Interacting two-level defects as sources of fluctuating high-frequency noise in superconducting circuits

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Fluctuations in relaxation rate it is possible to fully characterise the properties of these structuations in relaxation relaxions level-structure and coherence times [15, 16].

- \mathbf{T} the relation rate is the relation of the rate $\frac{1}{2}$ Timo nooded to obtain single - Time needed to obtain single value of $T₁$ is ~min
- confidence interval of the fits, the fits, the dotted red line indicates in distribution \mathcal{L}_t the mean value of the mean value of the mean value of the measurements and the dashed black and the dashed black line is a moving average over 10 samples to emphasize the the qubit, where it can be measured. - There is no apparent structure to fluctuations of $T₁$
- multivalued character of the jumps. Each individual point in this plane are two lever system - There are two level systems (TSL) inside the dieletric forming spectroscopically. Often, we observe telegraph-signal like Josephson junctions (needed to realize the qubit)

Qualitative description of fluctuating T₁ e description of fluctua technique can be found in Ref. 16 as well as appendix A. We description

$Charsa$ defects Charge defects

$$
\hat{H}_{\rm TLS} = -\frac{1}{2}\epsilon \sigma_z + \frac{1}{2}\Delta \sigma_x
$$

 $\mathbb{E}[\mathbf{R}^{\dagger}]$ assimatry of DOW Tunnal splitting and is the tunnel splitting. The Pauli-matrix *^z* here Assimetry of DQW Tunnel splitting

TSL coupling to the qubit via dipole moment~ σ . the wells. Diagonalizing Eq. (1) yields *H* TSL coupling to the qubit via dipole moment~ $\sigma_{\rm z}$

$$
\Gamma_1 \propto C(\omega_{10})
$$
 qubit splitting

$$
C(\omega) = \int dt e^{-i\omega t} \langle \sigma_z(t)\sigma_z(0) \rangle
$$

that the thermal excellent can be neglected. It is the neglected on \mathbb{R} TSL emsemble

- i) High-frequency TS (E>>T, $E \sim \omega_{10}$) a strongly coloured a strongly coloured and ω_{10}
- ii) Low-frequency TF (E<<T) $\begin{array}{ccc} & & & & & \text{if} & \$
- iii) Slowly fluctuating TF $\begin{array}{cccc} \begin{array}{cccc} \hline \end{array} & \begin{array}{cccc} \hline$

Qualitative description of fluctuating T₁ **Z** *dt* ^ei!*^t* ^h*z*(*t*)*z*(0)ⁱ *^C*(!) = ^Z *dt* ^ei!*^t* ^h*z*(*t*)*z*(0)ⁱ $\overline{}$ h 1 h*z*i = cos2 *i* hz
2 *hz*i hzml $fl₁24$ \sim sin \sim <u>IUII UI HUC</u> their coupling operator *^z* [14]. We obtain

$$
C(\omega) = \cos^2 \theta \left[1 - \left\langle \sigma_z \right\rangle^2 \right] \frac{2\gamma_1}{\gamma_1^2 + \omega^2} + \sin^2 \theta \left[\frac{1 - \left\langle \sigma_z \right\rangle}{2} \right] \frac{2\gamma_2}{\gamma_2^2 + (\omega + E)^2} + \sin^2 \theta \left[\frac{1 + \left\langle \sigma_z \right\rangle}{2} \right] \frac{2\gamma_2}{\gamma_2^2 + (\omega - E)^2}
$$

22 (f_{rance} T_{Fa}) + sin
+ sin **hoise** (from I Fs) e to thermal switching leading \sim TC $_{\odot}$ $11015e$ (II 0111 113) 11 **1990** al switching lea Low-freq noise (from TFs) due to thermal switching

