# Journal Club

**Spin Correlations as a Probe of Quantum Synchronization in Trapped Ion Phonon-Lasers**

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# Outline

1) Introduction
   - Basics of synchronization
   - Why ions in microtraps?
2) Trapped Ion Setup
3) Individual Ions
   - Self-oscillation
   - Correlation between spin and phonon degrees of freedom
4) Coupled Ions
5) Synchronization and Spin Correlations
6) Conclusions

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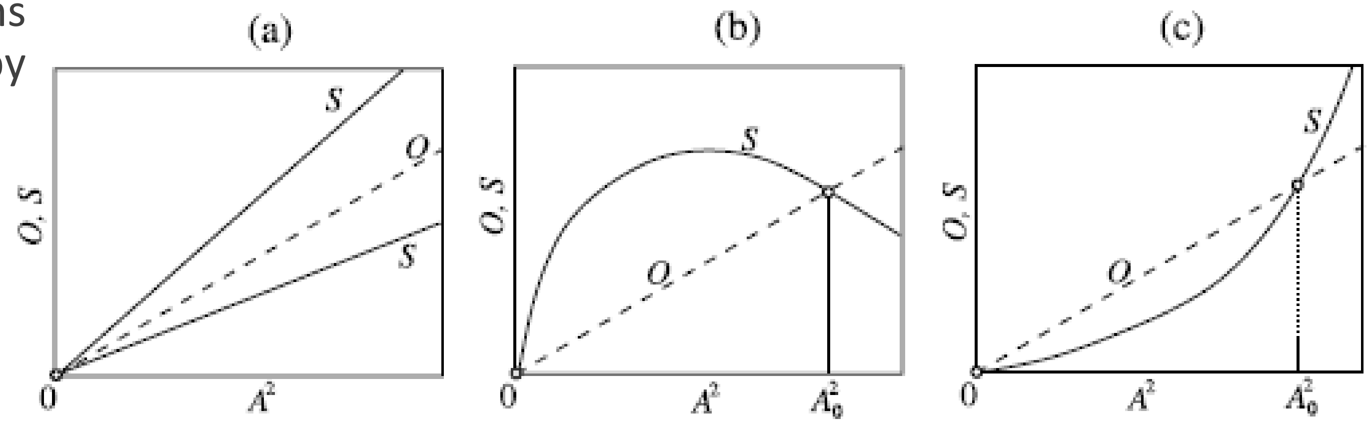
*Introduction.* Two macroscopic self-oscillators synchronize when their *relative phase* locks to a fixed value [1].

## Features of Self-Oscillations

- They do not damp, i.e., the repetitive motion of the system does not stop with the course of time.
- They oscillate “by themselves”, i.e., not because they are repetitively kicked from outside.
- The shape, amplitude and time scale of these oscillations are chosen by the oscillating systems alone, e.g., they are not easily changed by setting different initial conditions.

## Essentials for Self-Oscillations

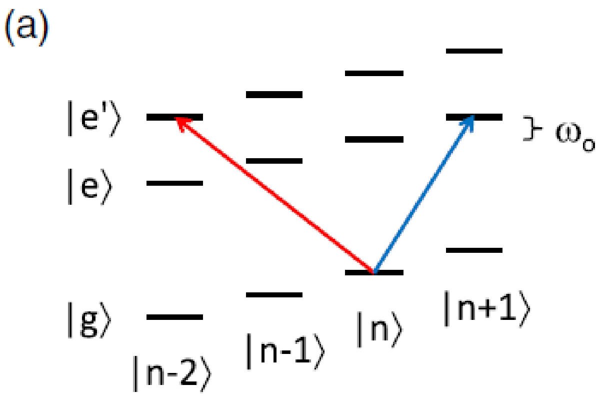
- Dissipation
- Power Source
- Non-linear



