Interacting two-level defects as sources of fluctuating high-frequency noise in superconducting circuits

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Clemens Müller, Jürgen Lisenfeld, Alexander Shnirman, and Stefano Poletto

Fluctuations in relaxation rate



- Time needed to obtain single value of T_1 is ~min
- There is no apparent structure to fluctuations of T₁
- There are two level systems (TSL) inside the dieletric forming Josephson junctions (needed to realize the qubit)

Qualitative description of fluctuating T₁

Charge defects



$$\hat{H}_{\rm TLS} = -\frac{1}{2} \epsilon \sigma_z + \frac{1}{2} \Delta \sigma_x$$

Assimetry of DQW Tunnel splitting

TSL coupling to the qubit via dipole moment~ σ_z

TSL emsemble

- i) High-frequency TS (E>>T, E~ ω_{10})
- ii) Low-frequency TF (E<<T)
- iii) Slowly fluctuating TF



Qualitative description of fluctuating T₁

$$C(\omega) = \cos^2 \theta \left[1 - \langle \sigma_z \rangle^2 \right] \frac{2\gamma_1}{\gamma_1^2 + \omega^2} + \sin^2 \theta \left[\frac{1 - \langle \sigma_z \rangle}{2} \right] \frac{2\gamma_2}{\gamma_2^2 + (\omega + E)^2} + \sin^2 \theta \left[\frac{1 + \langle \sigma_z \rangle}{2} \right] \frac{2\gamma_2}{\gamma_2^2 + (\omega - E)^2}$$

Low-freq noise (from TFs) due to thermal switching

High-freq noise (from TS) leading to relaxation of the qubit



The qubit relaxation via spin-flip process with TS

slow TFs change E of TS

- TFs influence Υ_2

Calculation

Coupling between TSLs
$$\hat{H} = \frac{1}{2} \sum_{\langle ij \rangle} g_{ij} \tilde{\sigma}_{z,i} \tilde{\sigma}_{z,j}$$

Energy splitting of TSL
$$\hat{E}_i$$
 =

$$\hat{E}_i = E_{i,0} - \sum_{\langle ij \rangle} g_{ij} \tilde{\sigma}_{z,j} \qquad \qquad E_{i,0} = \sqrt{\epsilon_i^2 + \Delta_i^2}$$

Relaxation rate of the qubit due to TSL

$$\hat{\gamma}_{q,i} = \cos^2 \theta_i \frac{2\gamma_{2,i}}{\gamma_{2,i}^2 + (\omega_{10} - \hat{E}_i)^2}$$

$$\begin{aligned} \hat{\gamma}_{q,i} &= \gamma_{q,i}^{(0)} + \gamma_{q,i}^{(1)} \sum_{\substack{\langle ij \rangle \\ 2\gamma_{2,i} \\ \gamma_{q,i}^{(0)} = \cos^2 \theta_i \frac{2\gamma_{2,i}}{\gamma_{2,i}^2 + (\omega_{10} - E_{i,0})^2}, \qquad \langle \tilde{\sigma}_z \rangle = \cos \theta \, \langle \sigma_z \rangle = \cos \theta \tanh \left(E/2k_B T \right) \\ \gamma_{q,i}^{(1)} &= \frac{\partial \gamma_{q,i}}{\partial E_i} \Big|_{E_i = E_{i,0}} = \cos^2 \theta_i \frac{4\gamma_{2,i}(\omega_{10} - E_{i,0})}{(\gamma_{2,i}^2 + (\omega_{10} - E_{i,0})^2)^2}. \end{aligned}$$

Distribution of parameters

Flat distribution of TLS barrier height

$$P(\Delta) \sim 1/\Delta \quad P(\gamma_1) \sim 1/\gamma_1$$

Assuming general distribution in $\epsilon P(\epsilon, \Delta) d\epsilon d\Delta = A \frac{\epsilon^{\alpha}}{\Delta} d\epsilon d\Delta$

Distribution of coupling constant assuming dipolar interaction between TSLs

$$P(g)dg = P(r)\frac{\partial r}{\partial g}dg = \rho_0 |g|^{-\frac{4}{3}} dg$$

coupling constant can take both positive and negative values

$$\oint dg \ g P(g) = 0$$

Calculation of $<\Gamma_1 > \text{ and } <\Gamma_1(t)\Gamma_1(0)>_{\omega}$

$$\begin{split} \langle \hat{\gamma}_{q,i} \rangle &= \gamma_{q,i}^{(0)} + \gamma_{q,i}^{(1)} \sum_{j} g_{j} \cos \theta_{j} \tanh \frac{E_{j}}{2T} \\ &= \gamma_{q,i}^{(0)} + \gamma_{q,i}^{(1)} \int dg \, d\theta \, dE \, P(g,\theta,E) g \cos \theta \tanh \frac{E}{2T} \end{split}$$

