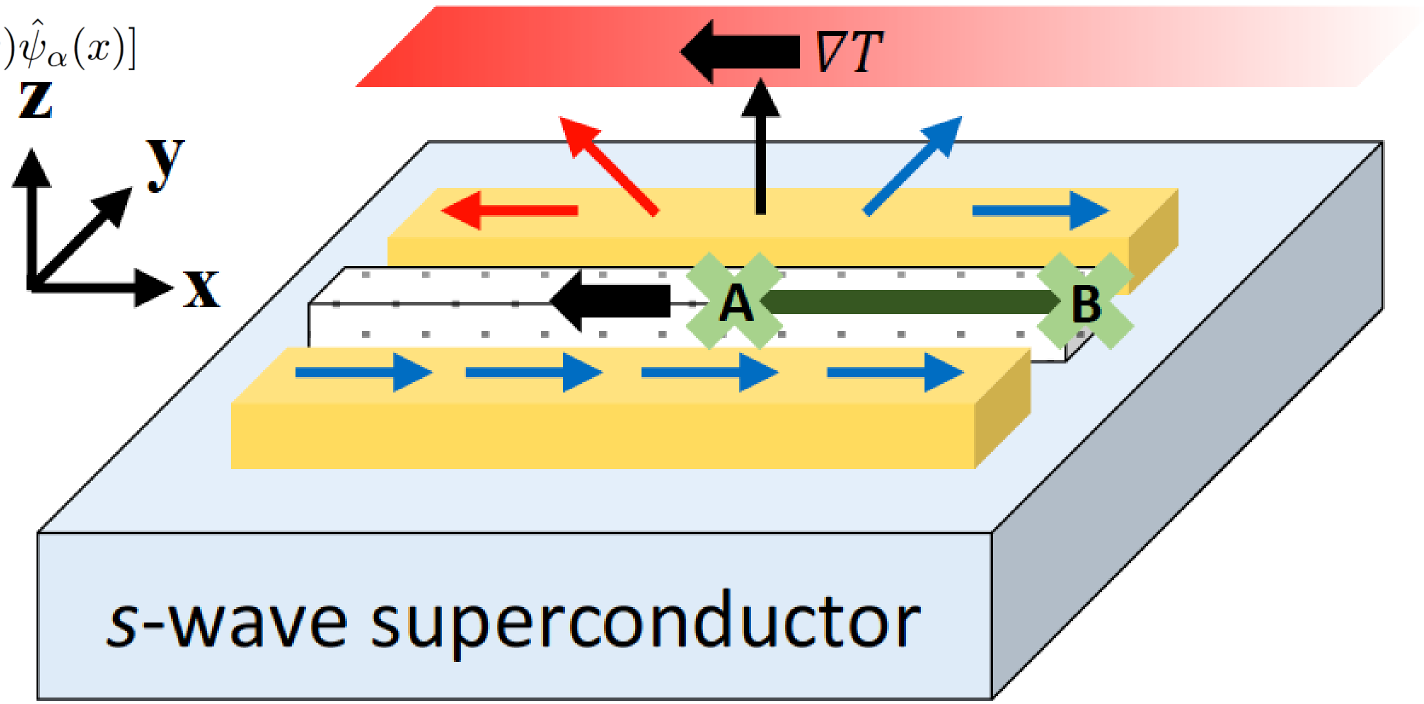
Device

BdG Hamiltonian:

$$H^{BdG} = \left(-\frac{\hbar^2}{2m}\partial_x^2 - \mu + i\alpha\partial_x\sigma_y \right)\tau_3 + \vec{\mathrm{M}}\cdot\vec{\sigma} + \Delta\tau_1$$

$$\Psi = (u_\uparrow, u_\downarrow, v_\downarrow, -v_\uparrow)^{\mathrm{T}}$$

$$\hat{\gamma}^\dagger = \int dx \sum_{\alpha=\uparrow,\downarrow} [u_\alpha(x)\hat{\psi}_\alpha^\dagger(x) + v_\alpha(x)\hat{\psi}_\alpha(x)]$$



Ingredients for MFs:

-Proximity induced exchange field **M**
-Proximity induced s-wave superconducting order parameter **Δ**
-Spin orbit coupling **α**