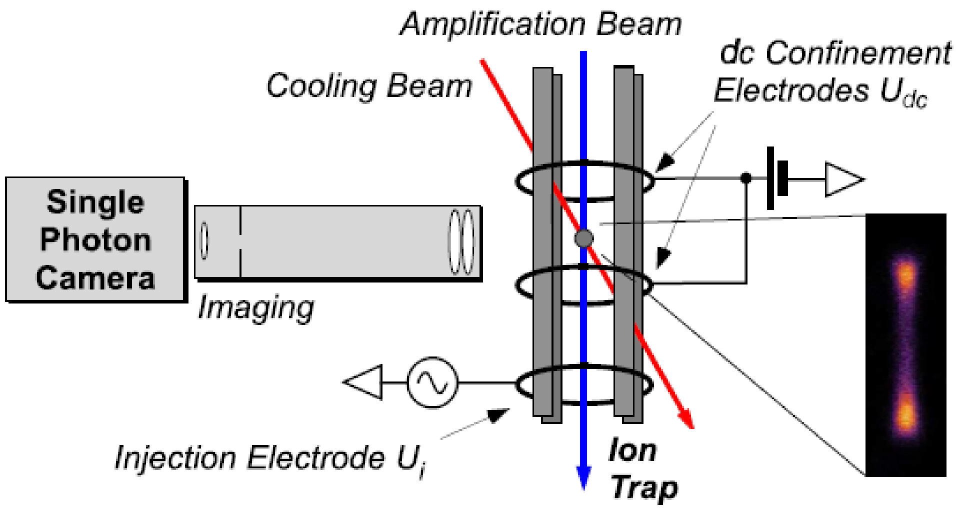

Balanov at el. – Synchronization: From Simple to Complex

Cold ions in microtraps provide a natural platform for exploring synchronization in the quantum regime [5]. The generation of self-oscillations in the motional state of ions, phonon-lasing, has already been observed [15]. Furthermore, precise control of trapping potentials of the individual ions can now be achieved with microtraps [16] allowing the vibrational frequencies of individual ions and the coupling between different ions to be tuned.

- Negative damping comes from exciting the blue sideband of $|g\rangle \rightarrow |e\rangle$
- Nonlinear damping comes from exciting the red double sideband of $|g\rangle \rightarrow |e'\rangle$

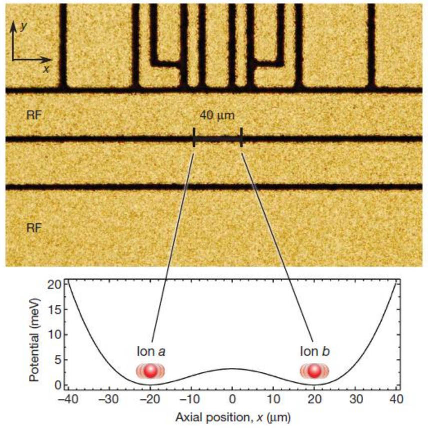



[5] Lee & Sadeghpour, Phys. Rev. Lett. 111, 234101 (2013)




[15] Knünz et al., Phys. Rev. Lett. 105, 013004 (2010)

- Two ions held in separated trapping potential are coupled via their Coulomb interaction




[16] Brown et al., Nature, 471, 234101 (2011)

## 1) Introduction

➢ Why ions in microtraps?

