JC on 2nd July, Kouki Nakata

Phys. Rev. Lett. **114**, 196601 (2015)/arXiv:1502.00347

``Thermal vector potential theory of transport induced by temperature gradient''

-Related work-

``Theory of Thermal Transport Coefficients''

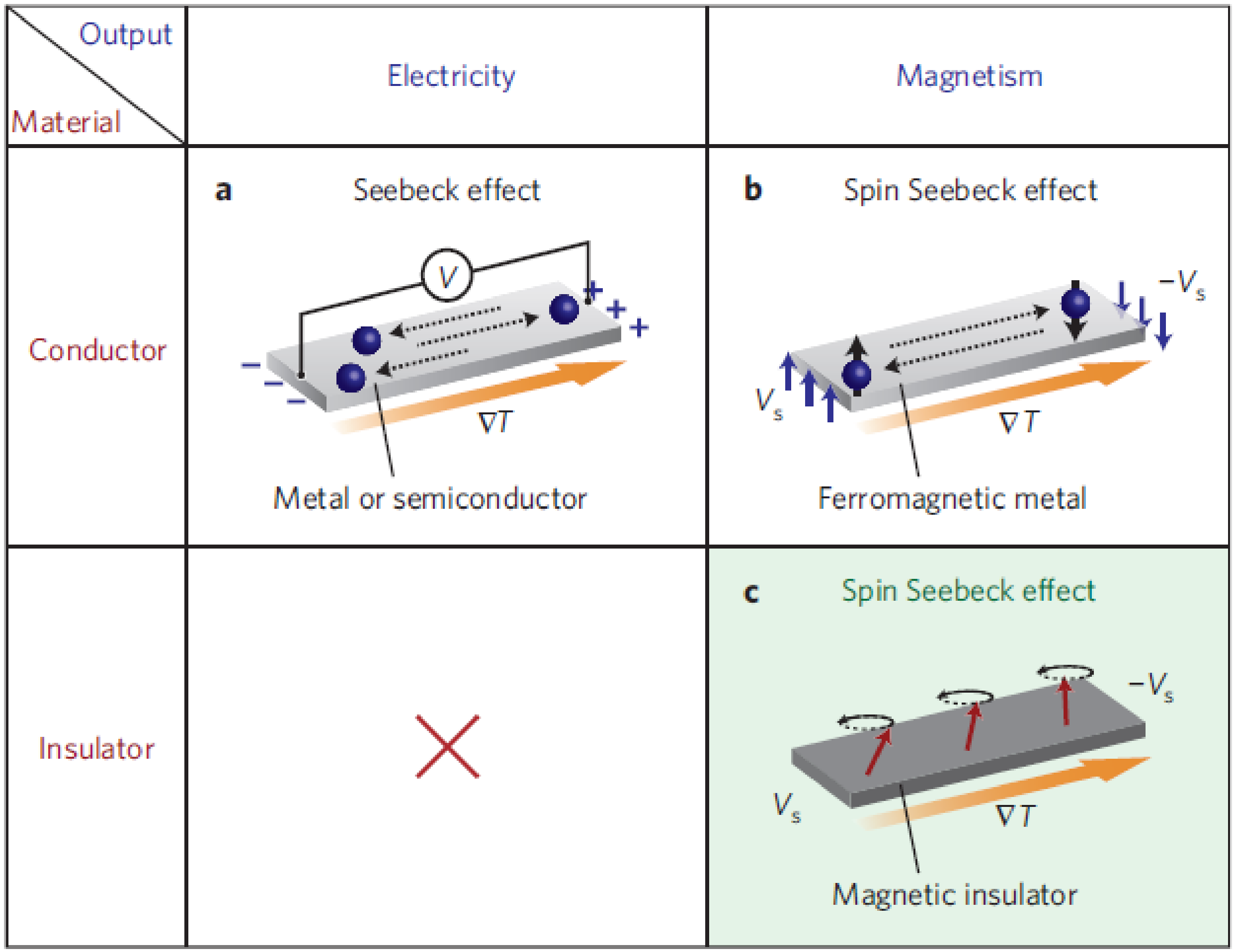
J. M. Luttinger, Phys. Rev. **135**, A1505 (1964)

✓ A microscopic formalism to calculate thermal transport coefficients is presented based on a thermal vector potential.

→ Time-derivative is related to a thermal force.

✓ The mathematical structure for thermal transport coefficients are shown to be identical with the electric ones if the electric charge is replaced by energy.

