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Classical Simulation of Quantum Error Correction in a Fibonacci Anyon Code

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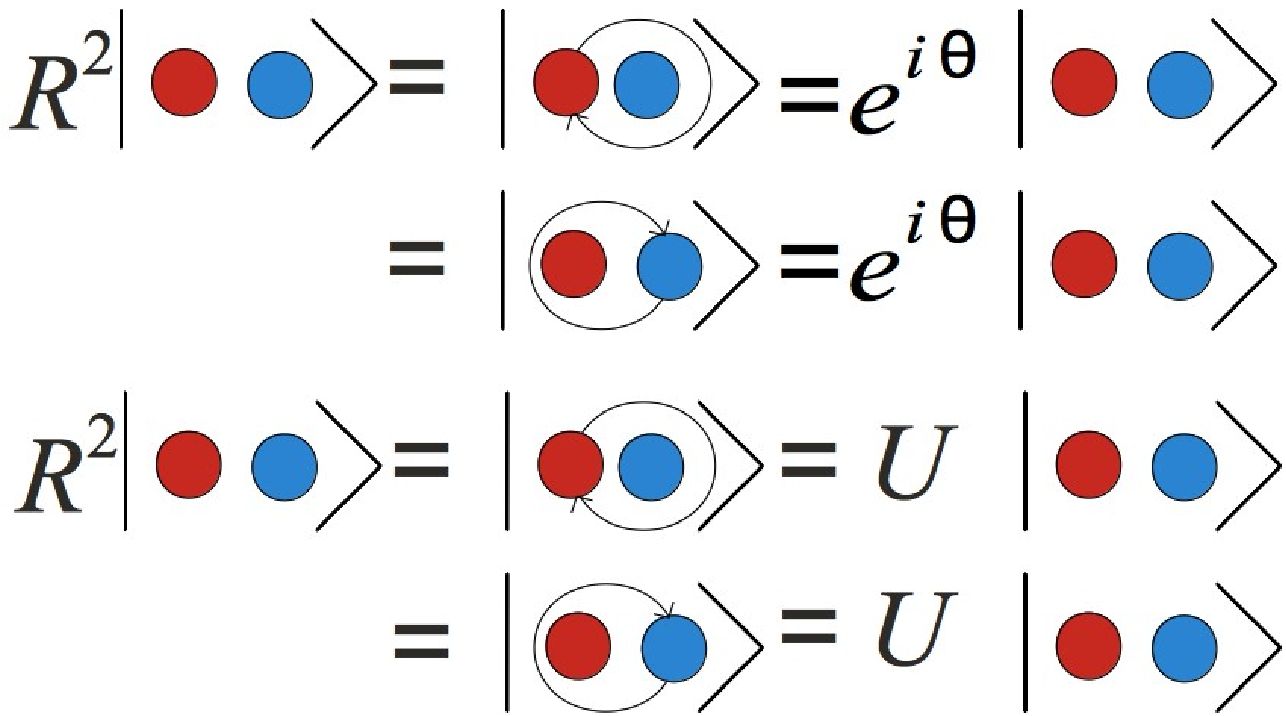
Classically simulating the dynamics of anyonic excitations in two-dimensional quantum systems is likely intractable in general because such dynamics are sufficient to implement universal quantum computation. However, processes of interest for the study of quantum error correction in anyon systems are typically drawn from a restricted class that displays significant structure over a wide range of system parameters. We exploit this structure to classically simulate, and thereby demonstrate the success of, an error-correction protocol for a quantum memory based on the universal Fibonacci anyon model. We numerically simulate a phenomenological model of the system and noise processes on lattice sizes of up to 128×128 sites, and find a lower bound on the error-correction threshold of approximately 12.5%, which is comparable to those previously known for abelian and (non-universal) nonabelian anyon models.