 $\begin{array}{ccc} \overline{1} & \cdots & \overline{$ switching leading to relaxation of the qubit ${\rm (from TFs)}$, ${\rm High\text{-}frac}$ noise (from ${\rm T}^{\rm c}$ \mathcal{V} 1 \mathcal{V} angle of the TLS. Eq. (2) describes the TLS. Eq. (2) describes the noise spectrum \mathcal{L} High-freq noise (from TS)

which acts on the qubit circuit from a single TLS's electronic from a single TLS's electronic from a single TLS's elec- \blacksquare complete the magnetic matrix \blacksquare \mathbb{Z} spin in process with \mathbb{Z} spin-flip process with TS

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1 decembrie: 1 decembrie:

parameter regimes. The first line describes low-frequency

and is most propounded for \mathbf{F}_2 and \mathbf{F}_3 is \mathbf{F}_3 and \mathbf{F}_2 is \mathbf{F}_3 and \mathbf{F}_4 are the theorem in \mathbf{F}_5 and \mathbf{F}_6 are the theorem is \mathbf{F}_7 and \mathbf{F}_8 are the theorem is \mathbf{F}_8 an

Calculation timescales corresponding to the experiments. As explained in the main text, we assume an interacbetween the two TLS dipoles, or $\mathcal{L}_{\mathcal{L}}$ dipoles, or $\mathcal{L}_{\mathcal{L}}$ dipoles, or $\mathcal{L}_{\mathcal{L}}$ mitigated via deformation of the surrounding atomic poetic portfolio atomic po-surrounding atomic po-surroundi
The surrounding atomic poetic poe \sim 100 μ mK. Fit parameters in (c) are the 100 μ \blacksquare Calculatio

Coupling between TSLs

\n
$$
\hat{H} = \frac{1}{2} \sum_{\langle ij \rangle} g_{ij} \tilde{\sigma}_{z,i} \tilde{\sigma}_{z,j}
$$

Energy splitting of TSL
$$
\hat{E}_i = E_{i,0} - \sum_{\langle ij \rangle} g_{ij} \tilde{\sigma}_{z,j}
$$
 $E_{i,0} = \sqrt{\epsilon_i^2 + \Delta_i^2}$

Relaxation rate of the qubit $\hat{\gamma}_{a,i} = c$ due to TSL $\gamma_{2,i}{}^2 + (\omega_{10})$

$$
\begin{array}{ll}\n\text{Relation rate of the qubit} & \hat{\gamma}_{q,i} = \cos^2\theta_i \frac{2\gamma_{2,i}}{\gamma_{2,i}^2 + (\omega_{10} - \hat{E}_i)^2}\n\end{array}
$$

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models can be both positive or negative, meaning the both positive or negative, meaning the both positive, mea
The both positive or negative, meaning the both positive or negative, meaning the both positive, meaning the b

g $\sqrt{}$ small coupling *P*(*g*)*dg* = *P*(*r*) by recent experiments, where it was used to experiments, where it was used to explain explain ex- \blacktriangledown small coupling constant g around the TS eigenenergy *Ei*, c.f. Eq (2). We defined \blacktriangledown small coupling constant g weak, **weak, and interaction between industry** the industry \mathbf{y} \blacklozenge small coupling constant g \mathbf{d} conclusions motivated above. We concentrate \mathbf{d} $\sqrt{ }$ sinan coupling constant ζ

 $\sum_{i=1}^{n}$

$$
\hat{\gamma}_{q,i} = \gamma_{q,i}^{(0)} + \gamma_{q,i}^{(1)} \sum_{\substack{i,j > \\ \gamma_{q,i}^{(0)} = \cos^2 \theta_i}} g_{ij} \tilde{\sigma}_{z,j} + O(g^2)
$$
\n
$$
\gamma_{q,i}^{(0)} = \cos^2 \theta_i \frac{2\gamma_{2,i}}{\gamma_{2,i}^2 + (\omega_{10} - E_{i,0})^2}, \qquad \langle \tilde{\sigma}_z \rangle = \cos \theta \langle \sigma_z \rangle = \cos \theta \tanh \left(E/2k_B T \right)
$$
\n
$$
\gamma_{q,i}^{(1)} = \frac{\partial \gamma_{q,i}}{\partial E_i} \Big|_{E_i = E_{i,0}} = \cos^2 \theta_i \frac{4\gamma_{2,i}(\omega_{10} - E_{i,0})}{\left(\gamma_{2,i}^2 + (\omega_{10} - E_{i,0})^2 \right)^2}.
$$