Thermal Transport



K. Uchida et al., Nat. Mater. 9, 894 (2010). K. Uchida et al., Nature 455, 778 (2008).
J. Xiao et al., Phys. Rev. B 81, 214418 (2010). H. Adachi et al., Phys. Rev. B 83, 094410 (2011).
S. Hoffman et al., Phys. Rev. B 88, 064408 (2013).
J. Flipse et al., Phys. Rev. Lett. 113, 027601 (2014).

The Main Purpose

[Temperature gradient] = [Statistical mechanical quantity]

✓ Boltzmann factor → Statistical mechanical (i.e., thermally) averaged value

[Hamiltonian] = [(Quantum) Mechanical quantity]

✓ Heisenberg's E.O.M → time-revolution of physical quantities

⬇

To provide the Hamiltonian that includes the temperature gradient

⬇

✓ **Evaluate thermal coefficient by using the purely Hamiltonian formalism**

→ i.e., a perturbation theory or green's functions

by treating thermal gradient as an external field

Hamiltonian Formalism

Single system (i.e., bulk)

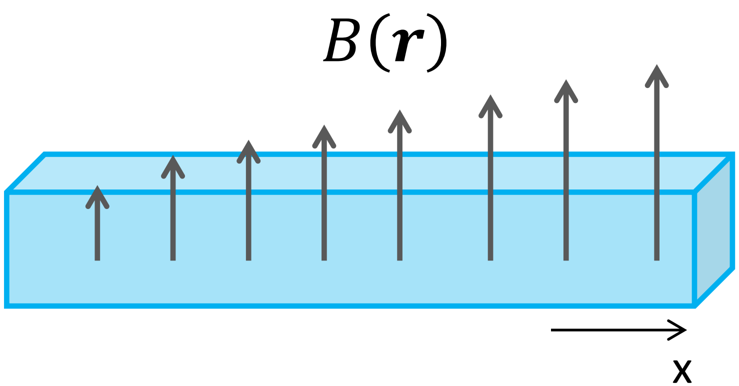
$$\mathcal{H} = \int d\boldsymbol{r}\, B(\boldsymbol{r}) a^\dagger a$$

$$\left[a(\boldsymbol{r}), a^\dagger(\boldsymbol{r}')\right] = \delta(\boldsymbol{r} - \boldsymbol{r}')$$

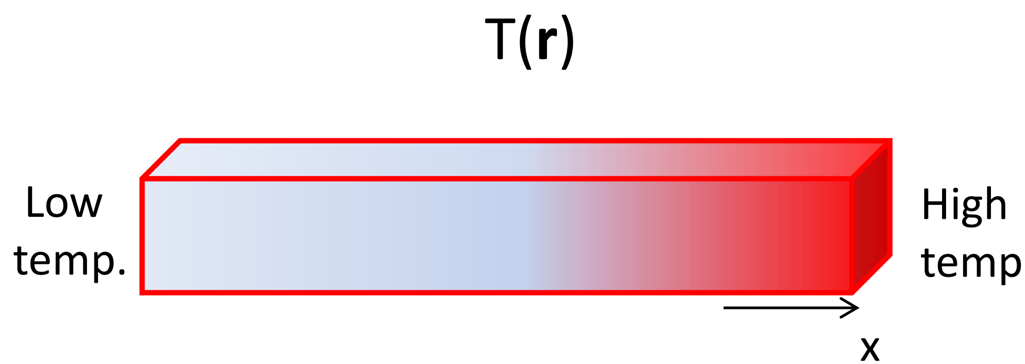
$B = \mu$; chemical potential

$$\mathcal{H} = \int d\boldsymbol{r}\, T(\boldsymbol{r}) a^\dagger a$$

?



$$I_x = -G \partial_x B$$



$$J_x = -\kappa \partial_x T$$

Guiding Principle

To treat thermal gradient as an external field

``Luttinger's Principle''

J. M. Luttinger, Phys. Rev. **135**, A1505 (1964)

✓ **``Gravitational potential''**

→ **Rewrite ``Boltzmann factor''**

Luttinger's Principle

→ ``TRICK'' that has NO microscopic reasons

J. M. Luttinger, Phys. Rev. **135**, A1505 (1964)

- ✓ **Statistical Mechanics**

$$P(E) \propto e^{-\beta \int d\boldsymbol{r}\, H(\boldsymbol{r})}$$

$T = Constant \qquad T \neq T(\boldsymbol{r})$

$\mathcal{H} = \int d\boldsymbol{r}\, H(\boldsymbol{r})$

$\beta = 1/(k_B T)$

- ✓ **Key quantity; PRODUCT**

$$\beta\mathcal{H} = \beta \int d\boldsymbol{r}\, H(\boldsymbol{r}) \equiv \mathcal{L} \int d\boldsymbol{r}\, \Psi(\boldsymbol{r})\varepsilon$$