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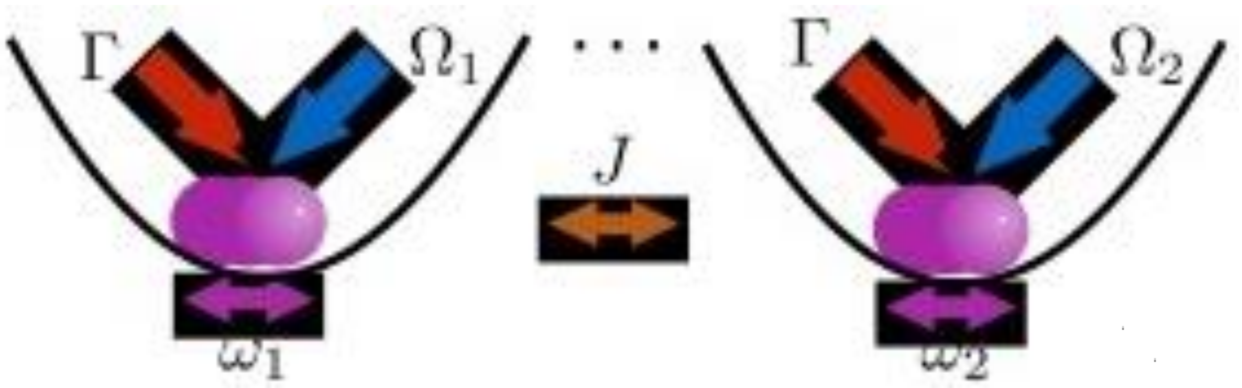
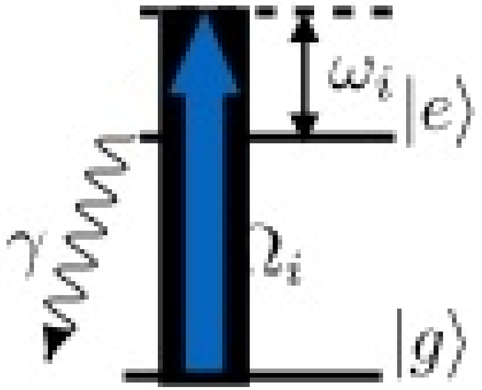
Each ion is in a microtrap [16] with frequency $\omega_{j=1,2}$. The quantized vibrational degree of freedom (phonons) are linearly damped at a rate $\Gamma$ using standing-wave lasers, as described in [17, 18]. Each ion's internal degree of freedom is driven by plane wave lasers with Rabi frequencies $\Omega_{j=1,2}$, which are set to be resonant with the first blue sideband transition.

**Assumptions**

- The dynamics of the internal degrees of freedom is assumed to occur on the fastest time scale.
- Lamb-Dicke Approximation: Keeping terms that change the phonon number by up to 1.
- RWA: $\omega_j \gg \Omega, \gamma, \Gamma, \Delta$




$$\dot{\rho} = -[H, \rho] + \sum_{j=1,2} \left\{ \gamma D[\sigma_j^-](\rho) + \Gamma D[a_j](\rho) \right\}$$

Damping terms

Internal degree of freedom

Number of phonons

Blue detuned laser - amplification

Coupling

$$D[L](\rho) \equiv L\rho L^\dagger - (L^\dagger L \rho + \rho L^\dagger L)/2$$

$$H \equiv \sum_{j=1,2} \left\{ (-1)^j \frac{\Delta}{4} \left( 2a_j^\dagger a_j - \sigma_j^z \right) + \Omega_j \left( a_j^\dagger \sigma_j^\dagger + a_j \sigma_j^- \right)/2 \right\} + J(a_2^\dagger a_1 + a_1^\dagger a_2)$$

# 2) Trapped Ion Setup

# Outline

*Individual Ions.* A prerequisite for synchronization is that each individual ion undergoes self-oscillations in their motion, so-called phonon lasing [15].

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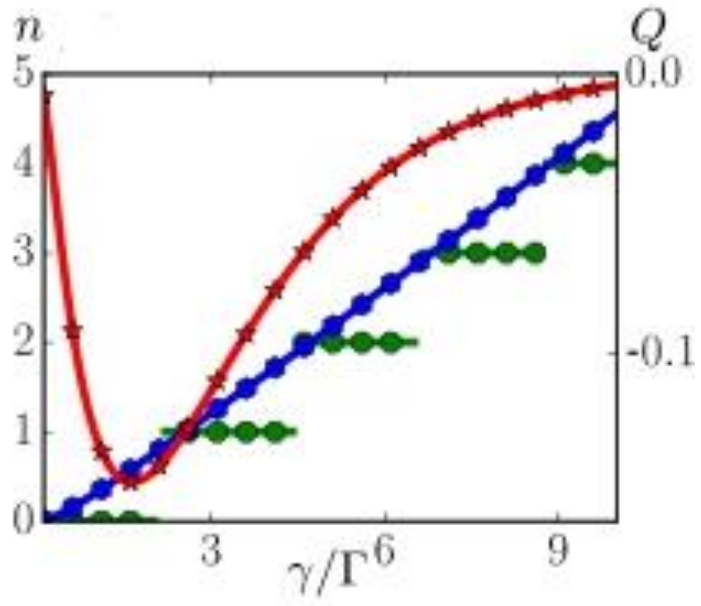
- For $\Omega^2 > \gamma\Gamma$ : The mean field equations of motion show a limit cycle solution with $\langle n \rangle = \dfrac{\gamma}{2\Gamma} - \dfrac{\gamma^2}{2\Omega^2}$ .