Topological Quantum Computation

Anyons are quasiparticles with complex exchange behaviour



Some can perform universal QC by braiding alone (Fibonacci,…)

Some require additional help (Majoranas,…)

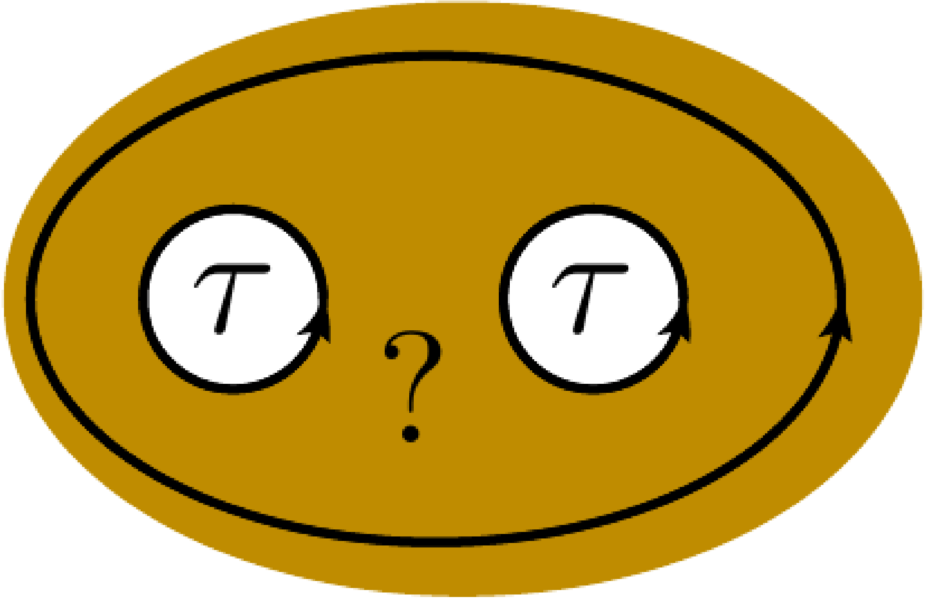
Fibonacci anyons

The Fibonacci model contains one non-trivial anyon type: $\{1, \tau\}$

Two (or more) anyons can be **fused** into a single composite

The result of fusing two τ anyons can be annihilation or a single τ , Depending on non-local degrees of freedom

$$\tau \times \tau = 1 + \tau$$



Fusion Space

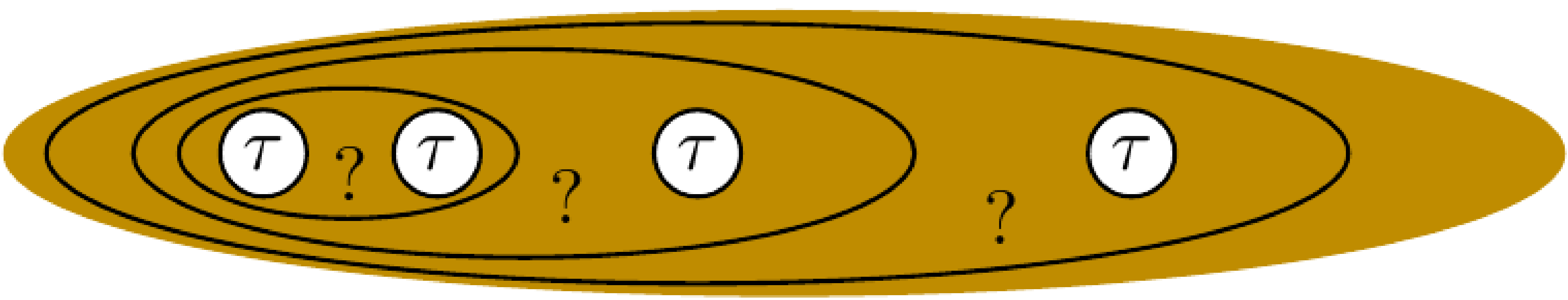
The non-local degrees of freedom form the **Fusion Space**

This is the space on which braiding operations act

It is where logical information is stored and processed

For Fibonacci anyons, the dimension grows according to the Fibonacci sequence

$$1, 1, 2, 3, 5, \ldots$$



Error Correction

Unwanted creation and transport of anyons can be suppressed by a gap

Even so, such errors will still happen for a big enough computation

Additional anyons affect and confuse the fusion space

Error correction needs to be done, and done often []

Non-Abelian error correction has so far considered only non-universal Models (Ising and Phi-Lambda) []

