arXiv:1503.08673

Open quantum system description of singlet-triplet qubits in quantum dots

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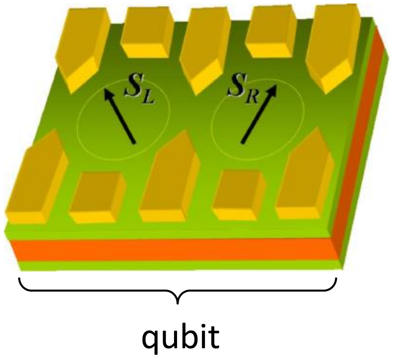
Journal Club, May 12, 2015

Singlet-Triplet Qubits (S-T_0)



Common approach:
Singlet-triplet qubits
in double quantum dots

Levy, PRL (2002)
Petta *et al*., Science (2005)

Basis states:

$$|S\rangle = \frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}$$

$$|T_0\rangle = \frac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{\sqrt{2}}$$

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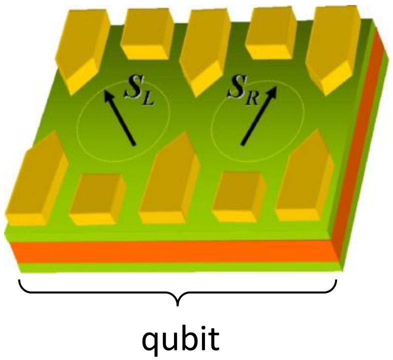

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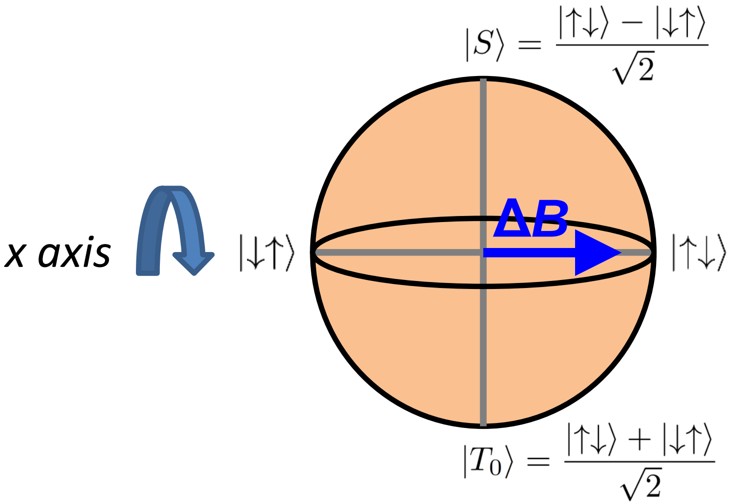
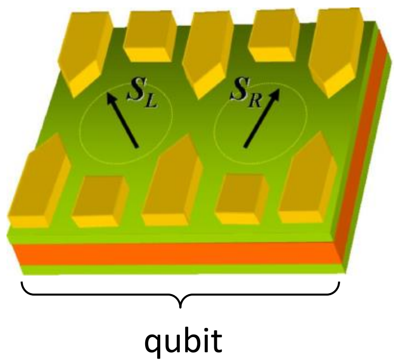


Magnetic field gradient

Singlet-Triplet Qubits (S-T_0)



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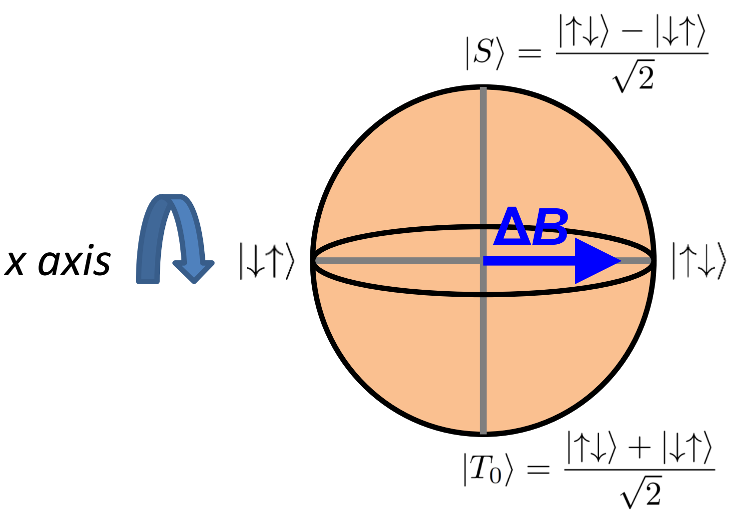

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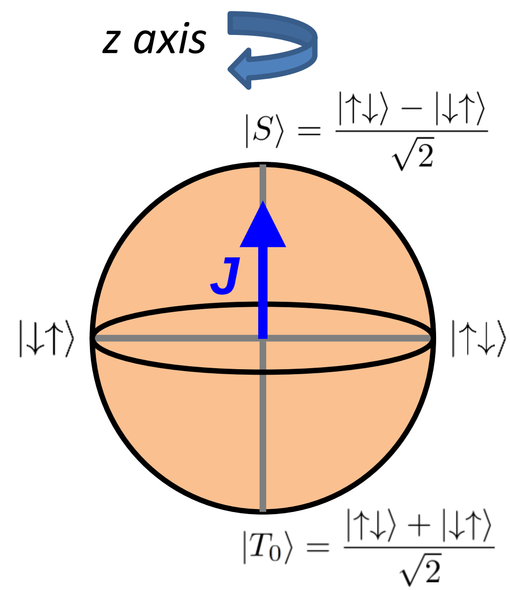
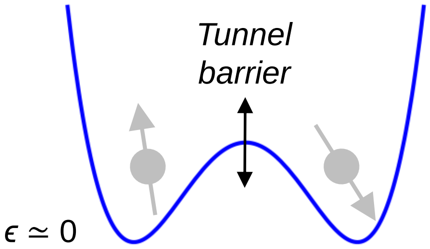


Exchange splitting

Exchange Coupling and Detuning

Control of *J* via tunnel barrier

Loss/DiVincenzo, PRA (1998)
Burkard/Loss/DiVincenzo, PRB (1999)



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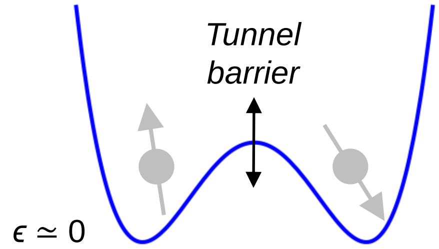

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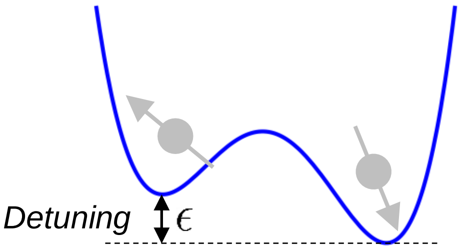
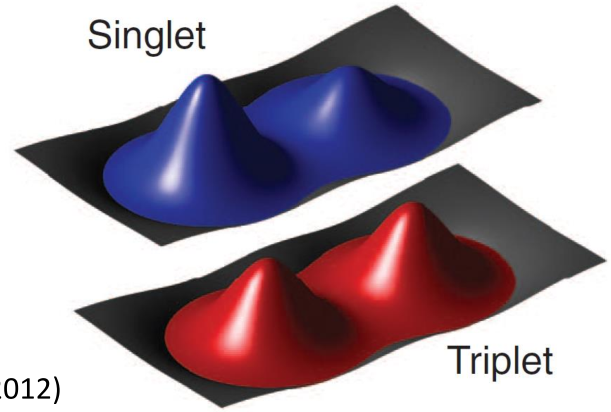


