Topological Superconductivity and High Chern Numbers in 2D Ferromagnetic Shiba Lattices

Joel Röntynen and Teemu Ojanen*
O. V. Lounasmaa Laboratory (LTL), Aalto University, P. O. Box 15100, FI-00076 AALTO, Finland
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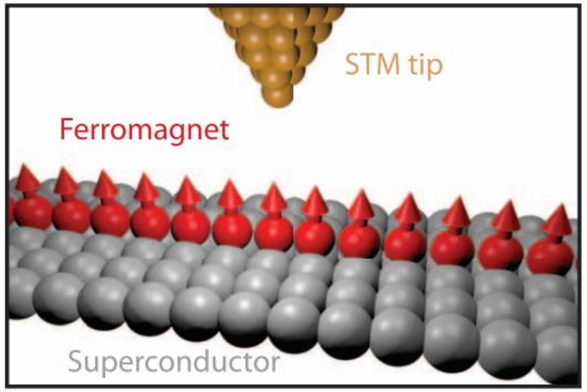
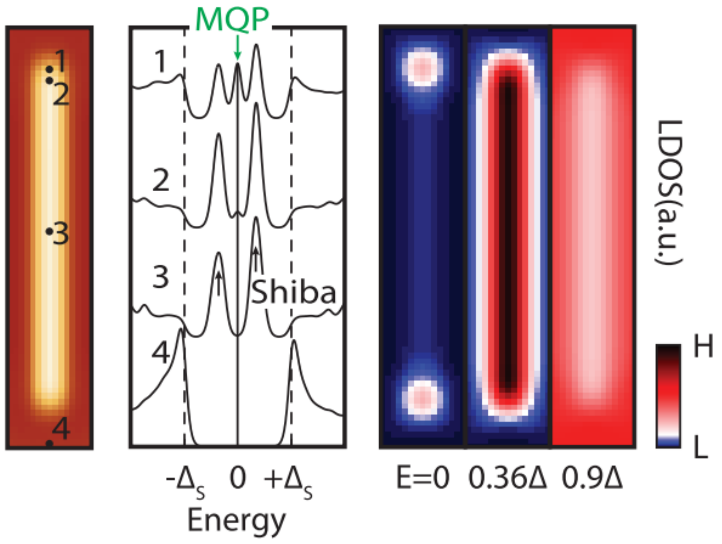
Inspired by the recent experimental observation of topological superconductivity in ferromagnetic chains, we consider a dilute 2D lattice of magnetic atoms deposited on top of a superconducting surface with a Rashba spin-orbit coupling. We show that the studied system supports a generalization of $p_x + ip_y$ superconductivity and that its topological phase diagram contains Chern numbers higher than $\xi/a(\gg 1)$, where ξ is the superconducting coherence length and a is the distance between the magnetic atoms. The signatures of nontrivial topology can be observed by STM spectroscopy in finite-size islands.

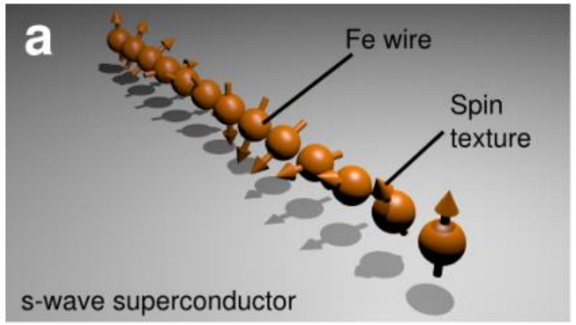
Paweł Szumniak

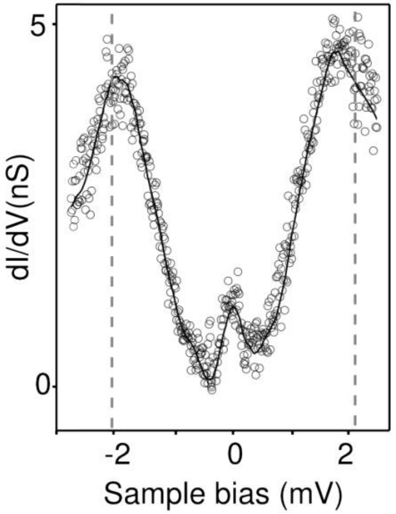
CMT Journal Club 23.06.2015

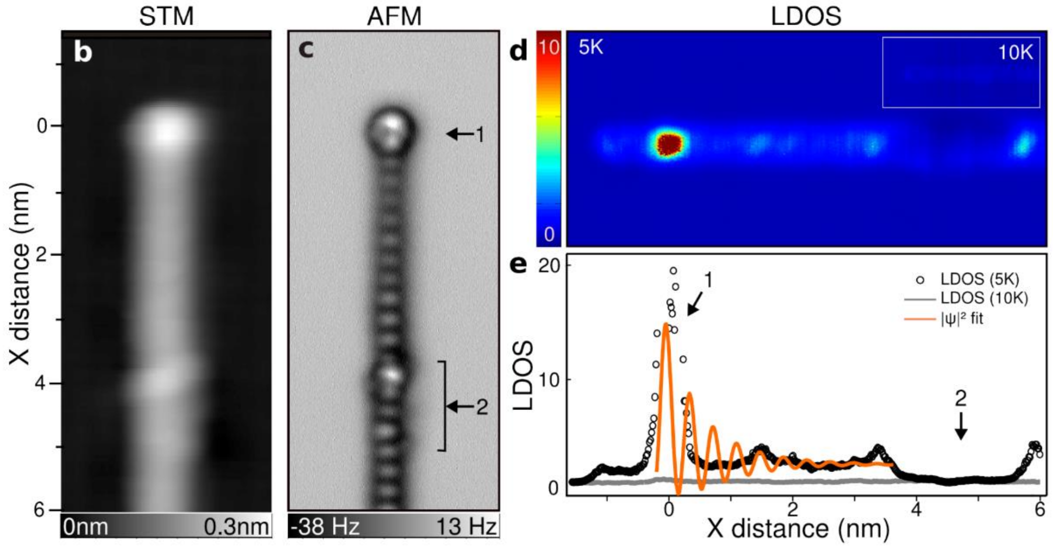
Motivation – recent experiments with 1D chains of magnetic impurities