„Topological gap"

$$E_g = |M| - \sqrt{\Delta^2 - \mu^2}$$

Topological gap, MF at DW position

„Topological gap”

$$E_g = |M| - \sqrt{\Delta^2 - \mu^2}$$

$$E_g = \begin{cases} E_g > 0 \text{ for } |M| > \sqrt{\Delta^2 - \mu^2} \text{ (topological phase)} \\ E_g < 0 \text{ for } |M| < \sqrt{\Delta^2 - \mu^2} \text{ (normal phase)} \end{cases}$$

A one-dimensional wire supports **MFs** at the **boundary between topological and nontopological regions.**

A spatially-varying exchange field induces the topological phase transition along the wire where |M| crosses $\sqrt{\Delta^2 - \mu^2}$.

A DW in a ferromagnet adjacent to the wire is a natural object to bring about such a position dependent field.

In general topological phase transition along the nanowire can be realized by spatially varying:

$$B(x),\ \Delta(x),\ \mu(x)$$

PRL 105, 177002 (2010)
Helical Liquids and Majorana Bound States in Quantum Wires
Yuval Oreg, Gil Refael,and Felix von Oppen

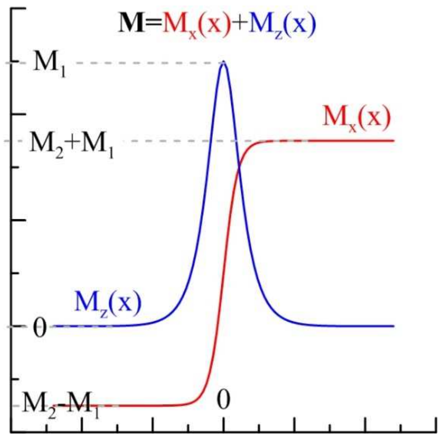
Magnetic Domain Wall

Energy of the ferromagnet

N. L. Schryer and L. R. Walker, J. Appl. Phys. 45, 5406 (1974)

$$U[\mathbf{m}(x)] = \int dx \left[A|\partial_x \mathbf{m}|^2 - K_x m_x^2 + K_y m_y^2\right]/2$$

A
K_x
K_y



Domain wall – topological soliton, minimazing U[**m**(x)]
With bounday conditions $m(\mathrm{x} \to \pm\infty) = \hat{\mathrm{x}}$

$$\mathbf{m}(x) = \tanh(x/\lambda)\hat{\mathbf{x}} + \mathrm{sech}(x/\lambda)\hat{\mathbf{z}}$$

Proximity induced exchange field:

$$\lambda = \sqrt{A/K_x}$$

$$\mathbf{M}(x) = M_1\left[\tanh(x/\lambda)\hat{\mathbf{x}} + \mathrm{sech}(x/\lambda)\hat{\mathbf{z}}\right] + M_2\hat{\mathbf{x}}$$

Spatially varying topological gap

$$E_g(x) = \sqrt{M_1^2 + M_2^2 + 2M_1M_2\tanh(x/\lambda)} - \sqrt{\Delta^2 + \mu^2}$$

Position of the Majorana $E_g(\mathrm{x}_0) = 0$

$$\mathrm{M}_1 = \mathrm{M}_2 \quad 2\mathrm{M}_1 > \sqrt{\Delta^2 + \mu^2}$$

$$\mathrm{x}_0 = \lambda\tan[\left(\Delta^2 + \mu^2\right)/\mathrm{M}_1^2 - 1]$$

How to control DW

- External magnetic field N. L. Schryer and L. R. Walker, J. Appl. Phys. 45, 5406 (1974)
- Spin polarized electric current (in itinerant ferromagnet) J. Slonczewski, J. Magn. Magn. Mater. 159, L1 (1996);L. Berger, Phys. Rev. B 54, 9353 (1996)