Example:
$\epsilon < 0$



Different charge distribution due to Pauli exclusion

Picture from
Shulman *et al.*, Science (2012)

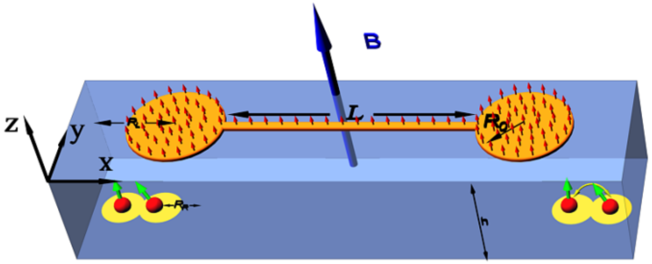
Two-Qubit Gates

- **Via exchange coupling and spin-orbit interaction**

  Klinovaja/Stepanenko/Halperin/Loss, PRB (2012)

- **Via ferromagnets (long-distance)**

  Trifunovic/Pedrocchi/Loss, PRX (2013) & arXiv:1305.2451



- **Via capacitive coupling through floating gates (long-distance)**

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  van Weperen *et al.*, PRL (2011)
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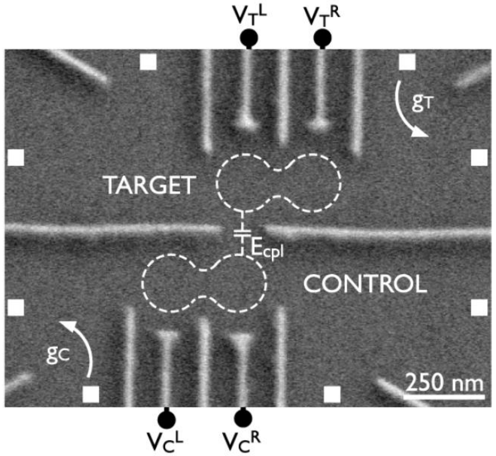

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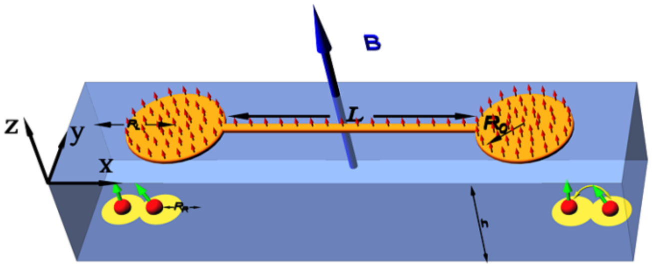

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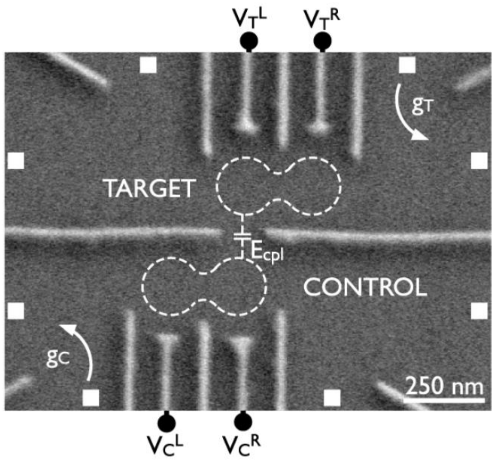

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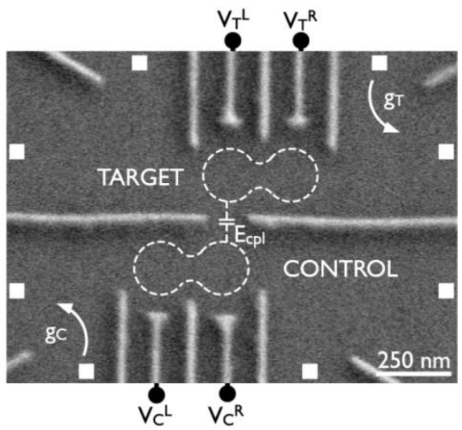
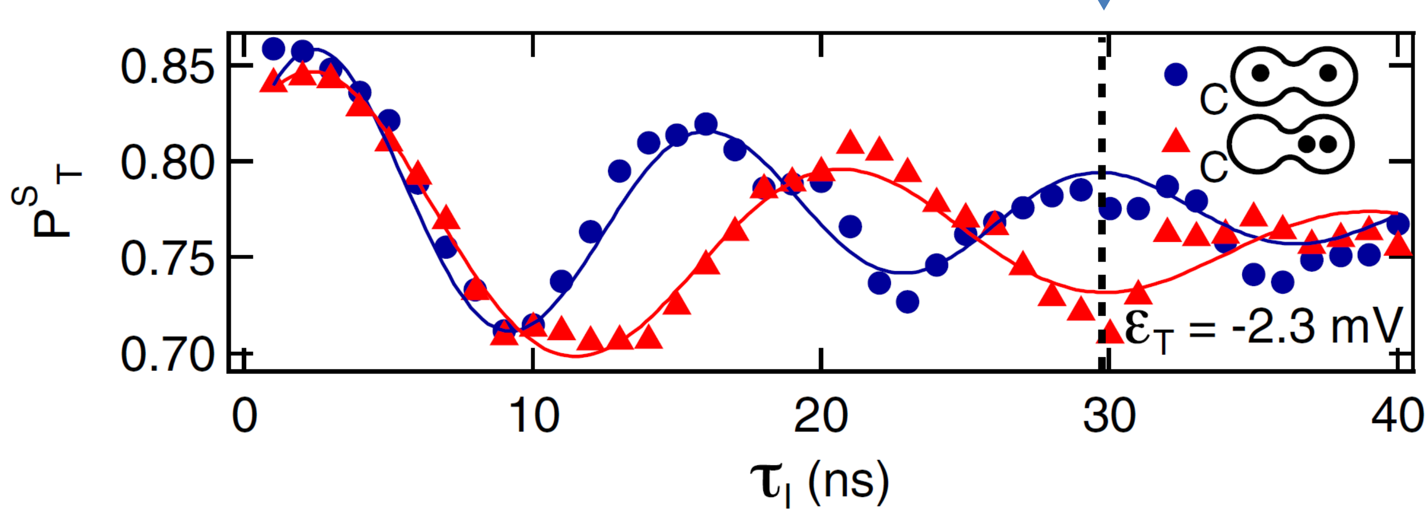
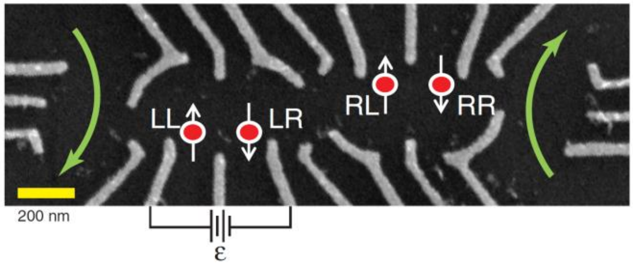