$n_a$ (blue line)

$n_m$ (green line)

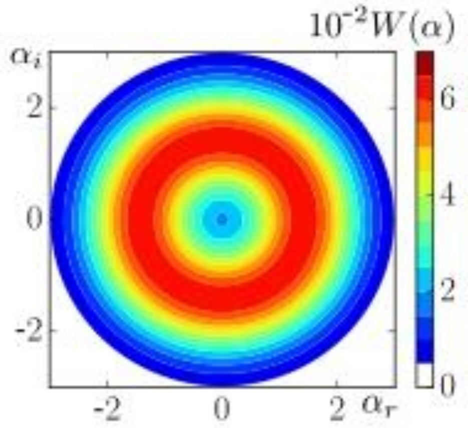
$$Q = \frac{\langle n^2 \rangle - \langle n \rangle^2}{\langle n \rangle} - 1$$ (red line)




# 3) Individual Ions

➢ Self-oscillation

Before investigating synchronization in coupled ions, we examine the correlations that build up between the spin and phonon degrees of freedom in an individual ion due to their strong coupling.

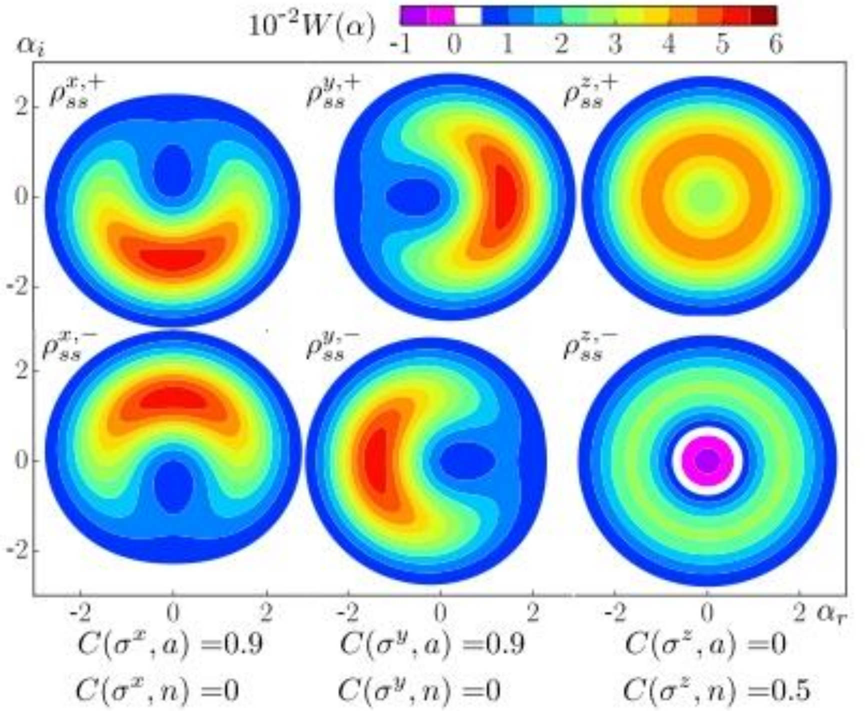



**Wigner distribution function**

- Non-zero average amplitude.
- No phase preference

**Wigner distribution function – After projection with one of Pauli-operator eigenstates**

- $\sigma_z$ - It's eigenstates are correlated with phonon number, but not with phase.
- $\sigma_x, \sigma_y$ - Their eigenstates are correlated with phase, but not with phonon number




**3) Individual Ions** ➢ Correlation between spin and phonon degrees of freedom

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*Coupled Ions.* We now consider how synchronization arises for two weakly coupled ions.