Distribution of parameters and the super-linear or super-linear or superdepends mainly on the size of the tunnelling barrier. In the s
In the size of tunneling barrier height barrier het barrier het barrière het barrière te staat barrière te staat barrière te
Since the tunneling endemondern het barrière te staat barrière te staat som en de staat som en som en de tunne
 ergy depends exponentially on the barrier, the resulttions from TLS with small level splitting, *E* . *kBT*, since bution of parameters are the ones directly observable in experiment.

Flat distribution of TLS barrier height
 ing distribution in TLS parameters is *P*(✏*,*) ⇠ 1*/*. The TLS relaxation rates are then also distributed log-

$$
\rightarrow P(\Delta) \sim 1/\Delta \rightarrow P(\gamma_1) \sim 1/\gamma_1
$$

 $P(\epsilon, \Delta) d\epsilon d\Delta = A \frac{\epsilon^{\alpha}}{\Delta}$ Assuming general distribution in ϵ $P(\epsilon, \Delta)$ $d\epsilon d\Delta = A \frac{d\epsilon}{\Delta} d\epsilon d\Delta$ ϵ^{α} al distribution in ϵ $P(\epsilon,\Delta) d\epsilon d\Delta = A \frac{d\epsilon}{\Delta} d\epsilon d\Delta$ Δ Δ seuming ge frequency *p q,i*ⁱ ⁼ (0) *q,i* ⁺ (1) *q,i* ^X *j ^g^j* cos ✓*^j* tanh *^E^j* $\sqrt{2}$

ing constant assuming dipolar interaction hetween TSI s ting constant assuming uipolal interaction between rols \sum istribution between in weak, *gij* ⌧ 2*,i*, we can expand this to first order as action. For example, for dipolar interaction with *|g|* ⇠ Distribution of coupling constant assuming dipolar interaction between TSLs Distril $=$ (0) $=$ *q,i* ⁺ (1) *q,i* Z *dg d*✓ *dE P*(*g,* ✓*, E*)*^g* cos ✓ tanh *^E* ²*^T ,*

$$
P(g)dg = P(r)\frac{\partial r}{\partial g}dg = \rho_0 |g|^{-\frac{4}{3}} dg
$$

✓ = arctan */*✏, we find ing constant can take both positive and negative value $\frac{1}{2}$ constant can take both positive and perstive values raphing constant can take both positive and hegative values coupling constant can take both positive and negative values

$$
\int dg\,gP(g)=0
$$

Calculation of $\langle \Gamma_1 \rangle$ and $\langle \Gamma_1(t) \Gamma_1(0) \rangle_{\omega}$ r arculation of r_1 and r_1 $($ U $)$ $($ U $)$ hˆ*q,i*(0)ˆ*q,i*(*t*)i! = *dt* ^ei!*^t* ^hˆ*q,i*(0)ˆ*q,i*(*t*)ⁱ = ⇣ (1) *q,i* ⌘²^X *gjg^l* h˜*z,j* (*t*)˜*z,l*(0)i! = ⇣ (1) *q,i* ⌘² ^Z

uniformly, *P*(1) ⇠ 1*/*1, since the tunnelling strength

$$
\langle \hat{\gamma}_{q,i} \rangle = \gamma_{q,i}^{(0)} + \gamma_{q,i}^{(1)} \sum_{j} g_j \cos \theta_j \tanh \frac{E_j}{2T}
$$

= $\gamma_{q,i}^{(0)} + \gamma_{q,i}^{(1)} \int dg d\theta dE F(g, \theta, E) g \cos \theta \tanh \frac{E}{2T}$

$$
\langle \hat{\gamma}_{q,i}(0)\hat{\gamma}_{q,i}(t) \rangle_{\omega} = \int dt \, e^{-i\omega t} \langle \hat{\gamma}_{q,i}(0)\hat{\gamma}_{q,i}(t) \rangle
$$