Charge-state conditional phase flip



Allows implementation of the CPHASE gate

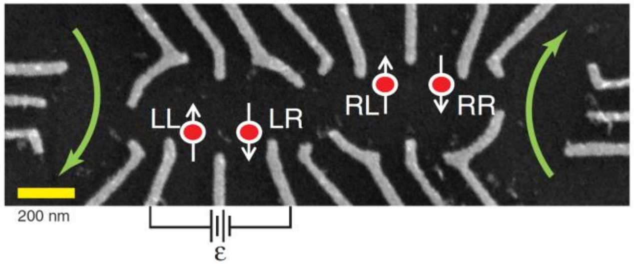
Shulman *et al.*, Science (2012)



Main results:

- **Experimental verification that the CPHASE gate is entangling**
- **(Charge) Noise is a big problem**

Shulman *et al.*, Science (2012)



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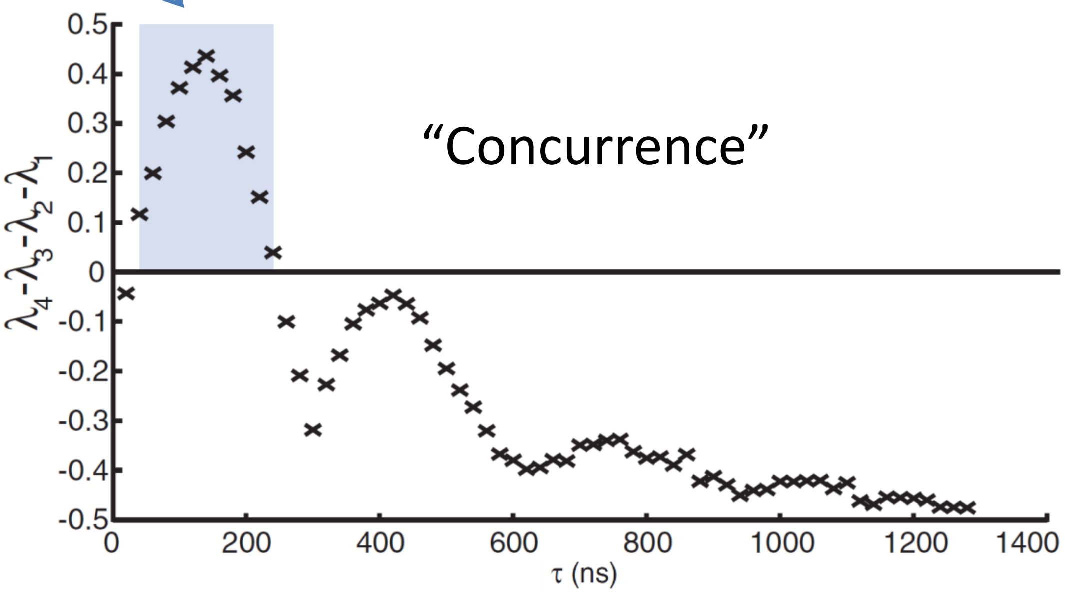
necessarily entangled

“Concurrence”



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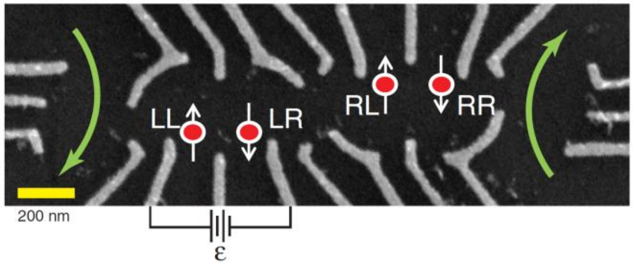

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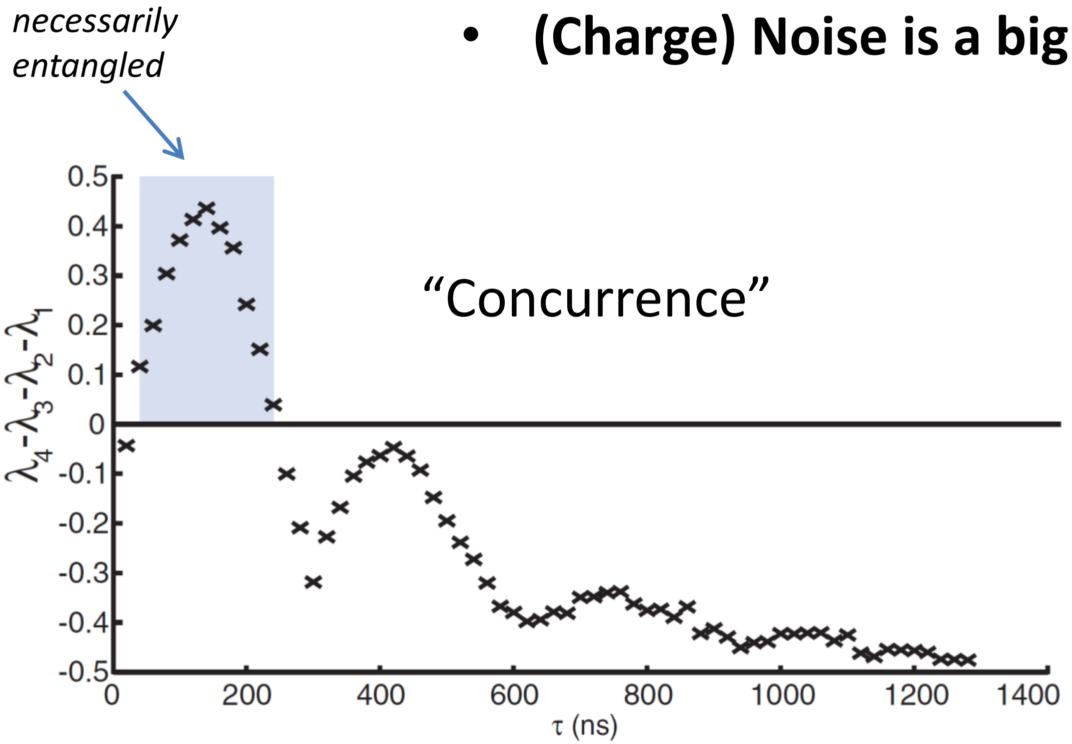

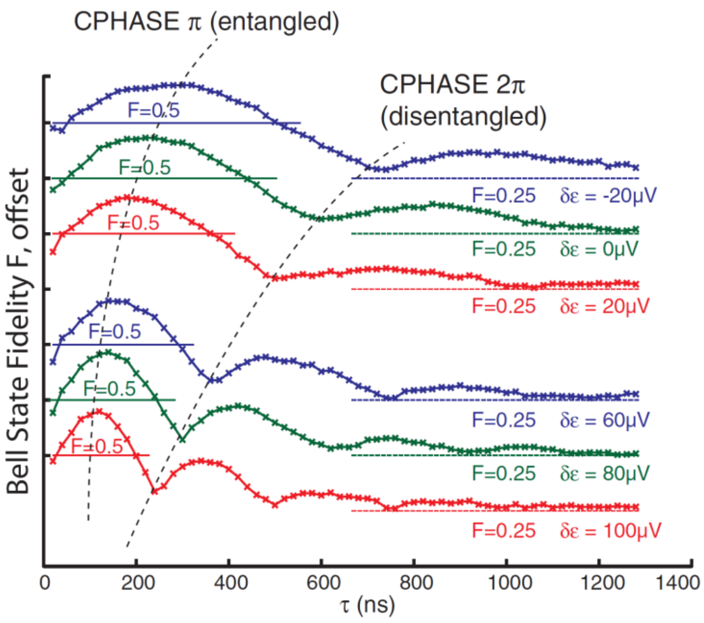

Concurrence

Hill/Wootters, PRL (1997)
Shulman *et al*., Science (2012)

Is the two-qubit state |ψ> entangled?