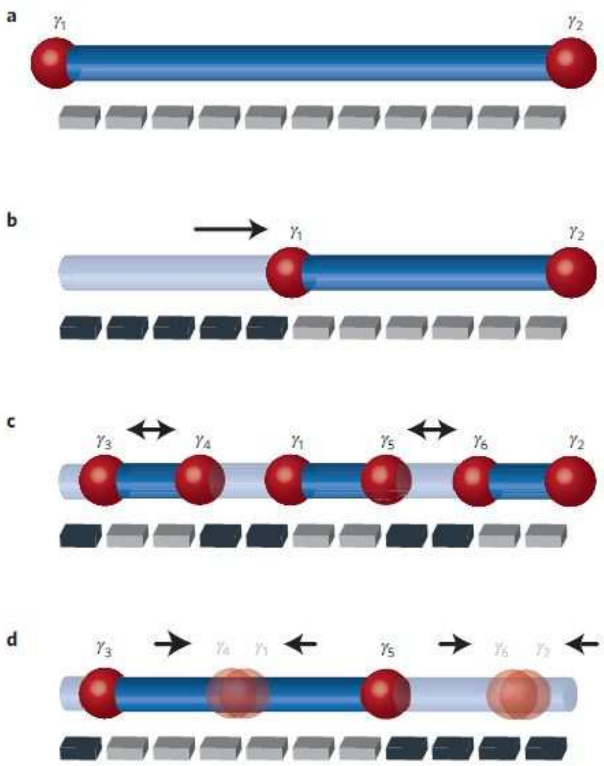
But

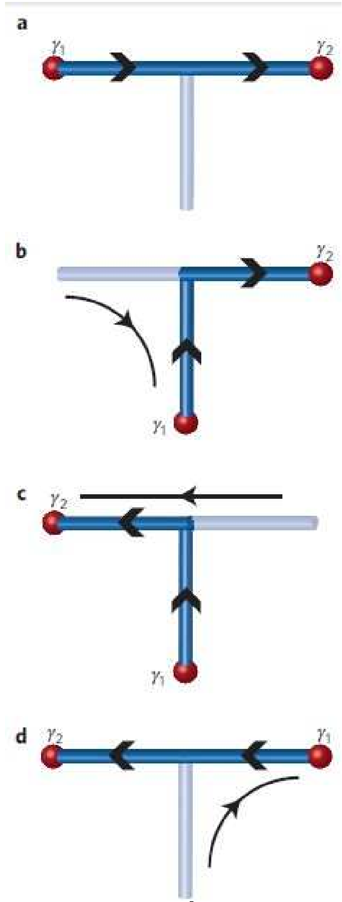
- These methods affects the electron in the nanowire by altering the Hamiltonian

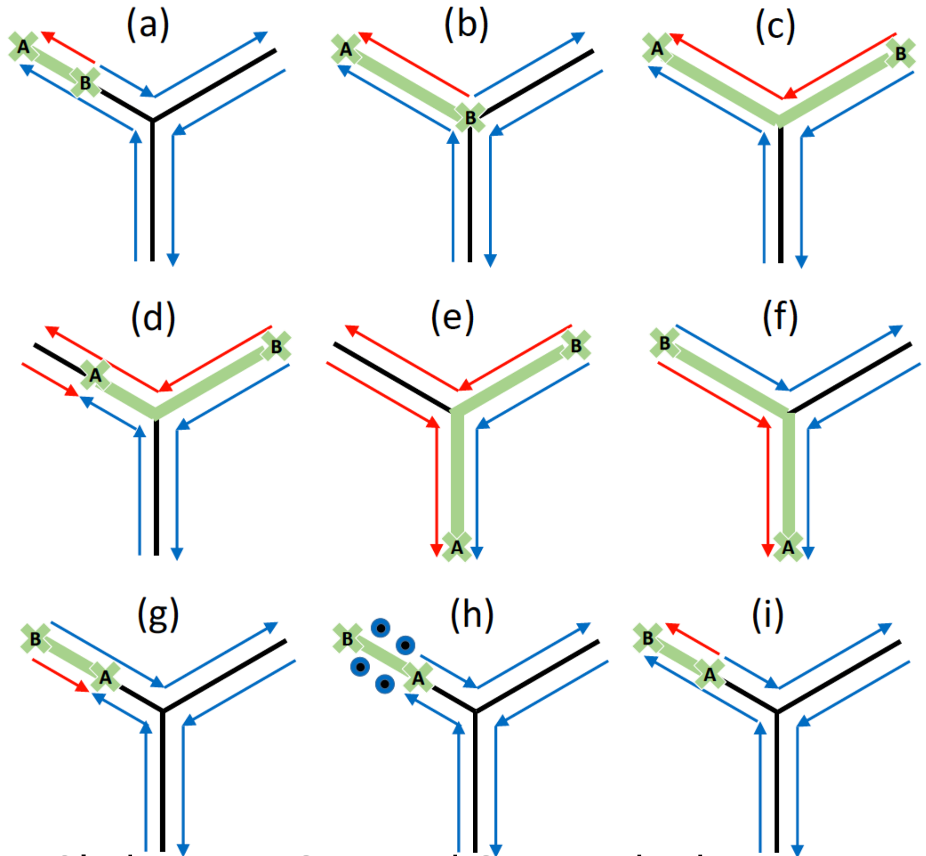
Authors propose instead:

- Induce motion of DW by thermal drag which is not intrusive for electrons D. Hinzke and U. Nowak, Phys. Rev. Lett. 107, 027205 (2011); P. Yan, X.Wang, and X.Wang, Phys. Rev. Lett. 107, 177207 (2011); A. A. Kovalev and Y. Tserkovnyak, Europhys. Lett. 97, 67002 (2012)

Braiding Majoranas







J. Alicea, Y. Oreg, G. Refael, F. von Oppen, and M. P. A. Fisher, Nature Physics 7, 412 (2011).

D. J. Clarke, J. D. Sau, and S. Tewari, Phys. Rev. B 84, 035120 (2011); B. Halperin, Y. Oreg, A. Stern, G. Refael, J. Alicea, and F. von Oppen, Phys. Rev. B 85, 144501 (2012)

Exact solution for Majoranas

Strong SOC regime $E_{SO}=\frac{m\alpha^2}{\hbar^2}\gg\max\left(\Delta,\left|\vec{M}\right|\right)$

Linearized BdG Hamiltonian density at **k**=0, and μ=0

$$\mathcal{H}^{\text{lin}}=-i\alpha\partial_x\sigma_2\tau_3+M_x(x)\sigma_1+M_z(x)\sigma_3+\Delta\tau_1$$

For $\mathrm{M}_z=0$, H^{lin} belongs to BDI symmetry class

Particle hole symmetry $\left\{H^{lin},\sigma_2\tau_2K\right\}=0$

Time reversal symmetry $\left\{H^{lin},\sigma_1\tau_1K\right\}=0$

Chiral symmetry $\left\{H^{lin},\sigma_3\tau_3K\right\}=0$

Block diagonalization by employing Majorana operators (instead of fermion operators).

$$\hat{\psi}_\downarrow=(\hat{\gamma}_\downarrow^A+i\hat{\gamma}_\downarrow^B)/\sqrt{2},\quad \hat{\psi}_\uparrow=(\hat{\gamma}_\uparrow^B+i\hat{\gamma}_\uparrow^A)/\sqrt{2}$$

$$\hat{\gamma}^+ = \int dx\sum_{\substack{\alpha=\uparrow,\downarrow\\ \beta=A,B}}\left[u_\alpha^\beta(x)\hat{\gamma}_\alpha^\beta(x)\right]$$

$$\tilde{\Psi}=(u^A,u^B)^{\mathrm{T}}\equiv(u_\uparrow^A,u_\downarrow^A,u_\uparrow^B,u_\downarrow^B)^{\mathrm{T}}\qquad \tilde{\Psi}=U\Psi$$

$$i\partial_t\tilde{\Psi}=(U\mathcal{H}^{\text{lin}}U^\dagger)\tilde{\Psi}$$

Two Dirac equations (Jackiw-Rebbi soliton with fermion number 1/2):

$$i\gamma^\mu\partial_\mu u^A+[-M_x(x)-\Delta]u^A=0$$
$$i\gamma^\mu\partial_\mu u^B+[M_x(x)-\Delta]u^B=0$$

R. Jackiw and C. Rebbi, Phys. Rev. D 13, 3398 (1976)

$$m^A(x)=-M_x(x)-\Delta \qquad m^B(x)=M_x(x)-\Delta$$

$$E_g(x)=\begin{cases}m^A(x) & \text{for } \mathrm{M}_x(x)>0\\ m^B(x) & \text{for } \mathrm{M}_x(x)<0\end{cases}$$

Solution

For the exchange field $\mathbf{M} = [M_1 \tanh(x/\lambda) + M_2]\hat{\mathbf{x}}$

(with help of supersymmetric quantum mechanics)