Universal models are thought to be too hard to classically simulate

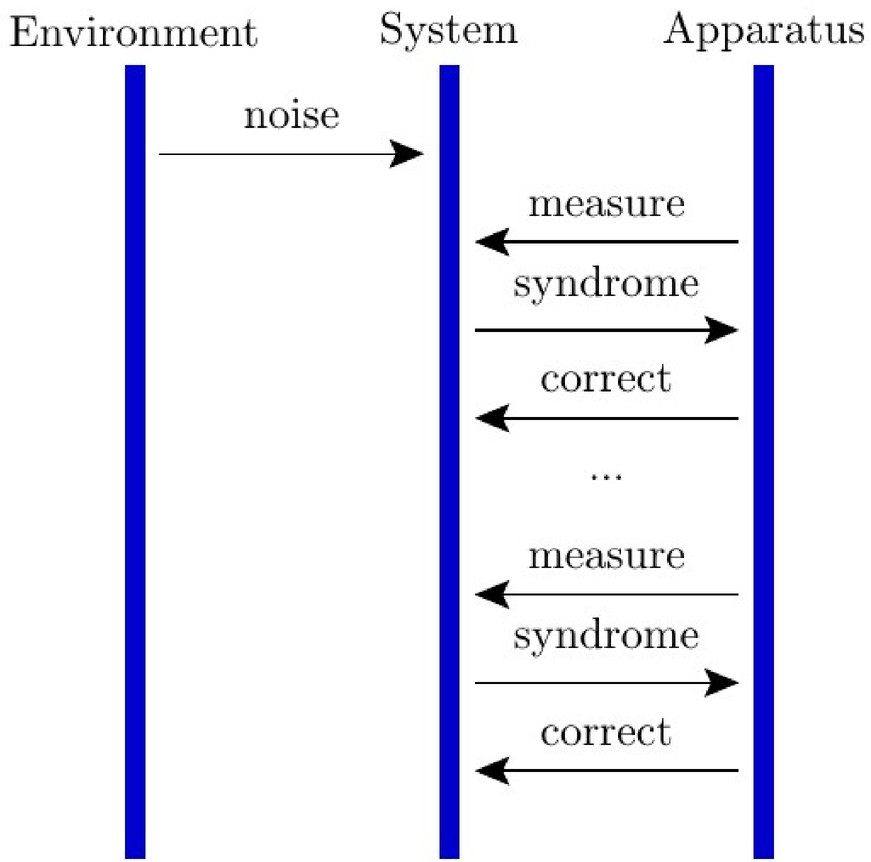
Here the authors prove otherwise, and do it for the Fibonacci model

Error Correction

Problem considered has a single burst of noise that creates anyons

Anyon positions are measured, and fusions are made in an attempt to annihilate them