\n
$$
= \left(\gamma_{q,i}^{(1)}\right)^2 \sum_{j,l} g_j g_l \langle \tilde{\sigma}_{z,j}(t)\tilde{\sigma}_{z,l}(0) \rangle_{\omega}
$$

\n
$$
= \left(\gamma_{q,i}^{(1)}\right)^2 \int dg \, d\theta \, dE \, d\gamma_1 \, P(g, \theta, E, \gamma_1) g^2 \cos^2 \theta \left[1 - \tanh^2\left(\frac{E}{2T}\right)\right] \frac{2\gamma_1}{\gamma_1^2 + \omega^2}
$$

\n
$$
\int dg \, g^2 P(g) \propto \text{const}
$$

\n
$$
\int dE P(E) \left(1 - \tanh^2\left(\frac{E}{2T}\right)\right) \approx \int_0^T dE E^{\alpha} = T^{\alpha+1}
$$

Calculation of $\langle \Gamma_1 \rangle$ and $\langle \Gamma_1(t) \Gamma_1(0) \rangle_{\omega}$ l culation of $\langle \Gamma_i \rangle$ and $\langle \Gamma_i(t) \Gamma_i(0) \rangle$ temperature dependence of the dependence of the dependence of the dependence of the departure rate due to a to
The departure due to a top dependence of the departure of the departure of the departure of the departure of t $\frac{1}{2}$ and $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ density, c.f. Eq. (2), as 22*,i* 2*,i*² + (!¹⁰ *^E*ˆ*i*)² *.* (S4) scription as a two-level system breaks down and *E*Max provides an upper bound on the TLS level-splitting. The TLS level-splitting. The TLS level-splitting. The TLS l The distribution of inter-TLS coupling strengths *gij* $\frac{1}{2}$ of $\frac{1}{2}$ / constraint $\frac{1}{2}$ \sim μ Γ μ \sim \sim \sim \sim \sim \sim we can now distinguish three regimes regimes \mathcal{L}_{max} tial detuning between our qubit and the high-frequency of the high-frequency of the high-frequency of the highmum possible coupling strength and thus by the mini-**Calculation of** \leq **and** \leq their interaction. Performing the average over the mix- $\Gamma_{1}(t)\Gamma_{1}(0)$ and the prefactor behavior behavior behavior behavior behavior behavior ω mum possible coupling strength and thus by the mini-**Calculation or Separate between TLS** $>$ and $\leq \Gamma$, (t) Γ , (0) $>$... and the prefactor behavior behavior behavior ω

 tr ly, dephasing rate due to bath of <code>TF</code> $\quad \gamma_2 \, \propto \, T^{\alpha+1}$ Similarly, dephasing rate due to bath of TF Δf for $\alpha=1$ $\mathbf{P} \mathbf{I}$ is the resulting $\mathbf{P}_2 \propto \mathbf{I}$ and \mathbf{I} Similarly dephasing rate due to bath of we can expand the canonical \mathbf{r} , we can expand the first order as \mathbf{r} \sim α α π α $+1$ $\gamma_2 \propto$ mum possible coupling strength and thus by the minisimilarly, dephasing rate que to bath of TS, ! = !¹⁰ *Ei,*0. For qubit and high-frequency TLS $\gamma_2 \propto T^{\alpha+1}$ $\frac{12}{2}$ \sim $\frac{1}{2}$ ing angle α also contributes a contributes a contributes a constant, with the exact α Similarly, dephasing rate due to bath c $I_n = \frac{1}{2}$ $\gamma_2 \propto T^{\alpha+1}$ $\sum_{i=1}^{n}$ value in depending race to bath of TF $\sim \infty$, $\gamma \alpha + 1$ $\frac{1}{2}$ α 1

their interaction. Performing the average over the mix-

ing small interaction strength between TLS, *g* ⌧ 2*,i*, averaged qubit relaxation rate we find ê *gij*˜*z,j* + *O*(*g*²)*,* (S5) ing angle α also contributes a contributes a constant, with the exact α value again depending on details of the microscopic TLS of the microscopic TLS of the microscopic TLS of the m In the far detuned regime, ! 2*,i*, we finally have