F. Cooper, A. Khare, and U. Sukhatme, Phys. Rep. 251, 267 (1995)

Dirac equations have two zero energy solutions:

$$\hat{\gamma}^A = \int dx\, e^{-(M_2+\Delta)x} \mathrm{sech}^{M_1\lambda}(x/\lambda)\hat{\gamma}^A_\uparrow(x)$$

$$\hat{\gamma}^B = \int dx\, e^{-(M_2-\Delta)x} \mathrm{sech}^{M_1\lambda}(x/\lambda)\hat{\gamma}^B_\uparrow(x)$$

Perturbations breaking the BDI time reversal symmetry split the degeneracy

$\left(-\frac{\hbar^2}{2m}\partial_x^2\right)\tau_3$ Hybridizes MFs with energy $\leq \hbar^2 \max\left(M_1^2, \Delta^2\right)/2m\alpha^2 \ll M_1, \Delta$

$\hat{f} = \left(\hat{\gamma}^A + i\hat{\gamma}^B\right)/2$ Physical fermion couples to

$$M_z(x) = M_z \mathrm{sech}(x/\lambda) \rightarrow 0(\lambda \rightarrow)$$

Normalizable MF solutions when $M_2 < M_1 - \Delta$, $M_2 < M_1 + \Delta$

Masses cross zero when

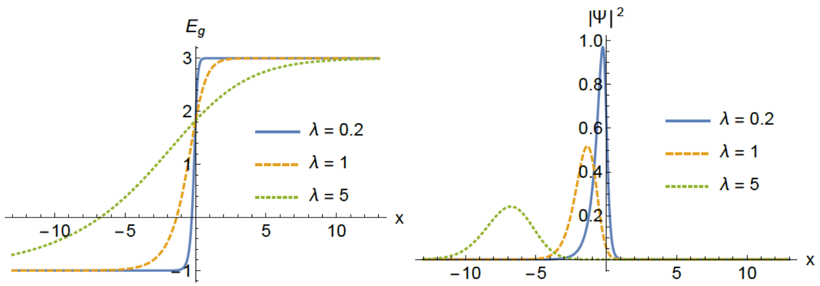
$$m^{A(B)}(x_0) = 0 \quad \text{for} \quad x_0 = \lambda \tan[-(M_2 \pm \Delta)/M_1 - 1]$$

For $M_1 = M_2 > \Delta/2 \quad x_0 = \lambda \tan[(\Delta^2)/M_1^2 - 1]$

M. Sato, Y. Takahashi, and S. Fujimoto, Phys. Rev. Lett. 103, 020401 (2009)

$$u^B_\uparrow(x) = \frac{e^{\Delta x}}{\sqrt[4]{1+e^{2x/\lambda}}} P^{-\nu}_{-1/2}\left(\frac{1}{\sqrt{1+e^{-2x/\lambda}}}\right)$$

$$u^A_\downarrow(x) = -(\nu+1/2)\frac{e^{\Delta x}}{\sqrt[4]{1+e^{2x/\lambda}}} P^{-\nu-1}_{-1/2}\left(\frac{1}{\sqrt{1+e^{-2x/\lambda}}}\right)$$



(a) (b)

Parameters

- **InAs** nanowires witg strong SOC $\alpha \sim$ 5meV nm
- **EuO** magnetic insulators $|\mathrm{M}| \sim 1\mathrm{meV}$
- **Nb** s-wave superconductors $\Delta \sim 0.5\mathrm{meV}$
- Thermally-driven motion of a DW has been observed in (**YIG**) yttrium iron garnet films, the DW moves at the velocity $v \sim 100\ \mu\mathrm{m/s}$ for $\nabla\mathrm{T} \sim 2\ \mu\mathrm{eV}/\mu\mathrm{m}$
- Rezultat temperature drop over the DW width λ=60nm smaller than the the induced topological gap ~ 200 μeV

Summary

- Magnetic domain Walls $\Rightarrow$ Spatially varying exchange field

$\Downarrow$

- Spatially varying topological gap $\Rightarrow$ Majorana bounded to DW

  (analytical solution in strong SOC regime)

$\Downarrow$

- Thermal Driven motion of DW $\Rightarrow$ Riding MF

$\Downarrow$

- Braiding two Majoranas in Y junctions

Thank you for your attention!

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