$$\begin{split} \left\langle \hat{\gamma}_{q,i}(0)\hat{\gamma}_{q,i}(t)\right\rangle_{\omega} &= \int dt \,\mathrm{e}^{-\mathrm{i}\omega t} \left\langle \hat{\gamma}_{q,i}(0)\hat{\gamma}_{q,i}(t)\right\rangle \\ &= \left(\gamma_{q,i}^{(1)}\right)^{2} \sum_{j,l} g_{j}g_{l} \left\langle \tilde{\sigma}_{z,j}(t)\tilde{\sigma}_{z,l}(0)\right\rangle_{\omega} \\ &= \left[\left(\gamma_{q,i}^{(1)}\right)^{2}\right) \int dg \,d\theta \,dE \,d\gamma_{1} \,P(g,\theta,E,\gamma_{1})g^{2}\cos^{2}\theta \left[1-\tanh^{2}\left(\frac{E}{2T}\right)\right] \frac{2\gamma_{1}}{\gamma_{1}^{2}+\omega^{2}} \\ & \checkmark \\ \int dg \,g^{2}P(g) \propto \mathrm{const} \qquad \int dE P(E) \left(1-\tanh^{2}\left(\frac{E}{2T}\right)\right) \approx \int_{0}^{T} dE E^{\alpha} = T^{\alpha+1} \end{split}$$

Calculation of $<\Gamma_1 > \text{ and } <\Gamma_1(t)\Gamma_1(0)>_{\omega}$

Similarly, dephasing rate due to bath of TF $\gamma_2 \propto T^{lpha+1}$

$\mathbf{\Psi}$

Temp. dependence of avg. relaxation rate $\langle \Gamma_1 \rangle \propto \frac{2\gamma_2}{\gamma_2^2 + \delta\omega^2} \propto \begin{cases} T^{-(\alpha+1)} &, & \delta\omega \lesssim \gamma_2 \\ T^{\alpha+1} &, & \delta\omega \gg \gamma_2 \end{cases}$

$$\gamma_{q,i}^{(1)} = \frac{\partial \gamma_{q,i}}{\partial E_i}\Big|_{E_i = E_{i,0}} = \cos^2 \theta_i \frac{4\gamma_{2,i}(\omega_{10} - E_{i,0})}{\left(\gamma_{2,i}^2 + (\omega_{10} - E_{i,0})^2\right)^2} \quad \clubsuit \quad \left\langle \hat{\gamma}_{q,i}(0)\hat{\gamma}_{q,i}(t) \right\rangle_{\omega} \propto \begin{cases} T^{-5(\alpha+1)} &, & \delta\omega \ll \gamma_{2,i} \\ T^{-3(\alpha+1)} &, & \delta\omega \sim \gamma_{2,i} \\ T^{2(\alpha+1)} &, & \delta\omega \gg \gamma_{2,i} \end{cases}$$

Frequency dependence $\langle \Gamma_{l}(t) \Gamma_{l}(0) \rangle_{\omega}$ [P(γ_{1}) ~ 1/ γ_{1}]

$$\int_{0}^{\gamma_{\text{Max}}} d\gamma_1 P(\gamma_1) \frac{2\gamma_1}{\gamma_1^2 + \omega^2} = \frac{2 \arctan \frac{\gamma_{\text{Max}}}{\omega}}{\omega} \propto \begin{cases} \frac{1}{\omega} & , \quad \omega < \gamma_{\text{Max}} \\ \frac{\gamma_{\text{Max}}}{\omega^2} & , \quad \omega > \gamma_{\text{Max}} \end{cases}$$

Experimental results

For their data, the temperature dependence of the fluctuation amplitude is inconclusive and does not give any indication if the model is accurate



7.096

7.098

7.100

7.102

7.104

7.106

 ω [GHz]

7.094

7.092

Alternative explanations

I) Fluctuations of the quasiparticle density in the superconductor

$\mathbf{\Psi}$

Quasiparticle tunnelling across the circuit's Josephson junctions can induce relaxation and dephasing, but the quasiparticle induced noise is flat at high-frequencies

$\mathbf{\Psi}$

to change the relaxation rate by I kHz $\rightarrow \delta N_{qp} \approx 1.5 \times 10^4$

2) the qubit level splitting was fluctuating as a function of time, e.g. due to changes in the critical current of the circuits Josephson junction

\mathbf{V}

This mechanism can however be ruled out since in our measurements the qubit is always resonantly excited

Conclusions

- A simple model of interacting TLS which offers a qualitative understanding of the observed fluctuations in relaxation time

 The model is grounded in experimental observations, grants a clear route towards further confirmation, and provides a way to verify and refine the existing microscopic TLS models

- Proposed model clearly indicates that parasitic TLS are a limiting factor in today's best performing superconducting circuits

→A better understanding of this decoherence source is thus vital for further improving the fidelity of superconducting quantum circuits

THE END