S. Nadj-Perge, I. K. Drozdov, J. Li, H. Chen, S. Jeon, J. Seo, A. H. MacDonald, B. Andrei Bernevig, and A. Yazdani, Science 346, 602 (2014)

Remy Pawlak, Marcin Kisiel, Jelena Klinovaja, Tobias Meier, Shigeki Kawai, Thilo Glatzel, Daniel Loss, Ernst Meyer, arXiv:1505.06078 (2015)

System

Ingridients:

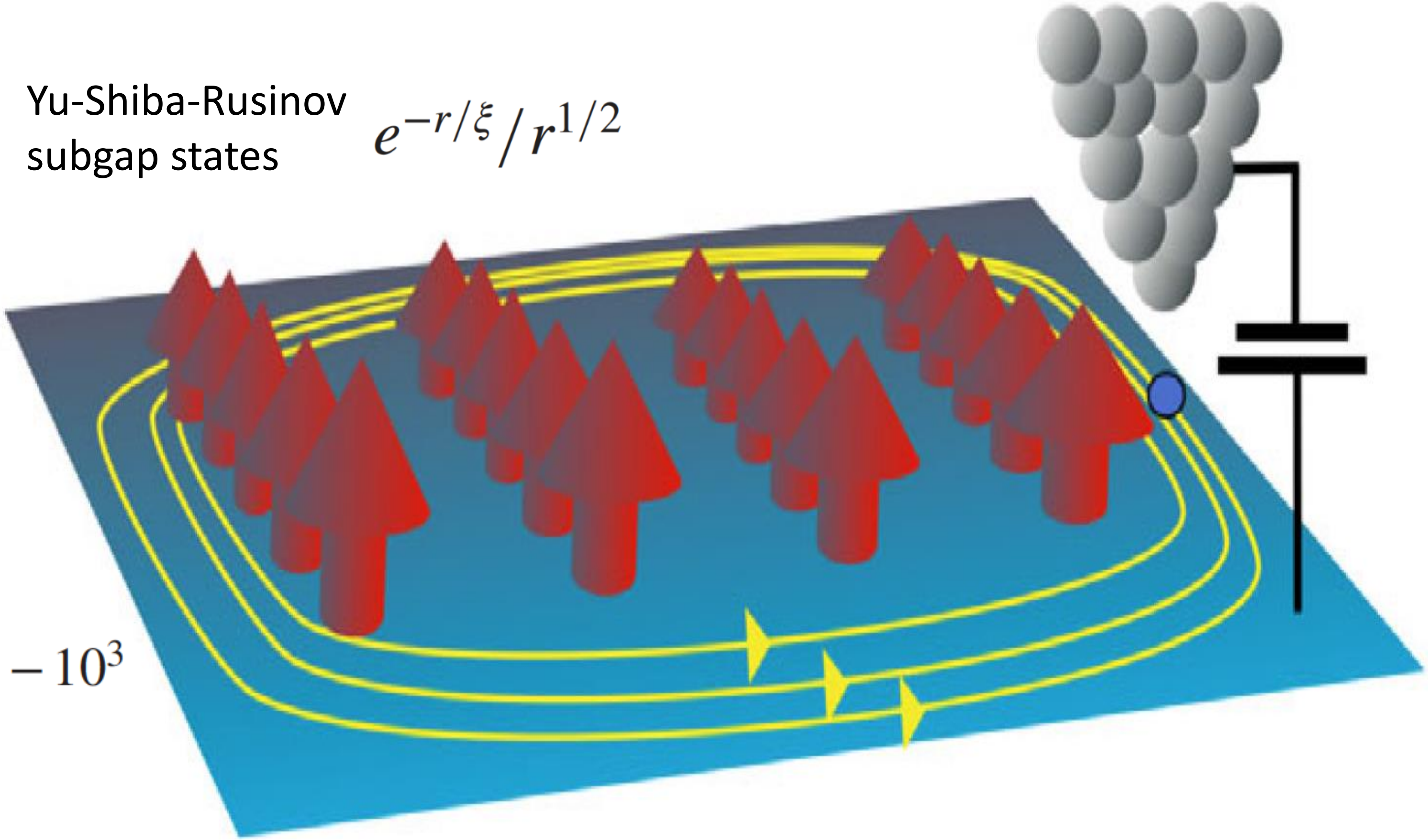
- 2D lattice of magnetic impurities (ferromagnetic order)
- S-type superconductor(surface)

Yu-Shiba-Rusinov subgap states $e^{-r/\xi}/r^{1/2}$

- Spin orbit interaction(Rashba)

Parameters:

- $a < \xi$ strong overlap with neighbours -> hybridization -> subgap band formation $\xi/a \sim 10 - 10^3$
- Isolated Shiba energy ε_0
- Hybridization energy of two impurities $\Delta/(k_F a)^{1/2}$



Theory

$$\mathcal{H} = \mathcal{H}^{(\mathrm{bulk})} + \mathcal{H}^{(\mathrm{imp})}$$

$$\mathcal{H}^{(\mathrm{bulk})}_{\mathbf{k}} = \tau_z \left[\xi_{\mathbf{k}}\sigma_0 + \alpha_R(k_y\sigma_x - k_x\sigma_y)\right] + \Delta\tau_x\sigma_0,$$

$$\mathcal{H}^{(\mathrm{imp})}(\mathbf{r}) = -J\sum_j \mathbf{S}_j \cdot \sigma\,\delta(\mathbf{r}-\mathbf{r}_j) \quad \xi_{\mathbf{k}} = \frac{\hbar^2 k^2}{2m} - \mu \quad \hat{\Psi} = (\hat{\psi}_\uparrow, \hat{\psi}_\downarrow, \hat{\psi}^\dagger_\downarrow, -\hat{\psi}^\dagger_\uparrow)^T$$

Ferromagnetic arrangement $\mathbf{S}_i = S\hat{e}_z$ $\quad \mathcal{H}(\mathbf{r})\Psi(\mathbf{r}) = E\Psi(\mathbf{r})$