All such measurements and manipulations are assumed to be perfect



Simulating Braiding

Anyons will braid as they are created and fused

Braiding Fibonacci anyons can efficiently realize universal QC

Universal QC is (thought to be) impossible to simulate on a classical computer

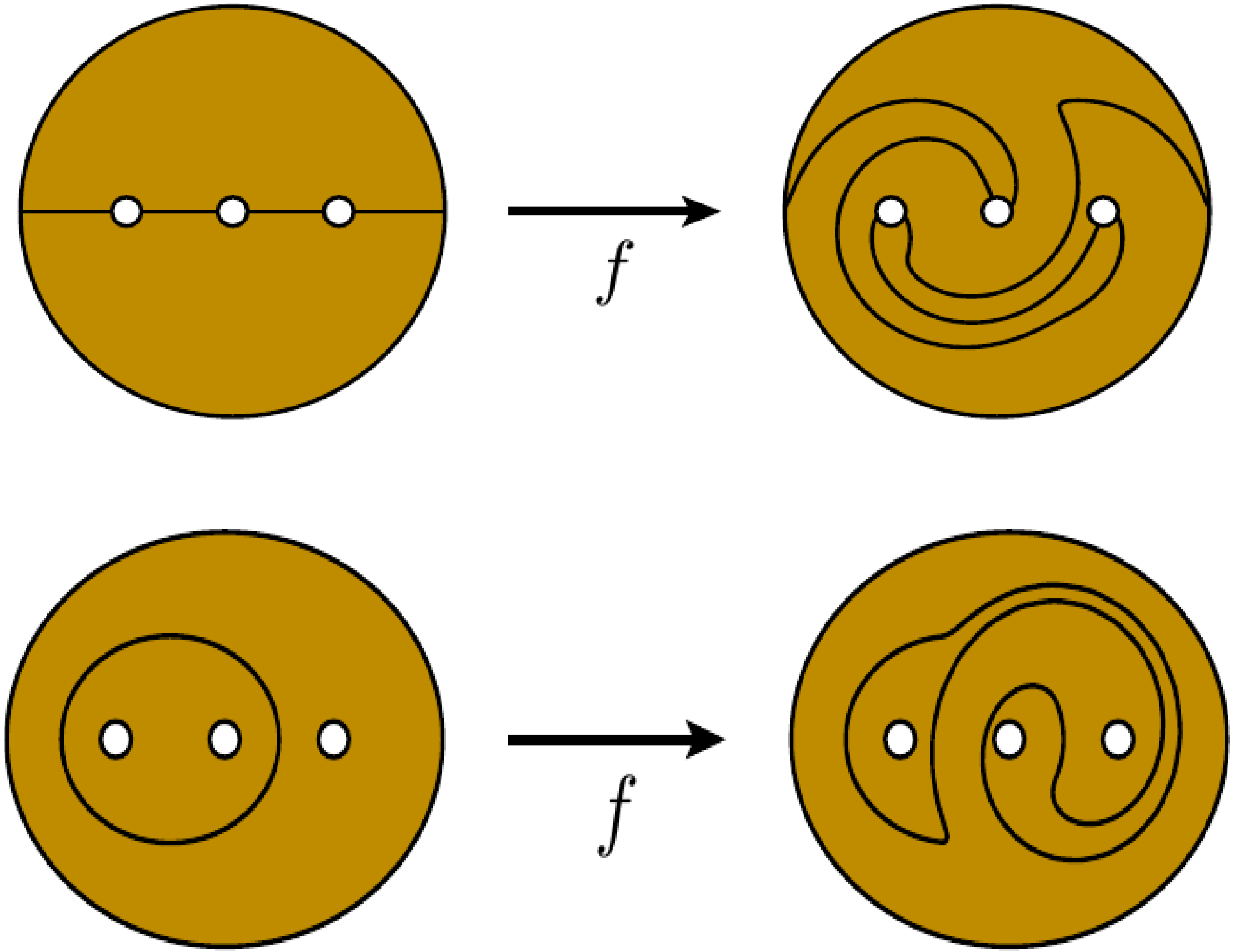
This suggests that error correction of universal anyons is hard to simulate

However, for low noise, anyons will form isolated clusters of size *log(L)*
(as in a percolation problem)

If these are simulated (and decoded) independently, the time required will
be *poly(L)*