$\mathcal{L}$; a constant

ε; local energy density

Ψ; gravitational potential;

←TERMINOLOGY

(I explain later)

$$\boldsymbol{\nabla}\Psi(\boldsymbol{r}) = \boldsymbol{\nabla}T/T$$

- ✓ **Boltzmann factor**

$$e^{-\beta \int d\boldsymbol{r}\, H(\boldsymbol{r})} \equiv e^{-\mathcal{L} \int d\boldsymbol{r} \Psi(\boldsymbol{r})\varepsilon}$$

POINT

J. M. Luttinger, Phys. Rev. **135**, A1505 (1964)

$$e^{-\beta \int d\boldsymbol{r}\, H(\boldsymbol{r})} \equiv e^{-\mathcal{L} \int d\boldsymbol{r}\, \Psi(\boldsymbol{r})\varepsilon}$$

✓ $\beta = k_{\mathrm{B}}T$

⬇

$$T \neq T(\boldsymbol{r})$$

✓ Uniform temperature

→ $\nabla T = 0$

✓ $\boldsymbol{\nabla}\Psi(\boldsymbol{r}) = \boldsymbol{\nabla}T/T$

⬇

$$T = T(\boldsymbol{r})$$

✓ Local temperature

→ $\nabla T \neq 0$

Gravitational Potential ?

→ No need to worry about the terminology ``Gravitational''

✓ **Key quantity; PRODUCT**

$$\beta\mathcal{H} = \beta\int d\boldsymbol{r}\, H(\boldsymbol{r}) \equiv \mathcal{L}\int d\boldsymbol{r}\, \Psi(\boldsymbol{r})\varepsilon$$

$\mathcal{L}$; a constant

ε; local energy density

***Ψ*; gravitational potential;** $\nabla\Psi(\boldsymbol{r}) = \nabla T/T$

In analogy to

✓ **Special relativity**

Albert Einstein
(Ph. D; Univ. of Zurich)

$$E = mc^2$$

✓ An energy density ε behaves as if it had a mass density ε/c^2, (as far as its interaction with a gravitation field goes.)

→ Call Ψ/c^2 or Ψ **``the gravitational potential''**

J. M. Luttinger, Phys. Rev. **135**, A1505 (1964)

➢ *(No positive reason to use gravitational, just say potential, enough I think.)*

Idea

J. M. Luttinger, Phys. Rev. **135**, A1505 (1964)

✓ **Key quantity; PRODUCT**

$$\beta\mathcal{H} = \beta \int d\boldsymbol{r}\, H(\boldsymbol{r}) \equiv \mathcal{L} \int d\boldsymbol{r}\, \Psi(\boldsymbol{r})\varepsilon$$

$\mathcal{L}$; a constant

ε; local energy density

***Ψ*; gravitational potential;** $\nabla\Psi(\boldsymbol{r}) = \nabla T/T$

✓ Just as the space- and time-varying external electric potential produced electric currents and density variations, so a varying gravitational field will produce, in principle, energy flows and temperature fluctuations.

✓ Clearly a varying will give rise to a varying energy density, which, in turn, will correspond to a varying temperature.

Similar Approach

- ``**Low-energy effective theory in the bulk for transport in a topological phase**''
  Barry Bradlyn, N. Read
  arXiv:1407.2911[Phys. Rev. B **91**, 125303 (2015)]

- ``**Heat transport as torsional responses and Keldysh formalism in a curved spacetime**''
  Atsuo Shitade
  arXiv:1310.8043[Prog. Theor. Exp. Phys. 2014, 123I01 (2014)]

⬇

✓ Analogy **of general relativity** that if an invariance under time translation is imposed locally.

→A vector potential arises from Luttinger's scalar potential.

→They are described by a gauge invariant theory.

→ **Wiedemann-Franz law**.

→ But still, The origin of the invariance under local time translation was not argued.