Applying methodology from
F. Pientka, L. I. Glazman, and F. von Oppen, Phys. Rev. B 88, 155420 (2013)
P. M. R. Brydon, H.-Y. Hui, and J. D. Sau, Phys. Rev. B 91, 064505 (2015)

$$[\mathbf{S}_i \cdot \sigma - J_E(0)]\Psi(\mathbf{r}_i) = -\sum_{j\neq i} J_E(\mathbf{r}_i - \mathbf{r}_j)\Psi(\mathbf{r}_j)$$

$$J_E(\mathbf{r}) = JS\int \frac{d\mathbf{k}}{(2\pi)^2} e^{i\mathbf{k}\cdot\mathbf{r}}[E - \mathcal{H}^{(\mathrm{bulk})}_{\mathbf{k}}]^{-1} \text{ and } S = |\mathbf{S}_j|$$

Renormalization of the gap Δ(r) should be taken into account

arXiv:1501.07901,
T. Meng, J. Klinovaja,
S. Hoffman, P. Simon, D. Loss
arXiv:1503.08762

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Single impurity give rise to two subgap bound states

$$\pm\varepsilon_0 = \pm\Delta[(1-\alpha^2)/(1+\alpha^2)] \quad \alpha = \pi JS\mathcal{N}$$

Deep-dilute impurity arrangement

$$\alpha \approx 1 \quad [1/(k_F a)^{1/2}] \ll 1$$

Shiba bands in two component basis of decoupled impurity states

$$\Psi'_j(\mathbf{r}_j) \equiv \Psi'_j = (u(\mathbf{r}_j)v(\mathbf{r}_j))^T$$

Eigenstates of single impurity problem

$$\varepsilon_0 \approx \Delta(1-\alpha) \quad -\varepsilon_0$$

Effective Hamiltonian

$$[\mathbf{S}_i \cdot \sigma - J_E(0)]\Psi(\mathbf{r}_i) = -\sum_{j\neq i} J_E(\mathbf{r}_i - \mathbf{r}_j)\Psi(\mathbf{r}_j)$$

$$H\Psi' = E\Psi'$$

$$\Rightarrow \quad H_{ij} = \begin{pmatrix} h_{ij} & \Delta_{ij} \\ (\Delta_{ij})^\dagger & -h_{ij} \end{pmatrix}$$

Long range hopping elements

$$h_{ij} = \begin{cases} \varepsilon_0 & i = j \\ -\frac{\Delta^2}{2}[\tilde{I}_1^-(r_{ij}) + \tilde{I}_1^+(r_{ij})] & i \neq j \end{cases},$$

$$\Delta_{ij} = \begin{cases} 0 & i = j \\ \frac{\Delta}{2}[\tilde{I}_0^+(r_{ij}) - \tilde{I}_0^-(r_{ij})]\frac{x_{ij}-iy_{ij}}{r_{ij}} & i \neq j \end{cases}$$

$$\mathbf{r}_i - \mathbf{r}_j \equiv (x_{ij}, y_{ij})$$

$$r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$$

$$k_F^\pm = k_F(\sqrt{1+\lambda^2} \mp \lambda)$$

$$\tilde{I}_0^\pm(r) = \frac{\mathcal{N}_\pm}{\mathcal{N}}\Re[iJ_1(k_F^\pm r + ir/\xi) + H_{-1}(k_F^\pm r + ir/\xi)],$$

$$\tilde{I}_1^\pm(r) = \frac{\mathcal{N}_\pm}{\mathcal{N}}\frac{1}{\Delta}\Re[J_0(k_F^\pm r + ir/\xi) + iH_0(k_F^\pm r + ir/\xi)]$$

Rashba modified coherence length

$$\xi = [(v_F\sqrt{1+\lambda^2})/\Delta]$$

Odd pairing symmetry that generalizes the $p_x + ip_y$ type pairing

$$\Delta_{ij} = \Delta(x_{ij} - iy_{ij})f(r_{ij})$$

$$f(r_{ij}) = [\tilde{I}_0^+(r_{ij}) - \tilde{I}_0^-(r_{ij})]/2$$

$$\Delta_{ij} = -\Delta_{ji}$$

Results - topological properties- Chern number

$$H_{ij} = \begin{pmatrix} h_{ij} & \Delta_{ij} \\ (\Delta_{ij})^{\dagger} & -h_{ij} \end{pmatrix}$$