We look for a signature of synchronization by calculating the relative phase distribution $P(\phi)$ from the steady state solution to the master equation when ions are in the lasing regime and weakly coupled: $J/\gamma = 1/10$.

**Relative Phase Distribution**

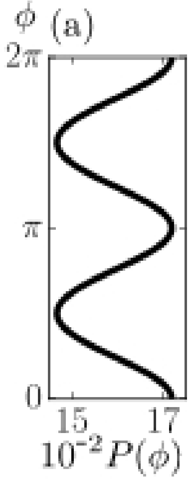
$$P(\phi) = \iint_0^{2\pi} \mathrm{d}\phi_1 \mathrm{d}\phi_2 \delta(\phi_1 - \phi_2 - \phi) \langle \phi_1, \phi_2 | \rho_{ss}^p | \phi_1, \phi_2 \rangle$$

**Measure of Synchronization:**

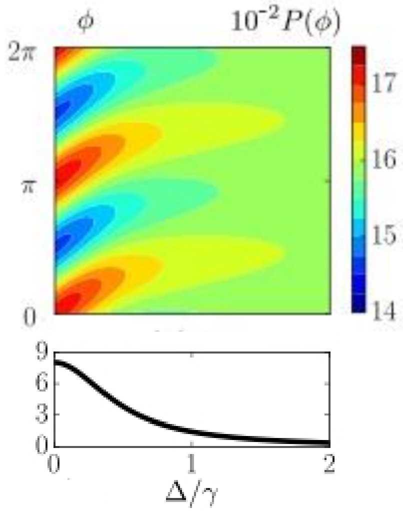
$$S = 2\pi \max[P(\phi)] - 1$$

$$\rho_{ss}^p = \mathrm{Tr}_s[\rho_{ss}] \quad |\phi_j\rangle = \frac{1}{\sqrt{2\pi}} \sum_{n=0}^{\infty} e^{i\phi_j n} |n\rangle$$

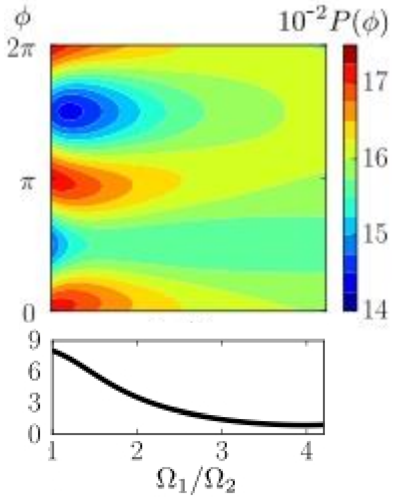
**Symmetric Case:** $\Delta = 0$ , $\frac{\Omega_1}{\Omega_2} = 1$




**Vary Detuning,** $\frac{\Omega_1}{\Omega_2} = 1$




**Vary Pumping Strength,** $\Delta = 0$




# 4) Coupled Ions

# Outline

the spin-phonon locking seen in Fig. 2(c) suggests we may be able to infer the presence of synchronization indirectly through measurements [19] of the spin degrees of freedom alone.

## Semi Classical Approximation

- Making the approximation $\langle \sigma_j^i a_j \rangle \approx \langle \sigma_j^i \rangle \langle a_j \rangle$ .
- Obtaining mean field relations $\langle \sigma_j^x \rangle \propto -\sin\phi_j$, $\langle \sigma_j^y \rangle \propto \cos\phi_j$.
- Correlation functions are $C(X,Y) = 0$ by definition.