Concurrence

Hill/Wootters, PRL (1997)
Shulman *et al*., Science (2012)

Is the two-qubit state $|\psi\rangle$ entangled?

- Take the density matrix $\rho = |\psi\rangle\langle\psi|$
- Calculate the matrix $R = \sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}}$

$$\tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$$

- Calculate the four eigenvalues λ_i of the matrix *R* and define the largest eigenvalue as λ_4
- Get the concurrence $C(\rho)$ through the relation

$$C(\rho) = \max\{0, \lambda_4 - \lambda_3 - \lambda_2 - \lambda_1\}$$

Concurrence

Hill/Wootters, PRL (1997)
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- Calculate the four eigenvalues λ_i of the matrix *R* and define the largest eigenvalue as λ_4
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  $$C(\rho) = \max\{0, \lambda_4 - \lambda_3 - \lambda_2 - \lambda_1\}$$

→ A **positive value of *C*** is a necessary and sufficient condition for **entanglement**

Castelano/Fanchini/Berrada, arXiv:1503.08673

The authors want to develop a (noise) model that reproduces the results of Shulman *et al.*

This model may then be used to describe the dissipative dynamics of singlet-triplet qubits in double quantum dots

Hamiltonian

$$\hat{H} = \hat{H}_{\text{2-qubit}} + \hat{H}_b + \hat{H}_{int}$$

$$\begin{aligned}\hat{H}_{\text{2-qubit}} = \frac{1}{2}\Bigg[& \left(J_1 \sigma_z^{(1)} \otimes \mathbf{I} + J_2\, \mathbf{I} \otimes \sigma_z^{(2)}\right) + \\ & \frac{J_{12}}{2}\left(\sigma_z^{(1)} \otimes \sigma_z^{(2)} - \sigma_z^{(1)} \otimes \mathbf{I} - \mathbf{I} \otimes \sigma_z^{(2)}\right) + \\ & \frac{1}{2}\left(\Delta B_{z,1} \sigma_x^{(1)} \otimes \mathbf{I} + \Delta B_{z,2}\, \mathbf{I} \otimes \sigma_x^{(2)}\right)\Bigg]\end{aligned}$$

Hamiltonian

$$\hat{H} = \hat{H}_{2\text{-qubit}} + \hat{H}_b + \hat{H}_{int}$$

$$\hat{H}_{2\text{-qubit}} = \frac{1}{2}\Bigg[\left(J_1\sigma_z^{(1)}\otimes\mathbf{I} + J_2\,\mathbf{I}\otimes\sigma_z^{(2)}\right) + \frac{J_{12}}{2}\left(\sigma_z^{(1)}\otimes\sigma_z^{(2)} - \sigma_z^{(1)}\otimes\mathbf{I} - \mathbf{I}\otimes\sigma_z^{(2)}\right) + \frac{1}{2}\left(\Delta B_{z,1}\sigma_x^{(1)}\otimes\mathbf{I} + \Delta B_{z,2}\,\mathbf{I}\otimes\sigma_x^{(2)}\right)\Bigg]$$

$$\hat{H}_b^C = \sum_k \omega_k b_k^\dagger b_k$$

$$\hat{H}_{\text{int}}^C = \left(\sigma_z^{(1)} + \sigma_z^{(2)}\right)\mathcal{L}$$

$$\mathcal{L} = B + B^\dagger \qquad B = \sum_k g_k b_k$$

Collective bath

$$\hat{H}_b^I = \sum_{i=1}^{2}\sum_k \omega_k^{(i)} b_k^{\dagger\,(i)} b_k^{(i)}$$

$$\hat{H}_{\text{int}}^I = \sigma_z^{(1)}\mathcal{L}^{(1)} + \sigma_z^{(2)}\mathcal{L}^{(2)}$$

$$\mathcal{L}^{(i)} = B^{(i)} + B^{\dagger\,(i)} \qquad B^{(i)} = \sum_k g_k^{(i)} b_k^{(i)}$$

Independent baths

Master Equation

Redfield equation

$$\frac{d\rho_I(t)}{dt} = -\int_0^t dt' \mathrm{Tr}_B \left\{ [H_I(t), [H_I(t'), \rho_B \rho_I(t)]] \right\}$$

Master Equation

Redfield equation

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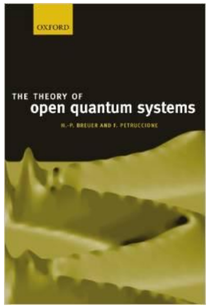


Breuer and Petruccione

The Theory of Open Quantum Systems

Chapter 3

Master Equation

reduced density matrix for the two qubits

Redfield equation

Markov approximation: t' → t

$$\frac{d\rho_I(t)}{dt} = -\int_0^t dt' \mathrm{Tr}_B \left\{ [H_I(t), [H_I(t'), \rho_B \rho_I(t)]] \right\}$$

$$H_I(t) = U^\dagger(t) U_B^\dagger(t) \hat{H}_{\text{int}} U_B(t) U(t)$$

$$U(t) = \exp\left(-i\hat{H}_{\text{2-qubit}} t\right)$$

$$U_B(t) = \exp\left(-i\hat{H}_b t\right)$$

Assumption: $\rho_B = \dfrac{1}{Z}\exp(-\beta\hat{H}_b)$

$$Z = \mathrm{Tr}_B\left[\exp(-\beta\hat{H}_b)\right]$$

$$\beta = 1/(k_B T)$$

oscillator bath initially decoupled

Parameters

Parameters and pulse sequences are chosen based on the experiment

Typical parameters are:

$$T = 50 \text{ mK}$$

$$J_1/2\pi \approx 280\text{MHz} \quad J_2/2\pi \approx 320\text{MHz}$$

$$\Delta B_{z,i}/2\pi \approx 30\text{MHz}$$

The qubit-bath coupling is parametrized via the spectral function

$$J(\omega) = \eta\omega \exp(-\omega/\omega_c)$$

Good agreement between theory and experiment found for η around 3 x 10^{-5}

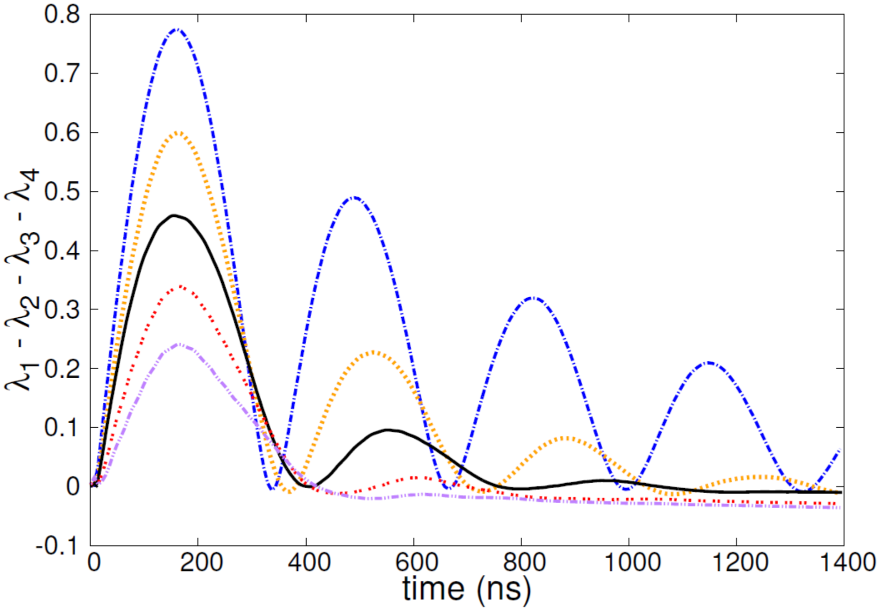
"After detailed analysis (not shown here) we find, as expected, that low cutoff frequencies are unable to reproduce the experimental data and we checked that all results presented in this work do not change significantly if ω_c > 2 x 10^4 MHz"