$$d_x(\mathbf{k}) = \Re \sum_j \Delta_{ij} e^{ik_x x_{ij} + ik_y y_{ij}},$$

$$d_y(\mathbf{k}) = \Im \sum_j \Delta_{ij} e^{ik_x x_{ij} + ik_y y_{ij}},$$

$$d_z(\mathbf{k}) = \sum_j h_{ij} e^{ik_x x_{ij} + ik_y y_{ij}},$$

$$H(\mathbf{k}) = \mathbf{d}(\mathbf{k}) \cdot \boldsymbol{\sigma}$$

Altland-Zirnbauer symmetry class D

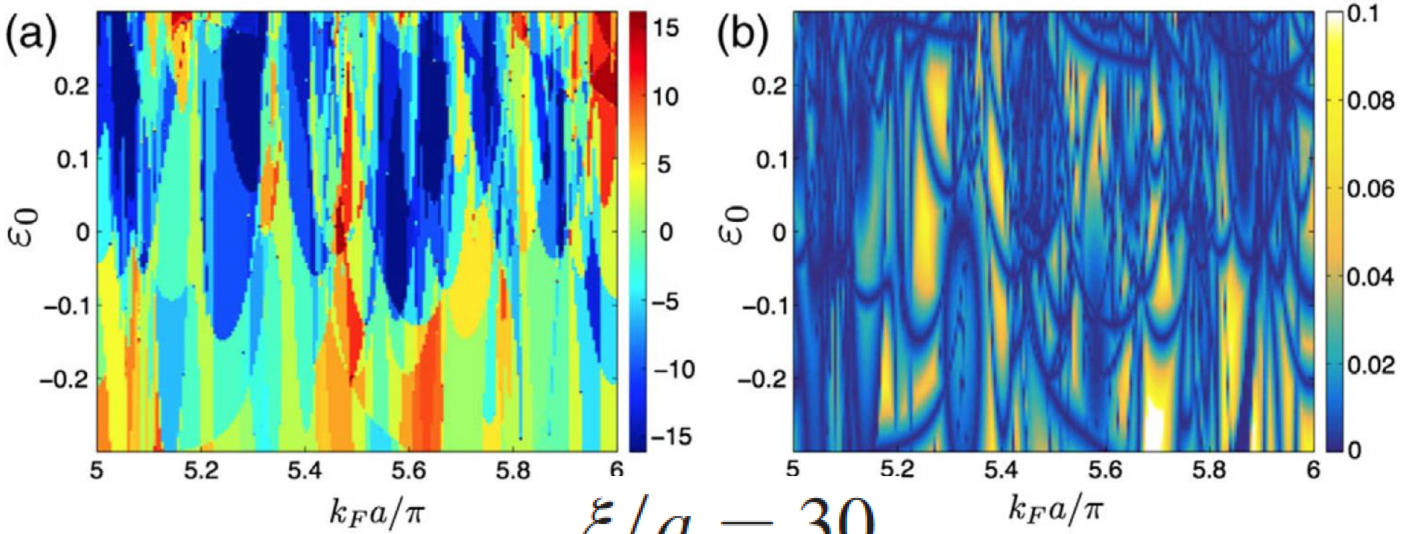
Chern number (how many times the vector $\hat{\mathbf{d}} = \mathbf{d}/|\mathbf{d}|$ wraps around the unit sphere) -> topological phase diagram

$$C = \frac{1}{4\pi} \int_{\mathrm{BZ}} d^2\mathbf{k} \, \frac{\mathbf{d} \cdot \partial_{k_x} \mathbf{d} \times \partial_{k_y} \mathbf{d}}{|\mathbf{d}|^3}$$

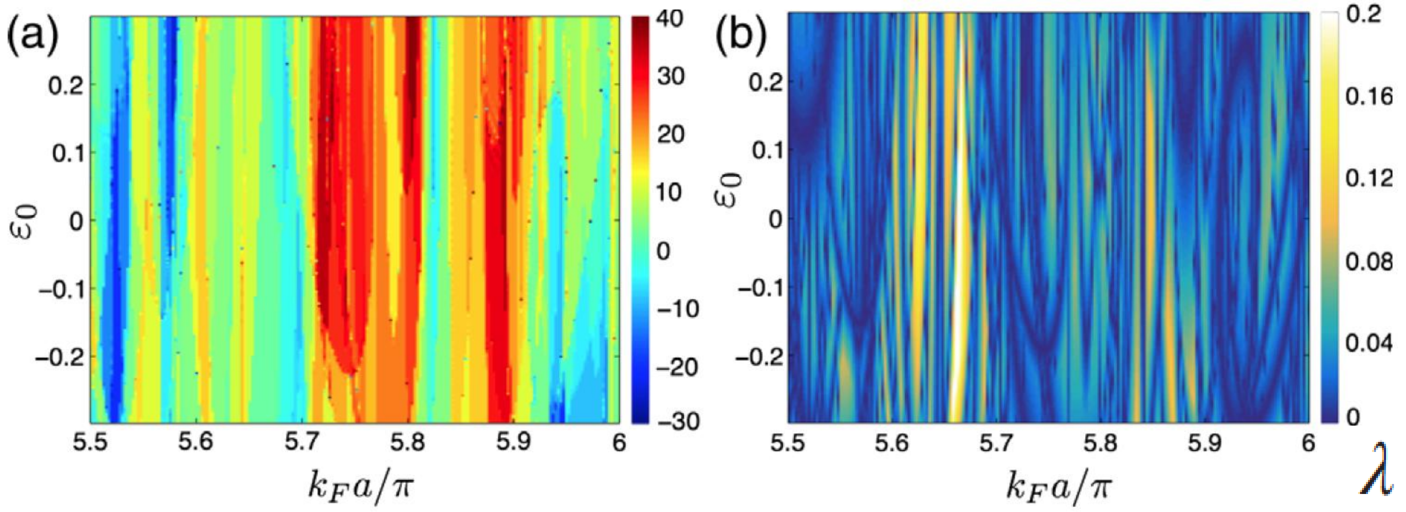
The bulk-boundary correspondence implies that topological states with Chern number C=q support **|q| branches of chiral gapless modes localized near the edge**. The **sign of C determines the chirality** of the edge modes. In **nonsuperconducting** systems **C determines a quantized Hall conductance** whereas in **superconducting** systems only the **thermal Hall conductance is quantized** and the edge states **are propagating Majorana modes**.

Chern number phase diagram

$\xi/a = 10$ $\qquad \min_k E(k)$



$\xi/a = 30$



Connection between long-range hopping and Chern number

n-th hopping in *x, y* give rise to such terms in d_i as to:

$\cos(nk_{x/y}a), \sin(nk_{x/y}a)$

n-th hopping terms decays as:

$[|\Delta|/(k_F a)^{1/2}](e^{-an/\xi}/n^{1/2})$

slow decay for $n < \xi/a$

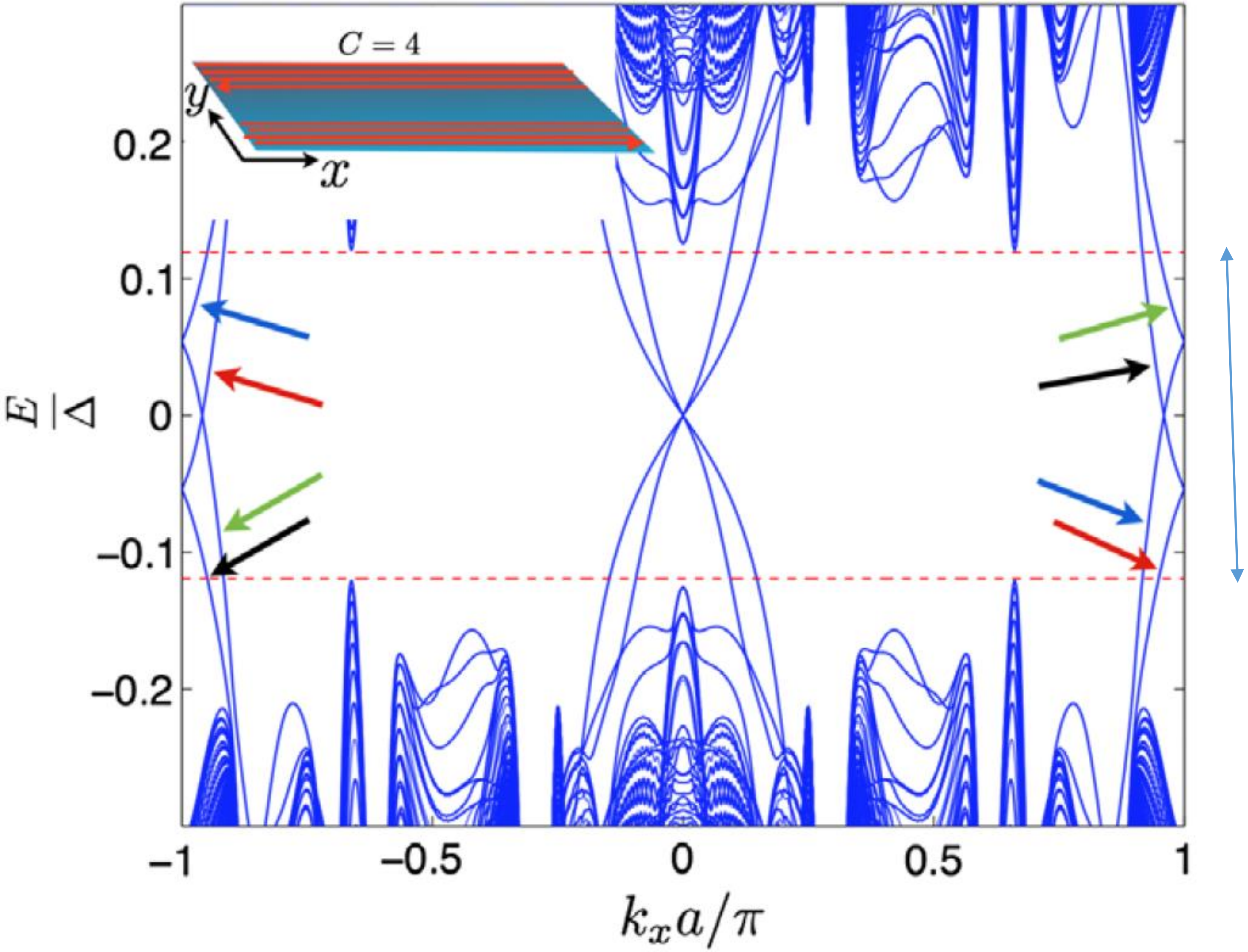
Competition between $\mathcal{O}(\xi/a)$ long range hopping terms

$$I_1^{\pm}(\mathbf{r}) \approx \frac{\mathcal{N}_{\pm}}{\mathcal{N}} \frac{1}{\sqrt{\Delta^2 - E^2}} \sqrt{\frac{2/\pi}{k_F^{\pm} r}} \cos\left(k_F^{\pm} r - \frac{\pi}{4}\right) e^{-r/\xi_E},$$

$$I_0^{\pm}(\mathbf{r}) \approx -\frac{\mathcal{N}_{\pm}}{\mathcal{N}} \left[\sqrt{\frac{2/\pi}{k_F^{\pm} r}} \sin\left(k_F^{\pm} r - \frac{3\pi}{4}\right) e^{-r/\xi_E} + \frac{2/\pi}{(k_F^{\pm} r)^2} \right]$$

$\lambda = 0.05$

Spectrum of the infinite strip



Chern number C=4

four chiral edge modes traversing the gap

Gap for the infinite system

$$\xi/a = 10,\ \varepsilon_0/\Delta = -0.25,$$
$$k_F a/\pi = 3.56,\ \lambda = 0.05$$

Observable consequences - LDOS

Topological edge modes supports a quantized thermal conductance

$$G_T = |C| \times G_0$$

$$G_0 = (\pi^2 k_B^2 T / 3h)$$

Such a measurement challenging at the moment

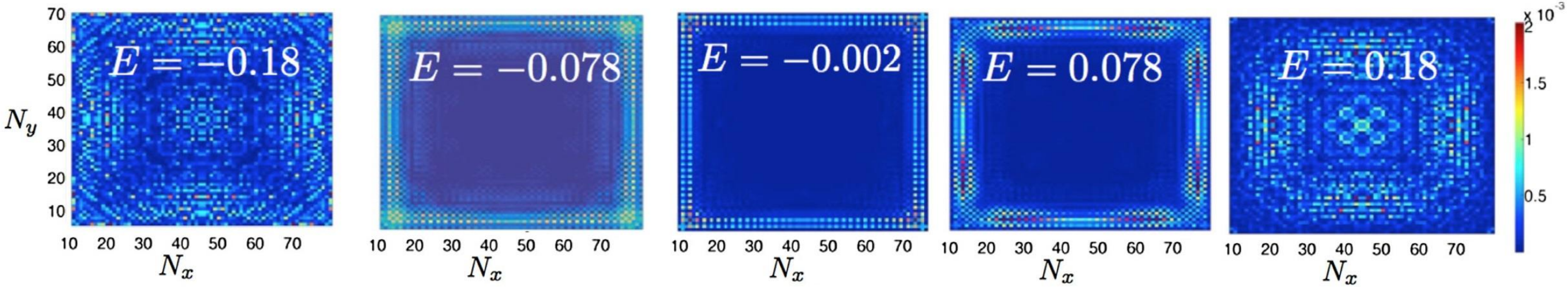
STM spectroscopy of LDOS or better AFM

$$N(\mathbf{r}, E) = \sum_n |u_n(\mathbf{r})|^2 \delta(E - E_n) + |v_n(\mathbf{r})|^2 \delta(E + E_n)$$

In the absence of magnetic atoms system is in trivial state and $N(\mathbf{r}, E) = 0$ for $|E| < \Delta$

LDOS near the center of the gap should reveal the existence of the topological edge states