^E^ˆ ⁼ *^E* ^P

we can now distinguish three regimes ℓ dependence of avg. relaxation rate $\langle \Gamma_1 \rangle \propto \frac{2\gamma_2}{\gamma_2{}^2+\delta\omega^2} \propto \begin{cases} T^{-(\alpha+1)} & , \quad \delta\omega \lesssim \gamma_2 \ \tau^{\alpha+1} & , \quad \delta\omega \gg \gamma_2 \end{cases}$ $\overline{{\gamma_2}^2 + \delta\omega^2} \,$ \propto $\int T^{-(\alpha+1)}$, $\delta\omega \lesssim \gamma_2$ $T^{\alpha+1}$, $\delta\omega \gg \gamma_2$ Temp. dependence of avg. relaxation rate Γ is a constant $2\gamma_2$ in $T^{-(\alpha+1)}$ in $\delta\omega \lesssim \gamma_2$ $\int_0^1 1/\sqrt{\alpha} \sqrt{\gamma_2^2 + \delta \omega^2} \propto \int T^{\alpha+1} \qquad \delta \omega \gg \gamma_2$ models can be both positive or negative, meaning the Z ۲. ✓ *E* ◆◆ Z *^T* ²*,i* and therefore ⇣ / ¹*/T*4(↵+1). $\delta_{11} \rangle \propto \frac{2\gamma_2}{\gamma_1} \propto \frac{1}{2} \int T^{-(\alpha+1)} \quad , \quad \delta \omega \lesssim \gamma_2 \quad ,$ $\int_1^1/\sqrt{\alpha} \sqrt{2^2 + \delta \omega^2} \sqrt{\alpha+1}$, $\delta \omega \gg 1$ *dEP*(*E*) 1 tanh2
1 tanh2 tanh2 tan 2*T* ⇡ $\overline{\mathbf{d}}$ *dEE*↵ = *T* ↵+1 *,* $\int T^{-}(\alpha+1)$ $\lambda_i, \lambda \geq 0$ $\alpha + 1$, *,* ! ⌧ 2*,i* ۱p 0 \blacksquare σ ^{2/2} $\int T^{-(\alpha+1)}$, $\delta \omega$

$$
\gamma_{q,i}^{(1)} = \frac{\partial \gamma_{q,i}}{\partial E_i} \Big|_{E_i = E_{i,0}} = \cos^2 \theta_i \frac{4 \gamma_{2,i} (\omega_{10} - E_{i,0})}{(\gamma_{2,i}^2 + (\omega_{10} - E_{i,0})^2)^2} \qquad \Longrightarrow \qquad \langle \hat{\gamma}_{q,i}(0) \hat{\gamma}_{q,i}(t) \rangle_\omega \propto \begin{cases} T^{-5(\alpha+1)} & , \quad \delta \omega \ll \gamma_{2,i} \\ T^{-3(\alpha+1)} & , \quad \delta \omega \gg \gamma_{2,i} \\ T^{2(\alpha+1)} & , \quad \delta \omega \gg \gamma_{2,i} \end{cases}
$$