→ ω_c = 2 x 10^4 MHz in the simulation

Results

Numerical solution:

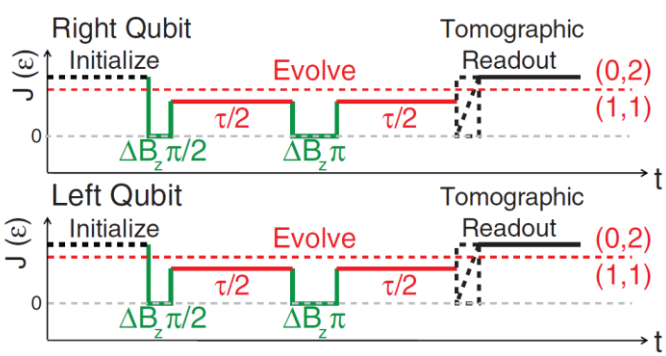
Common bath

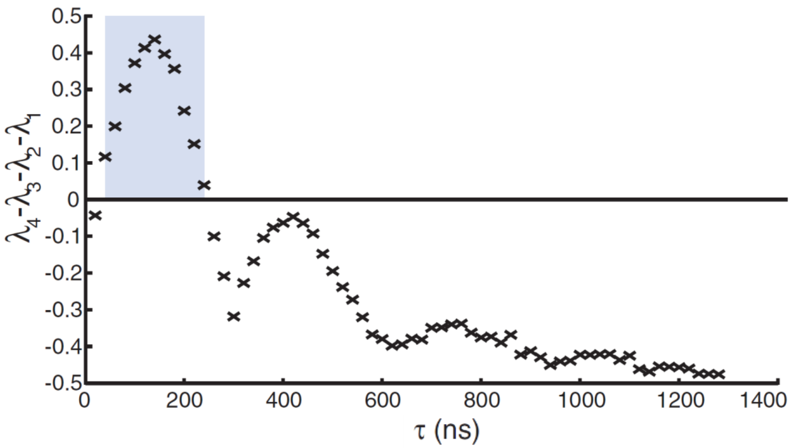


T = 50 mK

$\eta = 1 \times 10^{-5}$
$\eta = 2 \times 10^{-5}$
$\eta = 3 \times 10^{-5}$
$\eta = 4 \times 10^{-5}$
$\eta = 5 \times 10^{-5}$

Experiment:





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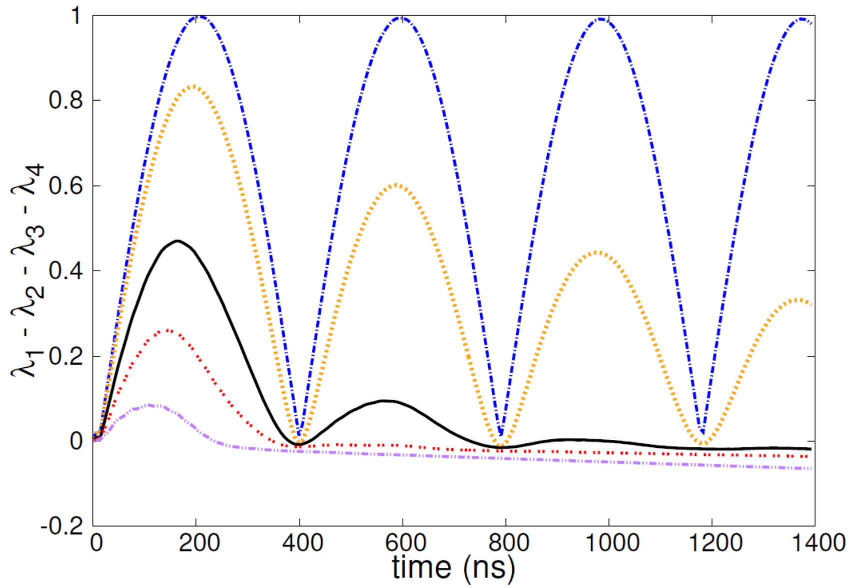
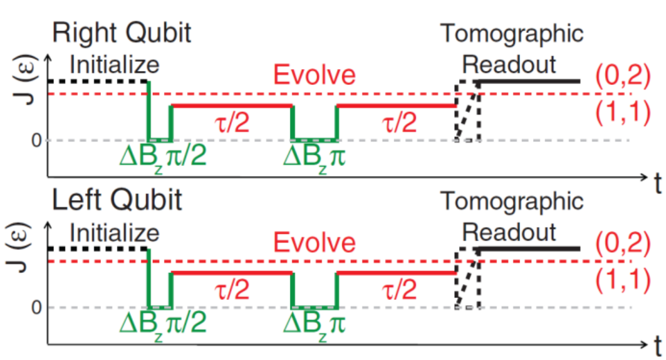
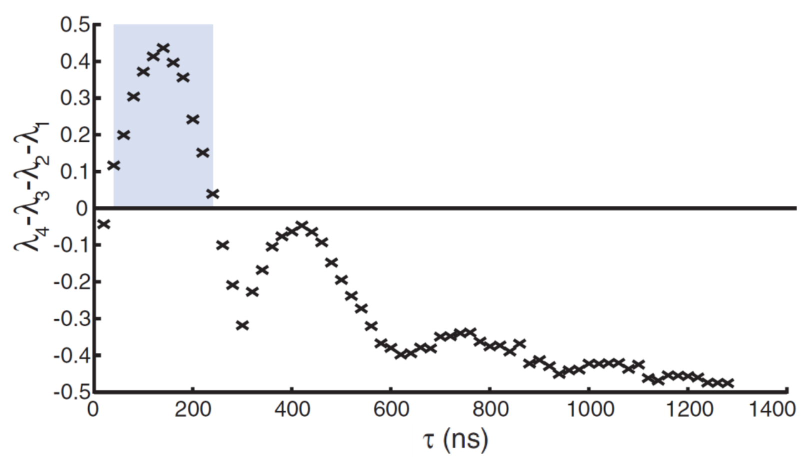


η = 3 x 10^{-5}

T = 0
T = 10 mK
T = 50 mK
T = 100 mK
T = 200 mK

Experiment:



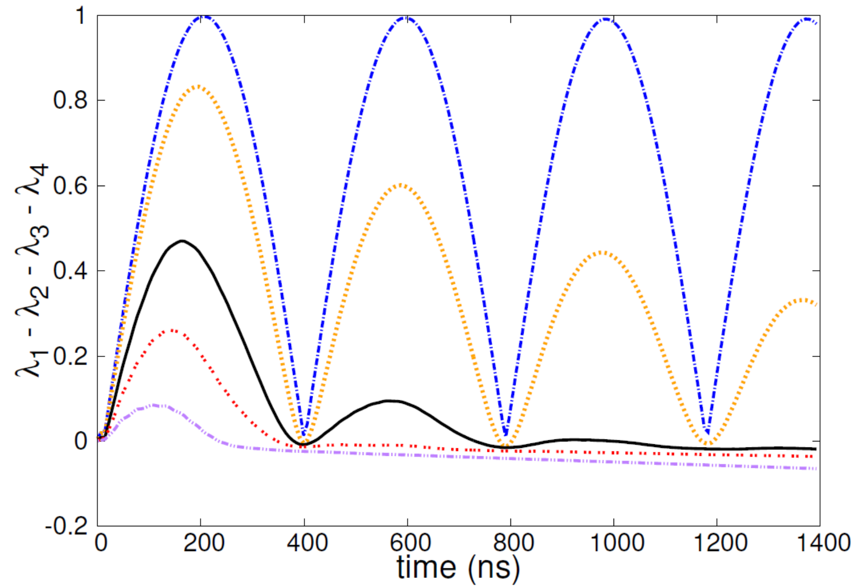


Results

Numerical solution:

Common bath

Does not describe the experiment!



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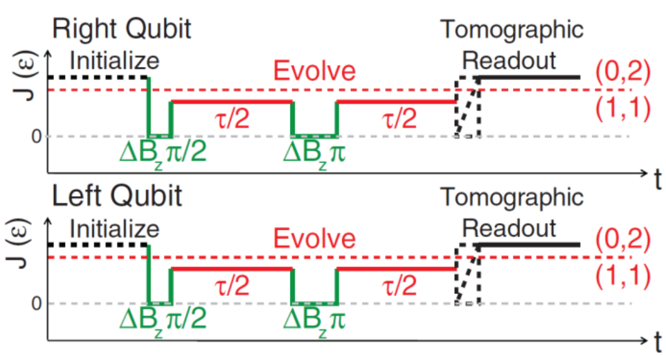
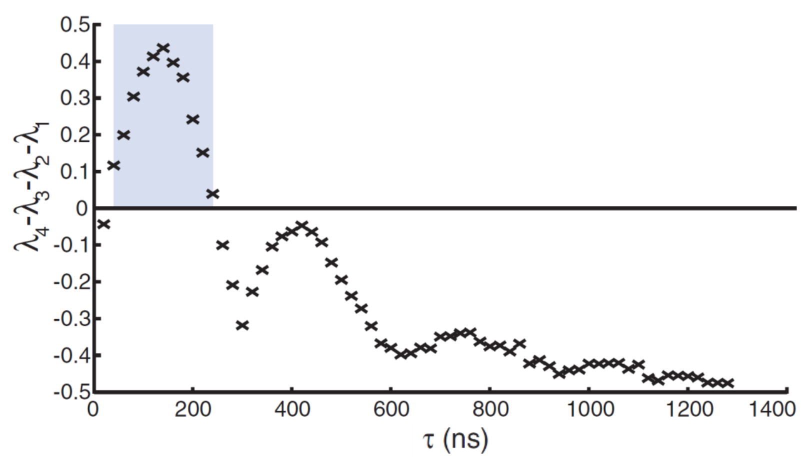
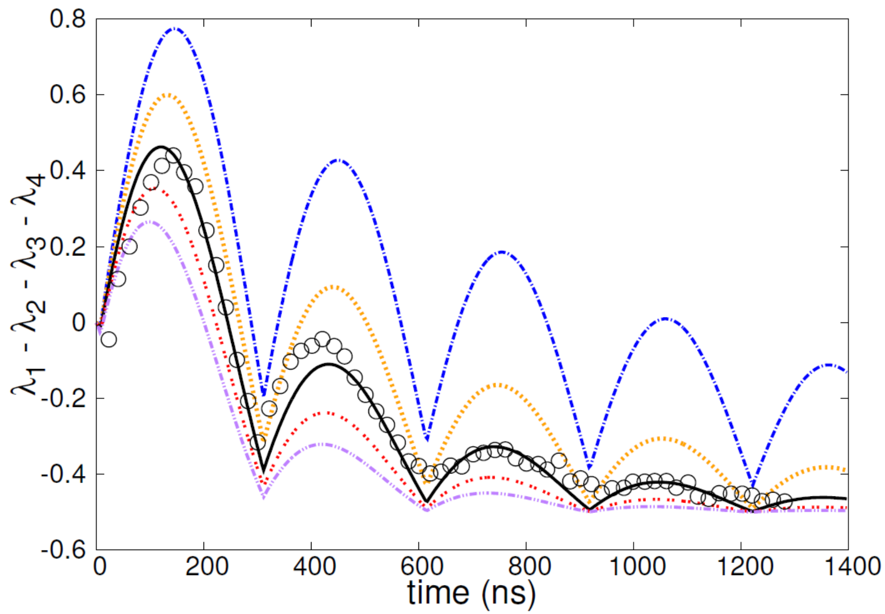




Results

Numerical solution:

Independent environments



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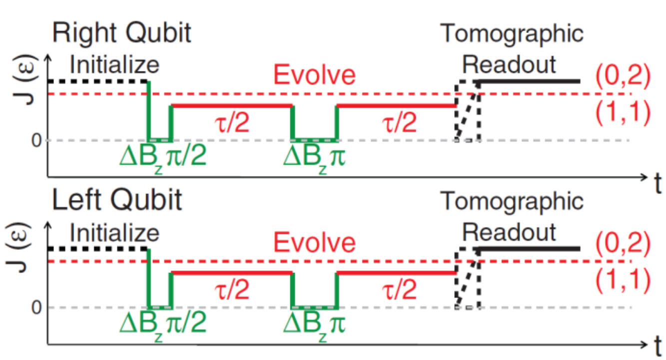