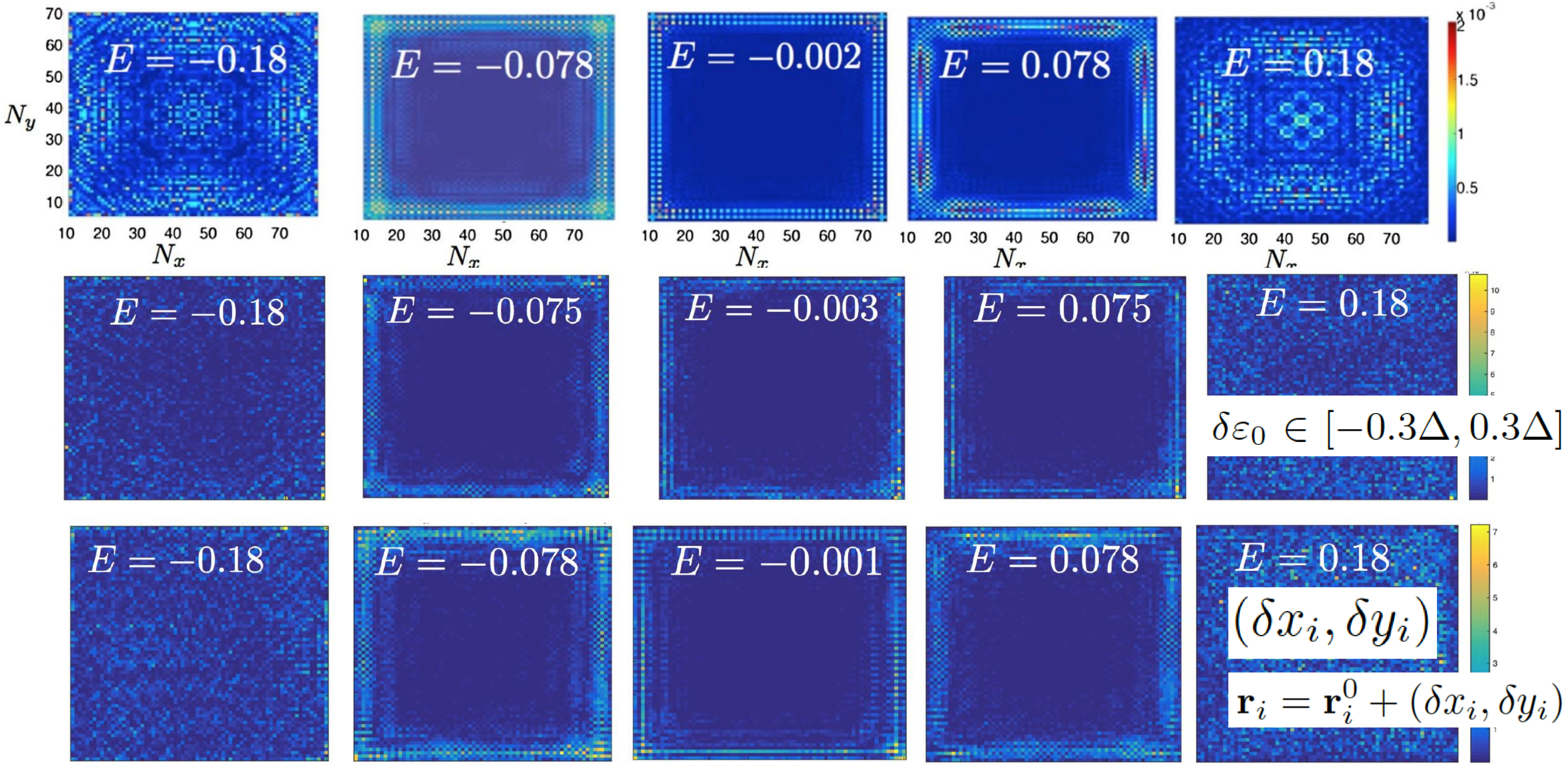
LDOS edge states for $|E| < \Delta$



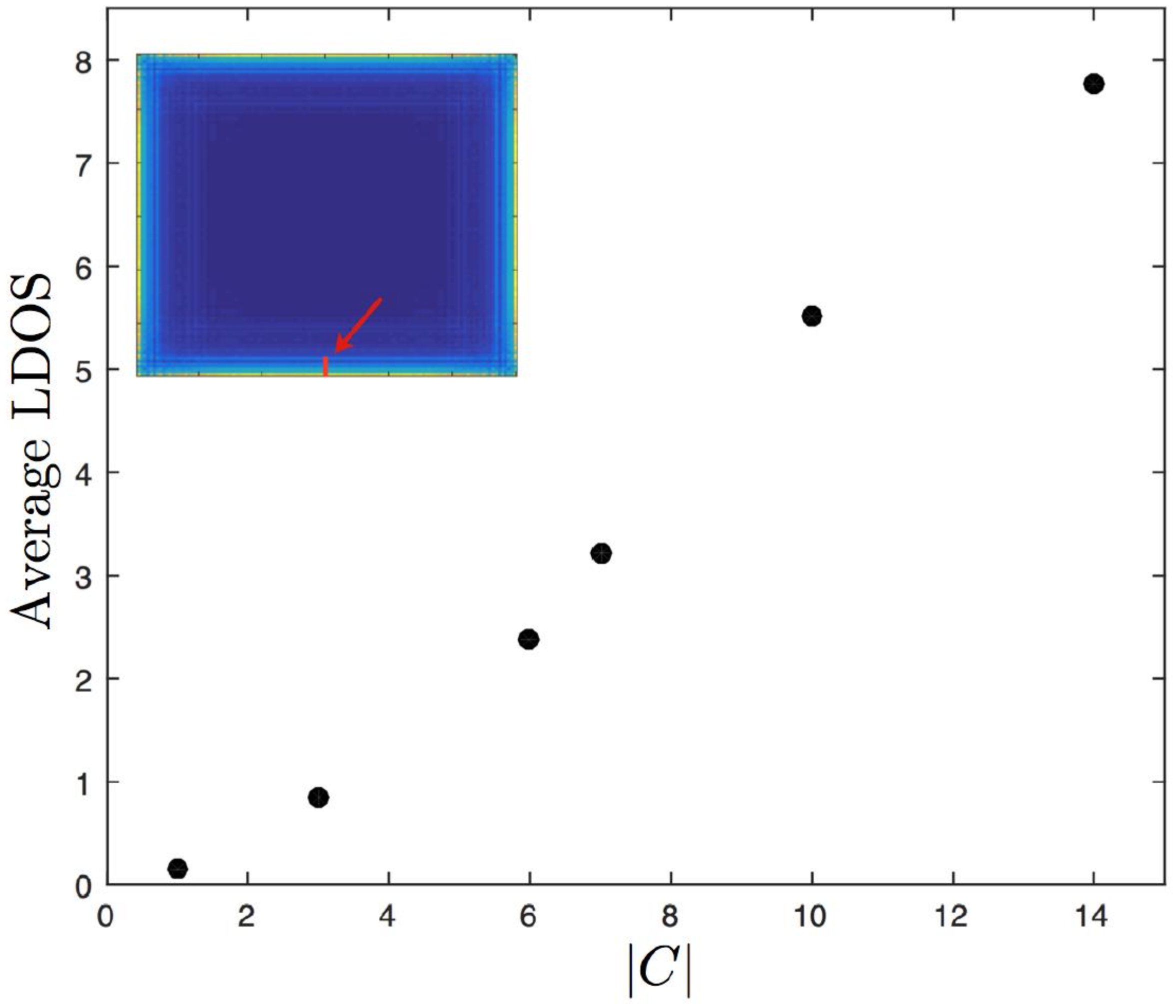
$C = 3 \qquad 70 \times 70$

$\xi/a = 10,\ \varepsilon_0/\Delta = -0.22,\ k_F a/\pi = 4.9,\ \lambda = 0.05$

LDOS edge states – disorder effects



Relation between LDOS and Chern numbers



Spatialy avaraged midgap LDOS at the edge

$$\xi/a = 5, \ \lambda = 0.05$$

$$\varepsilon_0/\Delta = 0.25$$

$$k_F a = 5.95 \, (C = -1)$$

$$k_F a = 9.96 \ (C = 3)$$

$$k_F a = 5.36 \ (C = 6)$$

$$k_F a = 5.49 \ (C = 7)$$

$$k_F a = 5.11 \ (C = 10)$$