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``Thermal vector potential theory of transport induced by temperature gradient''

Classical charged particles

An expectation from an example

$$H = \frac{(\boldsymbol{p} - e\boldsymbol{A})^2}{2m}$$

$$\dot{\boldsymbol{r}} = \frac{1}{m}(\boldsymbol{p} - e\boldsymbol{A}) \qquad \dot{\boldsymbol{p}} = 0$$

$$\boldsymbol{F} \equiv m\ddot{\boldsymbol{r}} = -e\dot{\boldsymbol{A}}$$

✓ In thermally-driven transport, it can be expected that a thermal force proportional to $\boldsymbol{\nabla} T$ is represented by a vector potential.

→ **\`\`Thermal vector potential A_T''**

STRATEGY

To propose a formalism describing thermal effects by a thermal vector potential

- ✓ Carry out a derivation of a **thermal vector potential form** of the interaction Hamiltonian by looking for a Hamiltonian equivalent to the **Luttinger's Hamiltonian**.

$$H_{A_T} \equiv H_{A_T}(A_T) \rightarrow H_L$$

A_T; thermal vector potential

$$H_{\mathrm{L}} = \int d^3 r \Psi \mathcal{E}.$$; Luttinger Hamiltonian

- ✓ Derive expressions for electric current and energy current by use of conservation laws.

Test Hamiltonian

$$H'_{\mathrm{L}}(t) = \int d^3r \int_{-\infty}^{t} dt' \boldsymbol{j}_{\mathcal{E}}(t') \cdot \nabla\Psi(\boldsymbol{r}, t)$$

$\boldsymbol{j}_{\varepsilon}$; energy current

$$\dot{H}'_{\mathrm{L}} = -\int dr\, \Psi(\nabla \cdot \boldsymbol{j}_{\varepsilon})$$

$$\dot{\mathcal{E}} = -\nabla \cdot \boldsymbol{j}_{\mathcal{E}}.$$

Energy conservation law

$$\dot{H}'_{\mathrm{L}} = \dot{H}_{\mathrm{L}}$$

Thermal Vector Potential

- ✓ To obtain the form between a vector potential A_T and energy current

$$H'_{\mathrm{L}}(t) = \int d^3 r \int_{-\infty}^{t} dt' \, \boldsymbol{j}_{\mathcal{E}}(t') \cdot \nabla \Psi(\boldsymbol{r}, t)$$

- ✓ Look for the Hamiltonian H_{A_T}

$$\int_{-\infty}^{\infty} dt \, H_{A_T} = \int_{-\infty}^{\infty} dt \, H'_{\mathrm{L}}(t).$$

$$H_{A_T} \equiv -\int d^3 r \, \boldsymbol{j}_{\mathcal{E}}(\boldsymbol{r}, t) \cdot \boldsymbol{A}_T(t)$$

Thermal vector potential; $\boldsymbol{A}_T(t) \equiv \int_{-\infty}^{t} dt' \nabla \Psi(t')$

→Temperature gradient; $\partial_t \boldsymbol{A}_T(\boldsymbol{r}, t) = \nabla \Psi(\boldsymbol{r}, t) = \frac{\nabla T}{T}.$

Hamiltonian Formalism

Single system (i.e., bulk)

$$\mathcal{H} = \int d\boldsymbol{r}\, B(\boldsymbol{r}) a^\dagger a$$

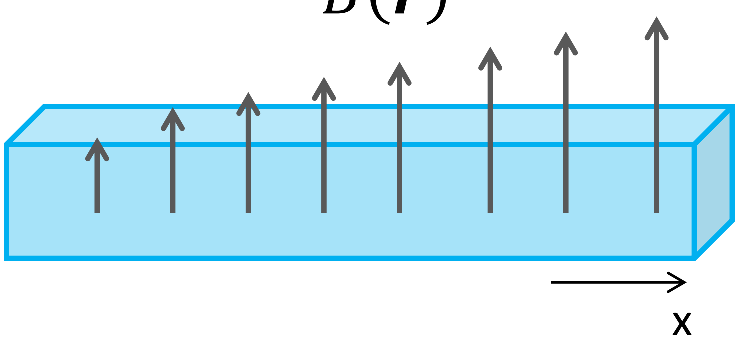
$$[a(\boldsymbol{r}), a^\dagger(\boldsymbol{r}')] = \delta(\boldsymbol{r} - \boldsymbol{r}')$$

$B = \mu$; chemical potential

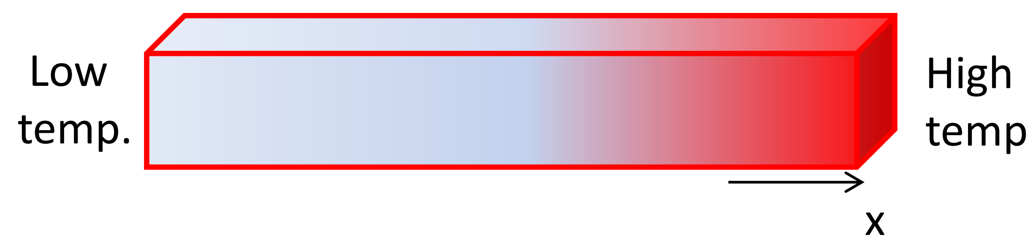
$$\mathcal{H} = \int d\boldsymbol{r}\, T(\boldsymbol{r}) a^\dagger a$$

?