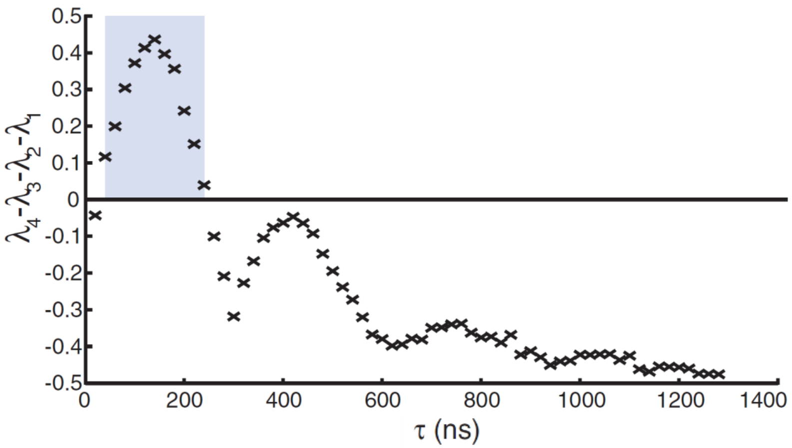

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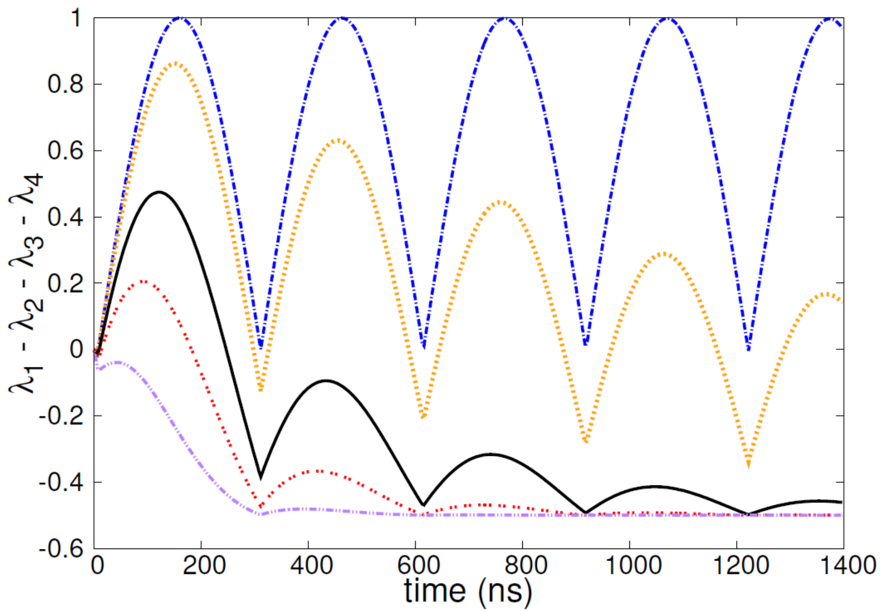

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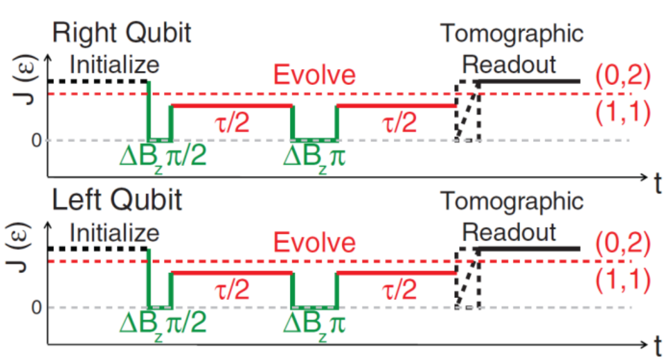

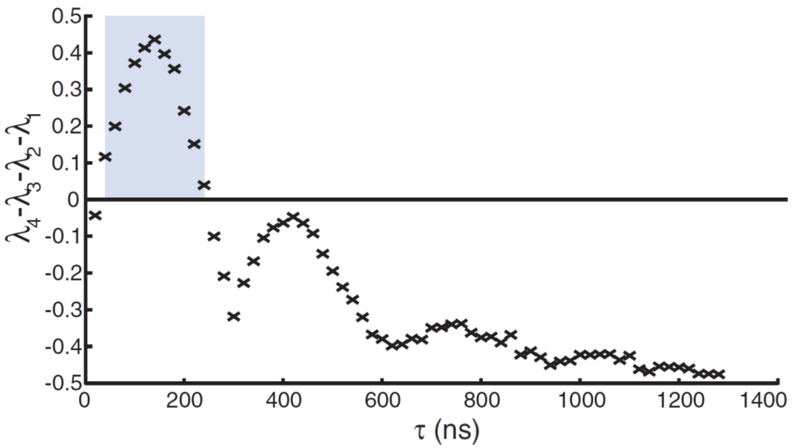

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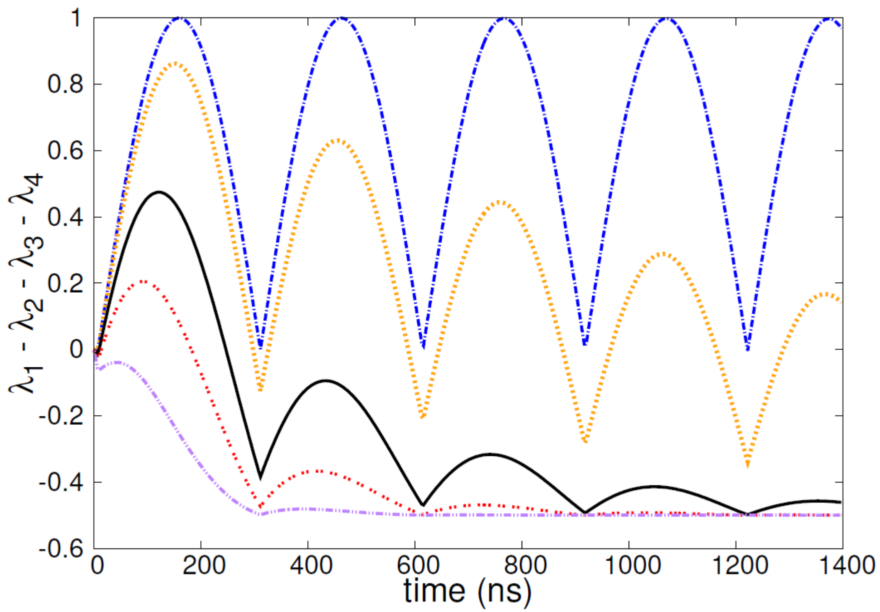

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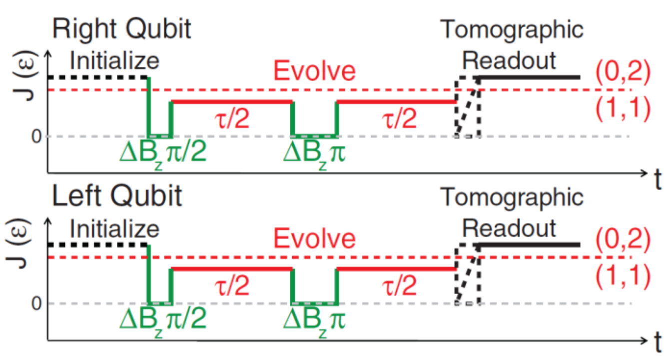
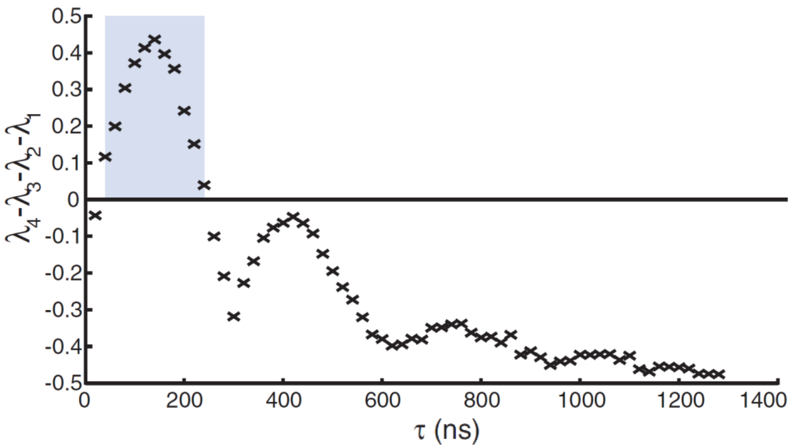
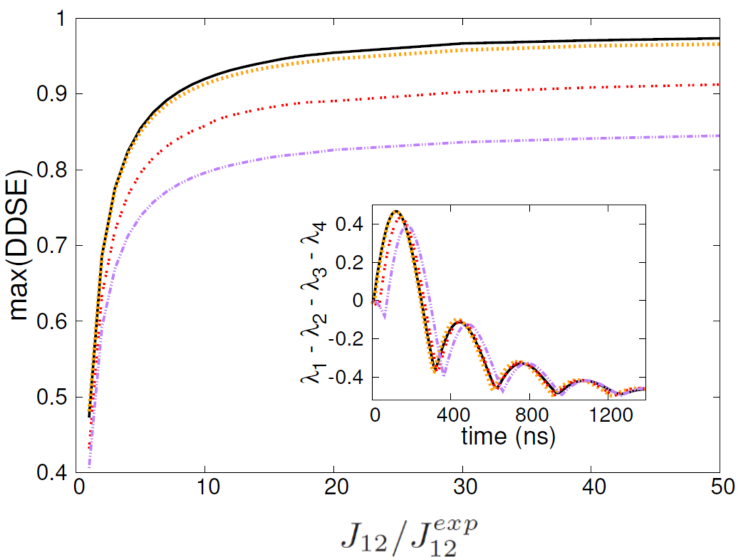