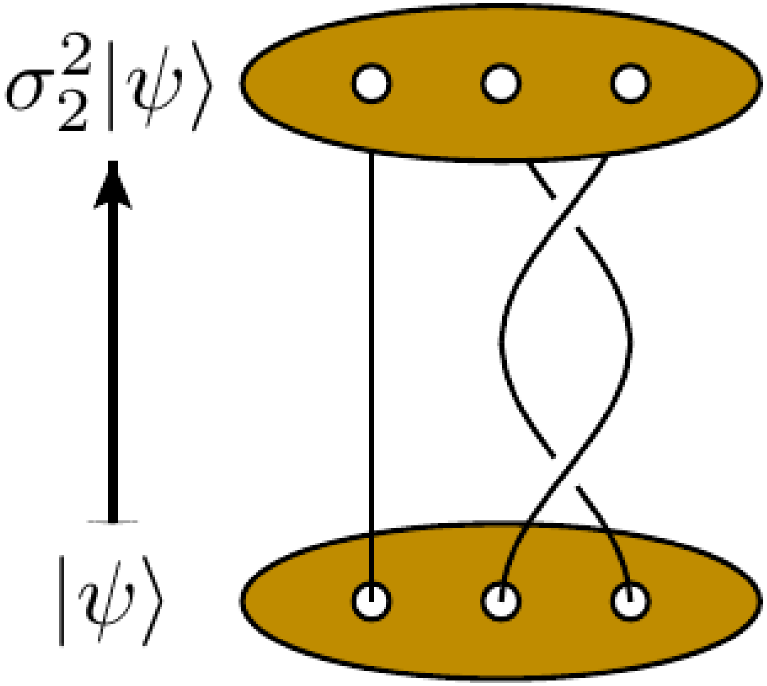
Simulating Braiding

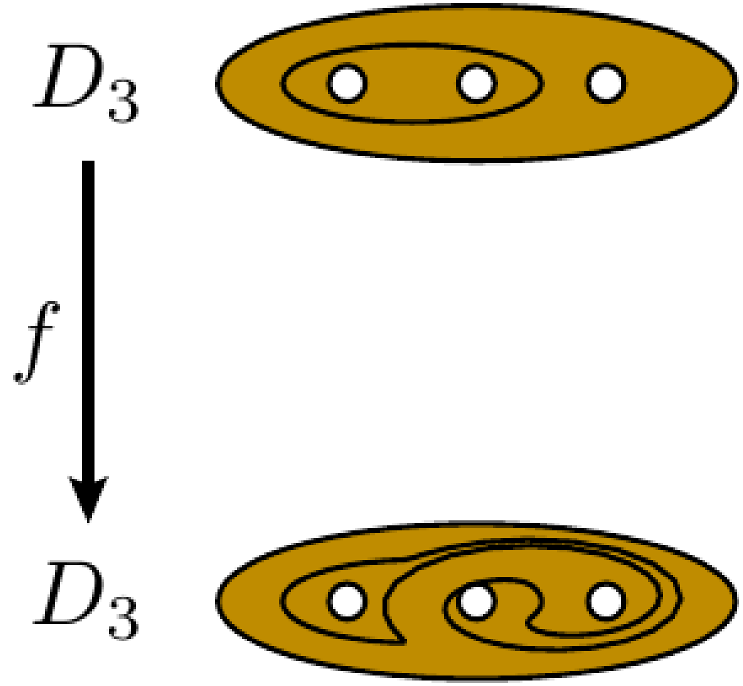
Braiding can be considered in a pictorial manner



Simulating Braiding

Braiding can be considered in a pictorial manner





Braid group acts on states: "Schrodinger picture"

Mapping class group acts on observables: "Heisenberg picture"