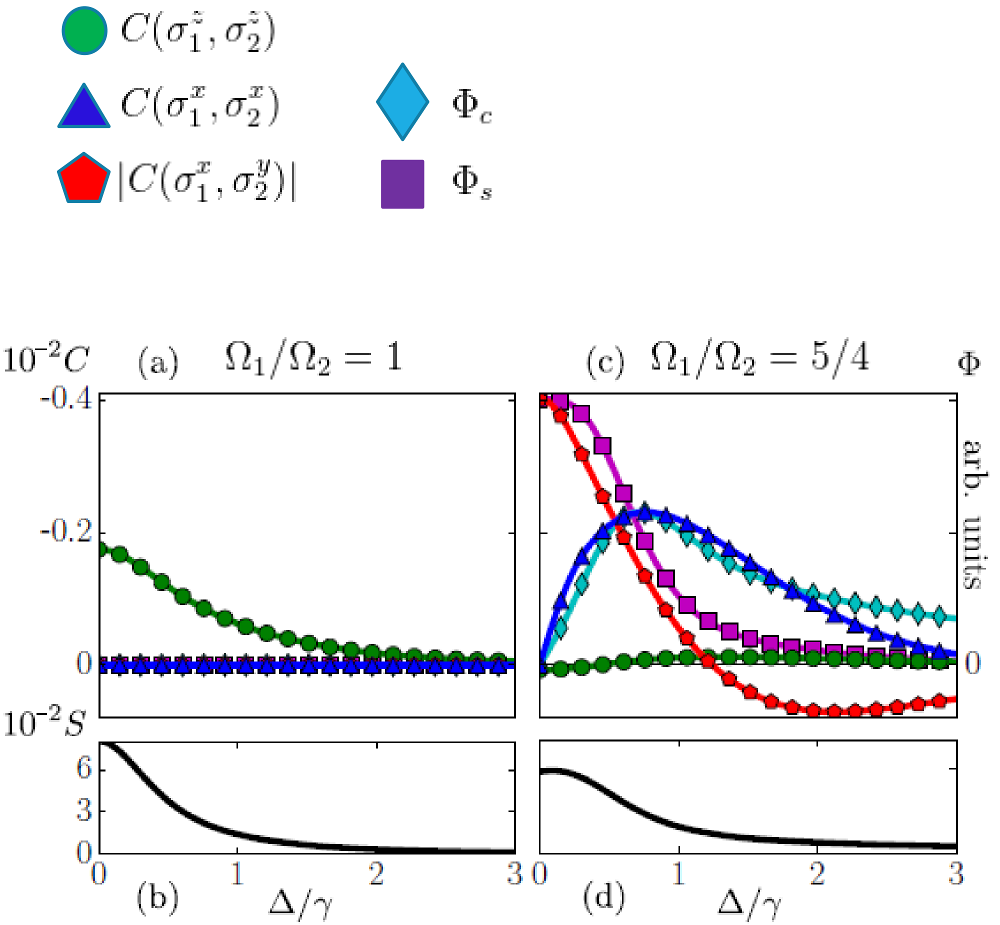
**However, making the assumption that** $\sigma_j^x \propto -\sin\phi_j$ **,** $\sigma_j^y \propto \cos\phi_j$ .

- Calculating expectation values by taking an average over the steady state phase distribution leads to:

$$\langle \sigma_j^x \rangle \propto 0 \qquad \langle \sigma_1^x \sigma_2^x \rangle \propto \int \mathrm{d}\phi \cos\phi P(\phi) \equiv \Phi_c$$

$$\langle \sigma_j^y \rangle \propto 0 \qquad \langle \sigma_1^x \sigma_2^y \rangle \propto -\int \mathrm{d}\phi \sin\phi P(\phi) \equiv \Phi_s$$

- $\Phi_{c,s} \neq 0$ is a sufficient condition for synchronization



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# Conclusions

- Two phonon-lasing ions undergo synchronization when they are weakly coupled.
- Strong correlation develop between the internal degrees of freedom and the phonons in each ion – This leads to correlations between the internal degrees of freedom of both ions when they are coupled.
- These correlations carry information about the relative phase distribution of the ions and could be used to infer the presence of synchronization.

# Thank you for listening!