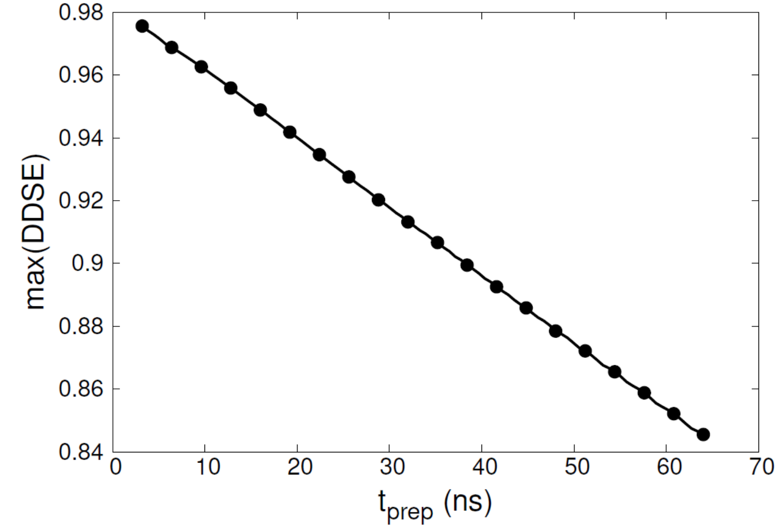
Results

Using the model based on independent environments, the authors show that **further improvements are possible:**



t_{prep} = 4 ms
t_{prep} = 8 ms
t_{prep} = 32 ms
t_{prep} = 64 ms

J_{12}/J_{12}^{exp} = 50



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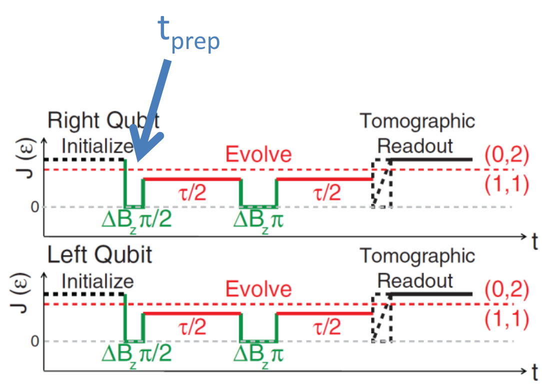

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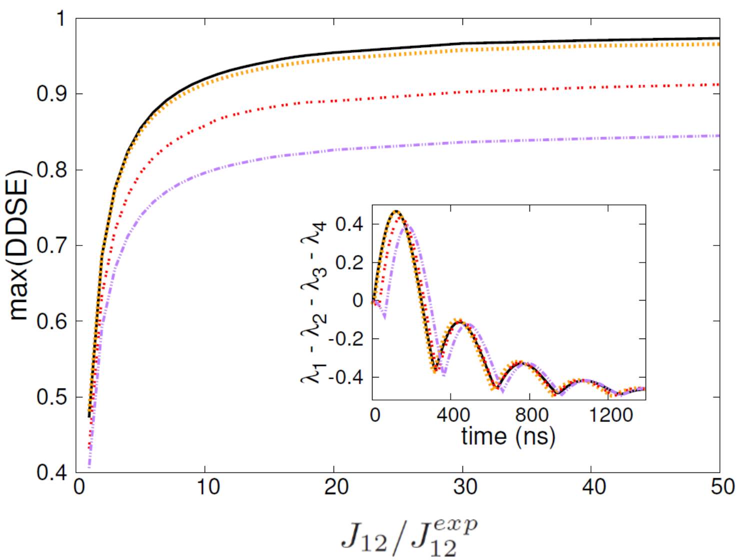

ns?!

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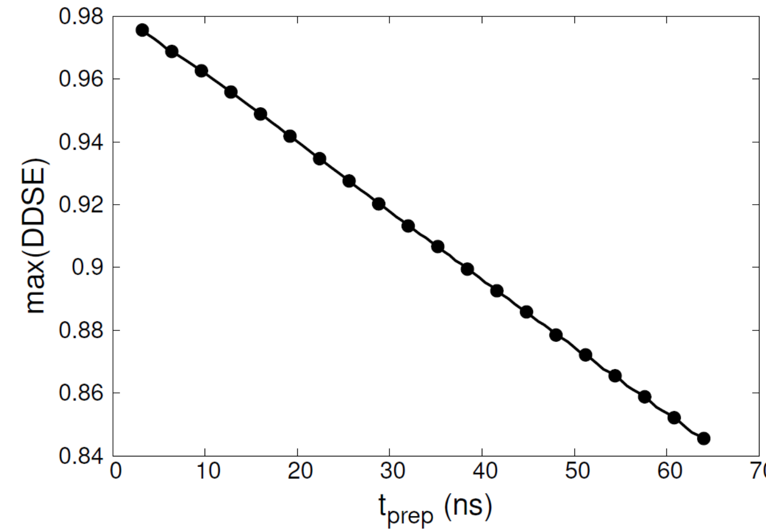
t_{prep} = 8 ms
t_{prep} = 32 ms
t_{prep} = 64 ms

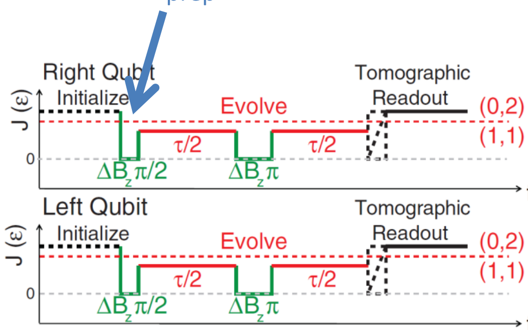
J_{12}/J_{12}^{exp} = 50



η = 3 x 10^{-5}

T = 50 mK

t_{prep}



Conclusions

- Using an open quantum system description, the authors develop a model to describe the data of Shulman *et al*., Science (2012)
- The model does (does not) describe the experiment when the qubits are coupled to independent environments (a common bath)
- The experimental results may be improved, particularly by increasing the capacitive coupling and by reducing the preparation time