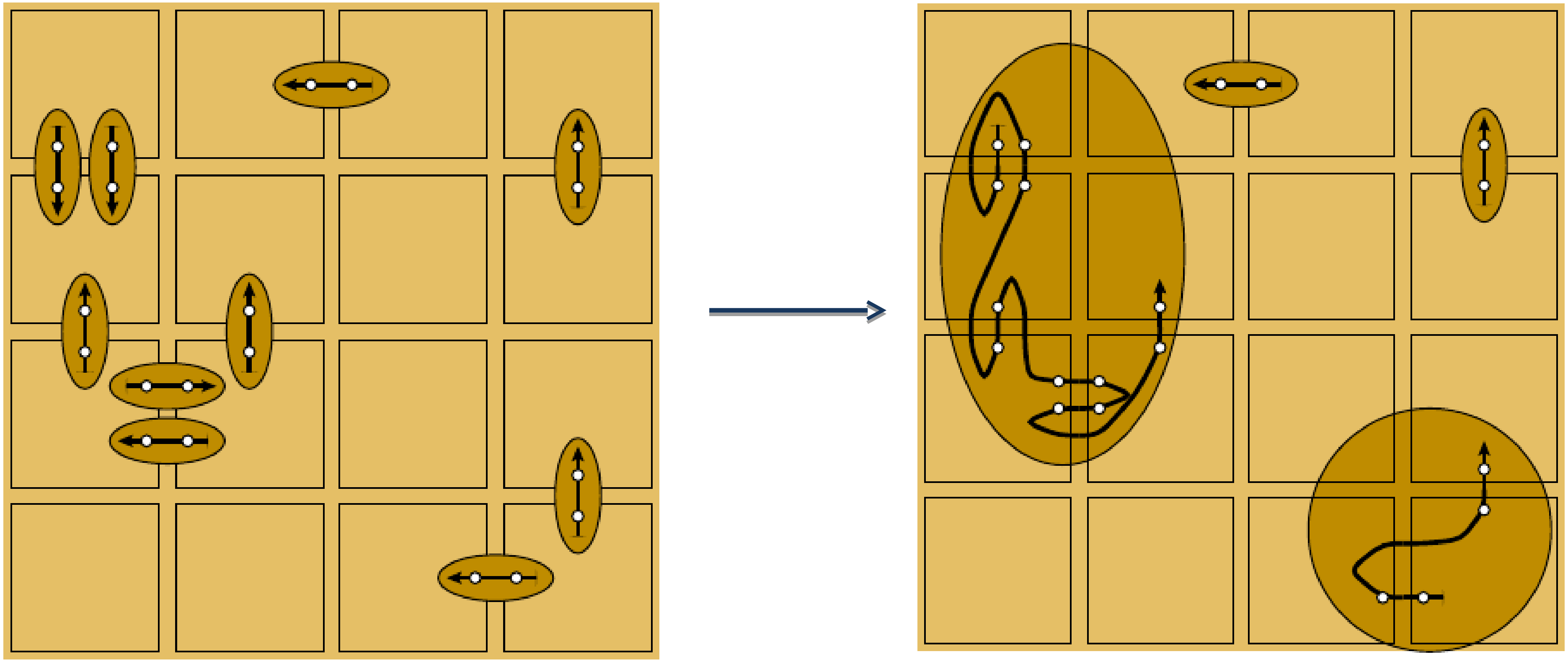
Simulating Errors

A continuous 2D space is considered, cut up into an LxL lattice

Errors are generated using a Poisson process with rate $1/t_{sim}$

The net anyon occupancy of each plaquette is then measured

Clusters of errors that effect the same plaquettes are simulated independently



Decoding

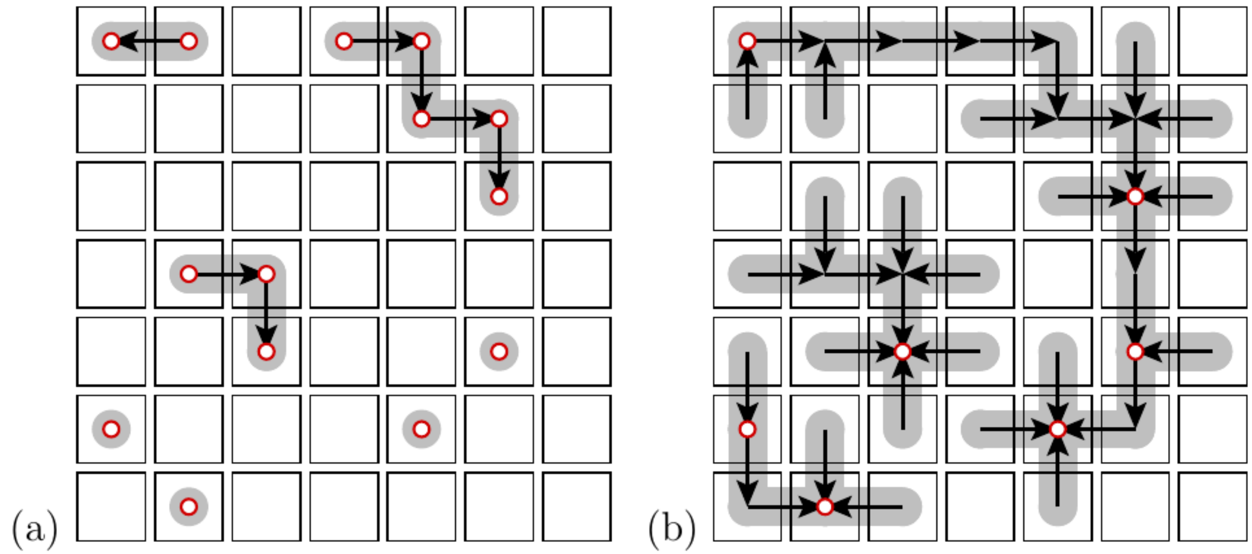


FIG. 2. The decoder works by maintaining a set of disjoint clusters as rooted trees. (a) At the initial clustering stage, these trees are formed from neighboring sites that contain charges. Within each cluster, the charges are transported to the root of the tree (chosen arbitrarily), and their combined charge measured. The direction of transport (towards the root) is denoted by arrows. (b) At each successive round, all trees are grown in every direction, and overlapping trees are joined. Again, any charges within a cluster are transported to the root of the tree and measured. All clusters with vacuum total charge are deleted.