ing Fr $\textsf{C}\textsf{C}\textsf{C}$ dependence $\textsf{C}\textsf{C}\textsf{C}\textsf{C}(\textsf{C})\textsf{C}\textsf{C}(\textsf{C})\geq \omega \qquad \textsf{C}\textsf{C}\textsf{C}\gamma_1\sim 1/\gamma_1\quad \textsf{C}\textsf{C}$ ²*,i* and therefore ⇣ *q,i* ⌘² perform the $P(\gamma_1) \sim 1/\gamma_1$ induced the Γ These equations will be the basis for further calculations will be the basis for further calculations. The basis for further calculations will be the basis for further calculations. The basis for further calculations of Performing the average F requency dependence F (t) F (0) \log these \log as \log is \log \log \log $P(\gamma_1) \sim 1/\gamma_1$ **l** \mathbf{F} (1) \mathbf{F} (1) \mathbf{F} Frequency dependence $\langle \Gamma_1(t) \Gamma_1(0) \rangle_{\omega}$ $[P(\gamma_1) \sim 1/\gamma_1]$ nelling TLS, *P*(1) ⇠ 1*/*1, the frequency dependence

$$
\int_0^{\gamma_{\rm Max}} d\gamma_1 P(\gamma_1) \frac{2\gamma_1}{\gamma_1^2 + \omega^2} = \frac{2 \arctan \frac{\gamma_{\rm Max}}{\omega}}{\omega} \propto \begin{cases} \frac{1}{\omega} & , \quad \omega < \gamma_{\rm Max} \\ \frac{\gamma_{\rm Max}}{\omega^2} & , \quad \omega > \gamma_{\rm Max} \end{cases}
$$

 \blacklozenge

find the temperature and frequency dependence of the *T*¹

(1)

(1)

q,i ⌘²

q,i ⌘²

/ ¹*/T*6(↵+1).

Experimental results

A, B [a.u.]

For their data, the temperature dependence of the fluctuation amplitude is For their data, the temperature dependence of the fluctuation amplit
inconclusive and does not give any indication if the model is accurate

092 mHz) for red (blue) dashed (blue) dashed in the control of the control of the control of the control of th
H

 $\overline{7.092}$ $\overline{7.094}$ $\overline{7.096}$ $\overline{7.098}$ $\overline{7.100}$ $\overline{7.102}$ $\overline{7.104}$ $\overline{7.106}$

which are weakly contained on the qubit, ω [GHz]

Alternative explanations TLS as sources of the fluctuations, which depends on a source of the fluctuations, which depends on a source o

I) Fluctuations of the quasiparticle density in the superconductor ing the theory of Ref. 38 we calculate the fluctuations in the fluctuations in the fluctuations in the fluctua
The fluctuations in the fluctu

Quasiparticle tunnelling across the circuit's Josephson junctions can induce relaxation and dephasing, but the quasiparticle induced noise is flat at high-frequencies quasiparticle increase noise to musical laxation rate by 1 kHz as *nqp* ⇡ ⁰*.*5*/µ*m³, see appendix

 \blacklozenge

\blacklozenge \Box the number of q and q and q and q and q and q there of the one of

to change the relaxation rate by 1 kHz n rate by 1 kHz $\qquad \blacktriangleright \qquad \delta N_{qp} \approx 1.5 \times 10^4$

structured noise spectrum as background, the quasipar-

 $\mathbf w$ are quasiparticle density required to e $\mathbf w$

2) the qubit level splitting was fluctuating as a function of time, e.g. due to changes in the critical current of the circuits Josephson junction ting was fluctuating as a function of time, e.g. due to time, e.g. due to time, e.g. due to time, e.g. due to
The contract under the contract of time, e.g. due to time, e.g. due to time, e.g. due to time, e.g. due to tim

changes in the critical current of the circuits Josephson junction \mathcal{S} . Together with the observed structure struc \blacklozenge

ture in the noise spectrum $\mathbf{1}_{\text{max}}$ this would also explained also tuled out since in our ineasurements the qubit is always resonantly excited This mechanism can however be ruled out since in our measurements the

Conclusions

A simple model of interacting TLS which offers a qualitative understanding of the observed fluctuations in relaxation time

The model is grounded in experimental observations, grants a clear route towards further confirmation, and provides a way to verify and refine the existing microscopic TLS models

Proposed model clearly indicates that parasitic TLS are a limiting factor in today's best performing superconducting circuits

 \rightarrow A better understanding of this decoherence source is thus vital for further improving the fidelity of superconducting quantum circuits

THE END