$$k_F a = 3.92 \ (C = 14)$$

Summary

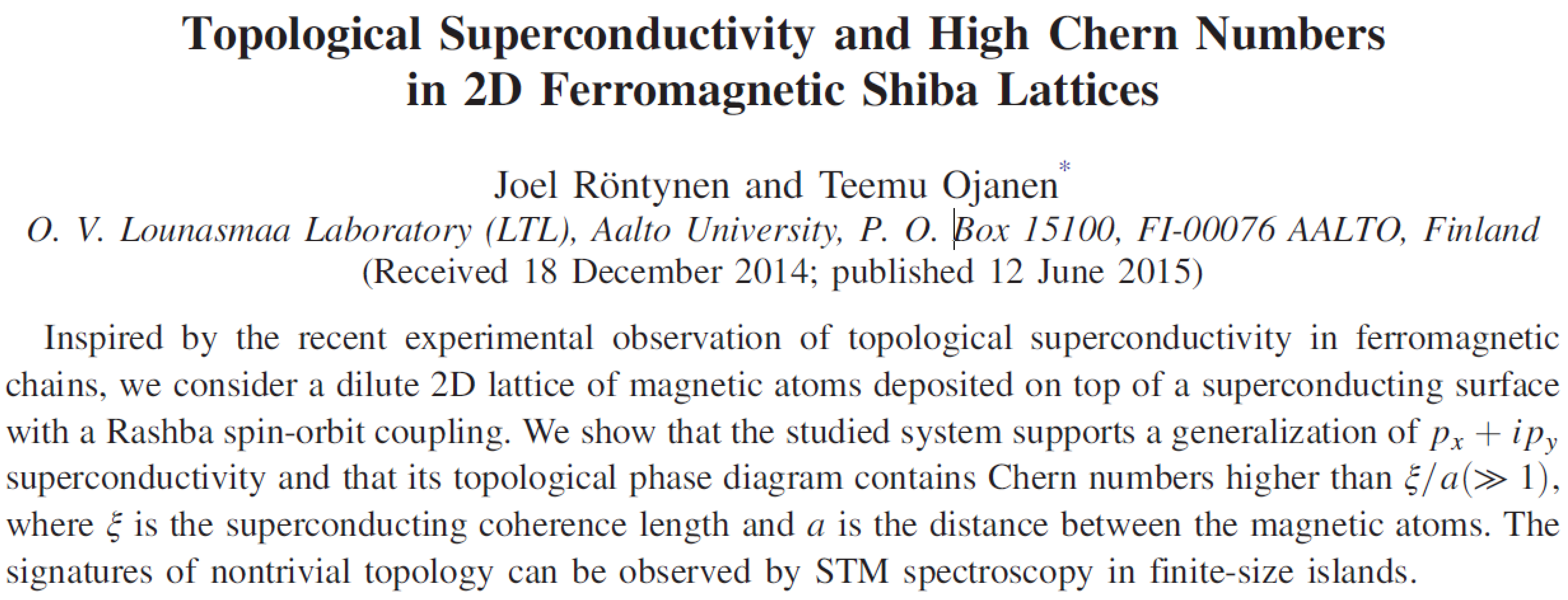
- 2D lattice of the magnetic impurities arranged in ferromagnetic order
- surface of s-wave superconductor
- spin orbit interaction

} Generalized 2D $p_x + ip_y$ superconductivity

- Long range hopping => high Chern numbers

- But main drawback of the model => renormalization of the superconducting gap not included which may affect/suppress the topological effects

Thank you for your attention!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Details