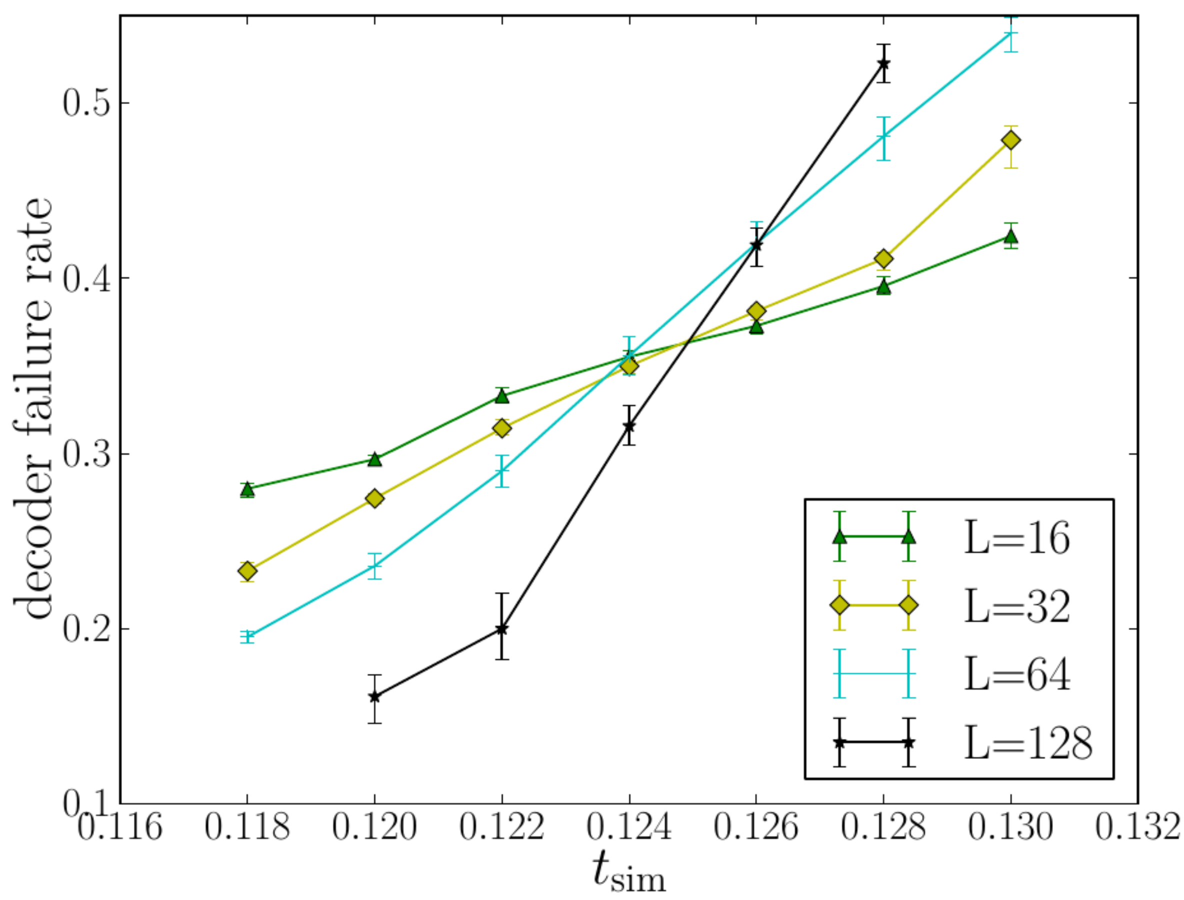
Closely located groups of anyons are fused according to Bravyi-Haah method

This is continued until all are removed

The effects on the fusion space is then (hopefully) trivial

Results

Results show that good error correction (decreasing logical error rate as L increases) is found below a threshold $t_{\mathrm{sim}} \simeq 0.125 \pm 0.003$



Decoder fails if it it affects the fusion space, or it takes too long and gives up

Conclusions

Interesting numerical studies can be done for complex (and universal) anyon models

Error correction thresholds for complex anyon models aren’t too different From Abelian ones