$B(\boldsymbol{r})$



T(**r**)



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$$J_x = -\kappa \partial_x T$$

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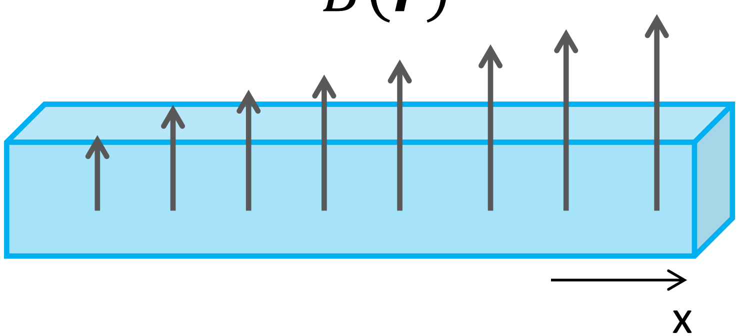

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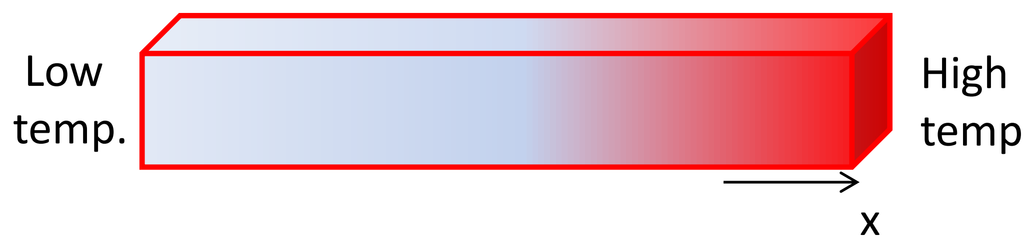

$$J_x = -\kappa\partial_x T$$

Application

Ferromagnetic insulator (i.e., magnonic bulk systems)

✓ By using the thermal vector potential Hamiltonian and Green function formalism

$$j_{\mathrm{m},i} = -\kappa \nabla_i T,$$

(i = x, y, z)

↓

Standard Boltzmann transport equation

[W. Jiang et al., Phys. Rev. Lett. **110**, 177202 (2013)]

arXiv:1505.01908

DISCUSSION

- ✓ In the electromagnetic case, the minimal form is imposed by a U(1) gauge invariance.
- ✓ For the thermal vector potential, in contrast, there is no gauge invariance in the strict sense since the energy conservation arises from a translational invariance with respect to time.

<u>Concerning steady state properties;</u>

[Luttinger's potential Ψ] ⇔[Thermal vector potential $\boldsymbol{A}_T$]

$$\partial_t \boldsymbol{A}_T(\boldsymbol{r},t) = \nabla\Psi(\boldsymbol{r},t) = \frac{\nabla T}{T}.$$

⬇

We may assign

$$\frac{\nabla T}{T} = \nabla\Psi + \dot{\boldsymbol{A}}_T.$$

⬇

A 'gauge invariance' as a result of the energy conservation law.

$\Psi \to \Psi - \dot{\chi}$ and $\boldsymbol{A}_T \to \boldsymbol{A}_T + \nabla\chi$ (χ is a scalar function).

SUMMARY

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``Theory of Thermal Transport Coefficients''

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✓ Based on the Luttinger's principle, thermal vector potential has been introduced and in terms of it, the Luttinger's Hamiltonian has been rewritten.

$$\partial_t \boldsymbol{A}_T(\boldsymbol{r},t) = \nabla\Psi(\boldsymbol{r},t) = \frac{\nabla T}{T}.$$

✓ Using the Hamiltonian formalism (i.e., Green functions), a coefficient has been evaluated and it has been verified that it reduces to the same result given by the standard Boltzmann transport theory.

✓ **Still, the microscopic origin of this formalism and the Luttinger's principle is lacking.**