Theory

Renormalization of the gap Δ(r) should be taken into account

arXiv:1501.07901v2
T. Meng, J. Klinovaja,
S. Hoffman, P. Simon, D. Loss

$$\mathcal{H} = \mathcal{H}^{(\mathrm{bulk})} + \mathcal{H}^{(\mathrm{imp})}$$

$$\mathcal{H}^{(\mathrm{bulk})}_{\mathbf{k}} = \tau_z \left[\xi_{\mathbf{k}}\sigma_0 + \alpha_R(k_y\sigma_x - k_x\sigma_y)\right] + \Delta\tau_x\sigma_0,$$

$$\mathcal{H}^{(\mathrm{imp})}(\mathbf{r}) = -J\sum_j \mathbf{S}_j\cdot\sigma\,\delta(\mathbf{r}-\mathbf{r}_j) \qquad \xi_{\mathbf{k}} = \frac{\hbar^2k^2}{2m} - \mu$$

$$\mathcal{H}(\mathbf{r})\Psi(\mathbf{r}) = E\Psi(\mathbf{r})$$

$$\left[E - \mathcal{H}^{(\mathrm{bulk})}(\mathbf{r})\right]\Psi(\mathbf{r}) = -J\sum_j \mathbf{S}_j\cdot\sigma\,\delta(\mathbf{r}-\mathbf{r}_j)\Psi(\mathbf{r}_j) \qquad \Psi(\mathbf{r}) = \int\frac{d\mathbf{k}}{(2\pi)^2}e^{i\mathbf{k}\cdot\mathbf{r}}\Psi_{\mathbf{k}}$$

$$\left[E - \mathcal{H}^{(\mathrm{bulk})}_{\mathbf{k}}\right]\Psi_{\mathbf{k}} = -J\sum_j \mathbf{S}_j\cdot\sigma\,e^{-i\mathbf{k}\cdot\mathbf{r}_j}\Psi(\mathbf{r}_j)$$

Methodology similar as in:
F. Pientka, L. I. Glazman, and F. von Oppen, Phys. Rev. B 88, 155420 (2013)
P. M. R. Brydon, H.-Y. Hui, and J. D. Sau, Phys. Rev. B 91, 064505 (2015)

Theory

$$\Psi(\mathbf{r}) = -\sum_j J_E(\mathbf{r}-\mathbf{r}_j)\,\hat{S}_j\cdot\sigma\,\Psi(\mathbf{r}_j) \qquad S=|\mathbf{S}|,\ \hat{S}=\mathbf{S}/S$$

$$J_E(\mathbf{r}) = JS\int\frac{d\mathbf{k}}{(2\pi)^2}e^{i\mathbf{k}\cdot\mathbf{r}}\left[E-\mathcal{H}_{\mathbf{k}}^{(\mathrm{bulk})}\right]^{-1}$$

RSOI lifts spin degeneracy and give rise to two helicity bands with dispersion $\xi_\pm = \xi_k \pm \alpha_R k$

Propagator splits into two helical sectors: $\left[E-\mathcal{H}_{\mathbf{k}}^{(\mathrm{bulk})}\right]^{-1} = \frac{1}{2}(G_- + G_+)$

$$G_\pm = \frac{(E\tau_0+\xi_\pm\tau_z+\Delta\tau_x)(\sigma_0\pm\sin\varphi\,\sigma_x\mp\cos\varphi\,\sigma_y)}{E^2-\xi_\pm^2-\Delta^2} \qquad \mathbf{k}=k(\cos\varphi,\sin\varphi)$$

Single magnetic impurity (Yu-Shiba-Rusinov states)

$$\Psi(\mathbf{r}) = -\sum_j J_E(\mathbf{r}-\mathbf{r}_j)\,\hat{S}_j\cdot\sigma\,\Psi(\mathbf{r}_j) \;\Longrightarrow\; \left[\mathbb{1} + J_E(\mathbf{0})\,\hat{S}\cdot\sigma\right]\Psi(\mathbf{0}) = 0$$

In the limit of deep impurities $|E| < \Delta$

$$\left[\mathbb{1} - \frac{\alpha}{\sqrt{\Delta^2 - E^2}}(E\tau_0 + \Delta\tau_x)\hat{S}\cdot\sigma\right]\Psi(\mathbf{0}) = 0$$

$|\tau_x -\rangle|\downarrow\rangle \quad |\tau_x +\rangle|\uparrow\rangle$

$\alpha = \pi J S \mathcal{N}$ Dimensionless impurity strength

$\mathcal{N} = \frac{1}{2\pi}\frac{m}{\hbar^2}$ Density of states at the Fermi level in the absence of SOI

$E = \Delta\frac{1-\alpha^2}{1+\alpha^2} \quad E = -\Delta\frac{1-\alpha^2}{1+\alpha^2}$

$\tau_x|\tau_x\pm\rangle = \pm|\tau_x\pm\rangle$

$\hat{S}\cdot\sigma|\uparrow\rangle = |\uparrow\rangle$

$\hat{S}\cdot\sigma|\downarrow\rangle = -|\downarrow\rangle$

2D lattice of magnetic impurities

$$\left[\mathbb{1} + J_E(\mathbf{0})\,\hat{S}_i\cdot\sigma\right]\Psi(\mathbf{r}_i) = -\sum_{j\neq i} J_E(\mathbf{r}_i-\mathbf{r}_j)\,\hat{S}_j\cdot\sigma\,\Psi(\mathbf{r}_j)$$

Limit of deep impurities $\varepsilon_0 \ll \Delta$

linearization of left hand side with regards to E and 1-α

$$\left[\mathbb{1} - (E/\Delta\,\tau_0 + \alpha\,\tau_x)\hat{S}_i\cdot\sigma\right]\Psi(\mathbf{r}_i) = -\sum_{j\neq i}\lim_{\substack{E\to 0\\ \alpha\to 1}} J_E(\mathbf{r}_i-\mathbf{r}_j)\,\hat{S}_j\cdot\sigma\,\Psi(\mathbf{r}_j)$$

Ferromagnetic arrangment $\mathbf{S}_i = S\hat{e}_z$

$$J_E(\mathbf{r}) = -\frac{\alpha}{2}\Big\{\left[I_1^-(\mathbf{r}) + I_1^+(\mathbf{r})\right](E\tau_0\sigma_0 + \Delta\tau_x\sigma_0) - \left[I_2^-(\mathbf{r}) - I_2^+(\mathbf{r})\right]\tau_z\sigma_x + \left[I_3^-(\mathbf{r}) - I_3^+(\mathbf{r})\right]\tau_z\sigma_y\Big\} + g(\tau_z, \sigma_{x/y}, \tau_x\sigma_{x/y})$$