Fractional topological phases in three-dimensional coupled-wire systems

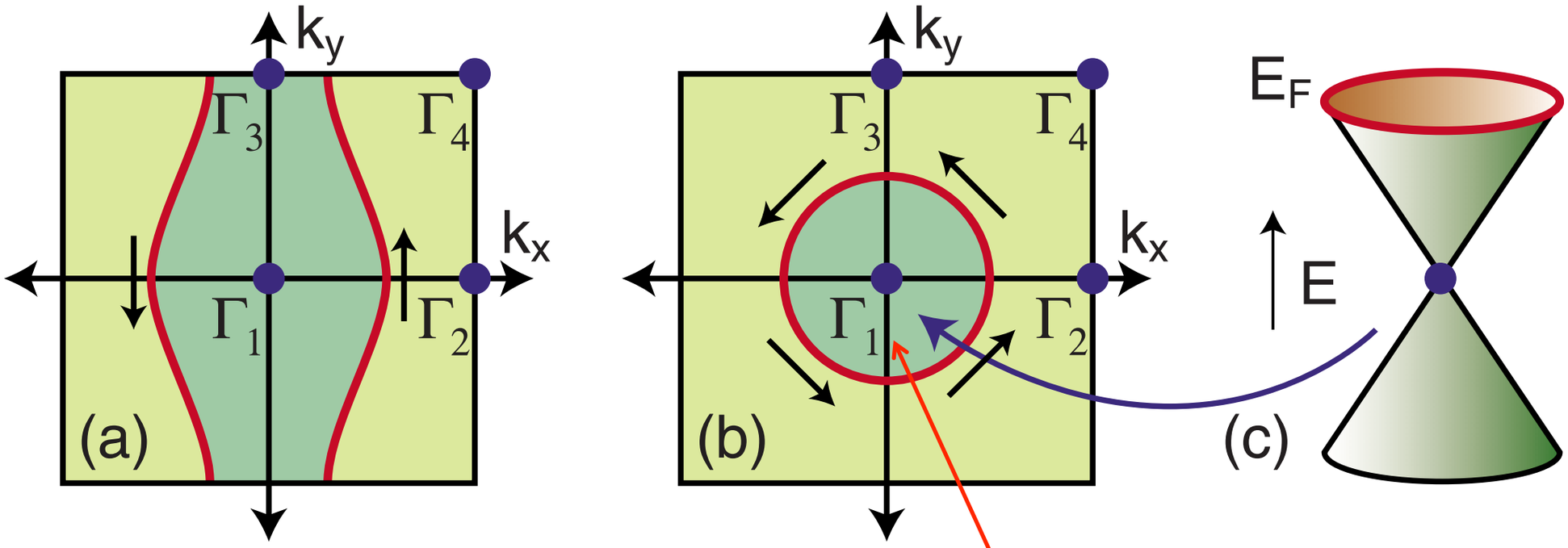
(arXiv:1506.01364)

Tobias Meng

JC, 21st of July

3D topological insulators [weak, strong]

insulating in the bulk, states present at the suface



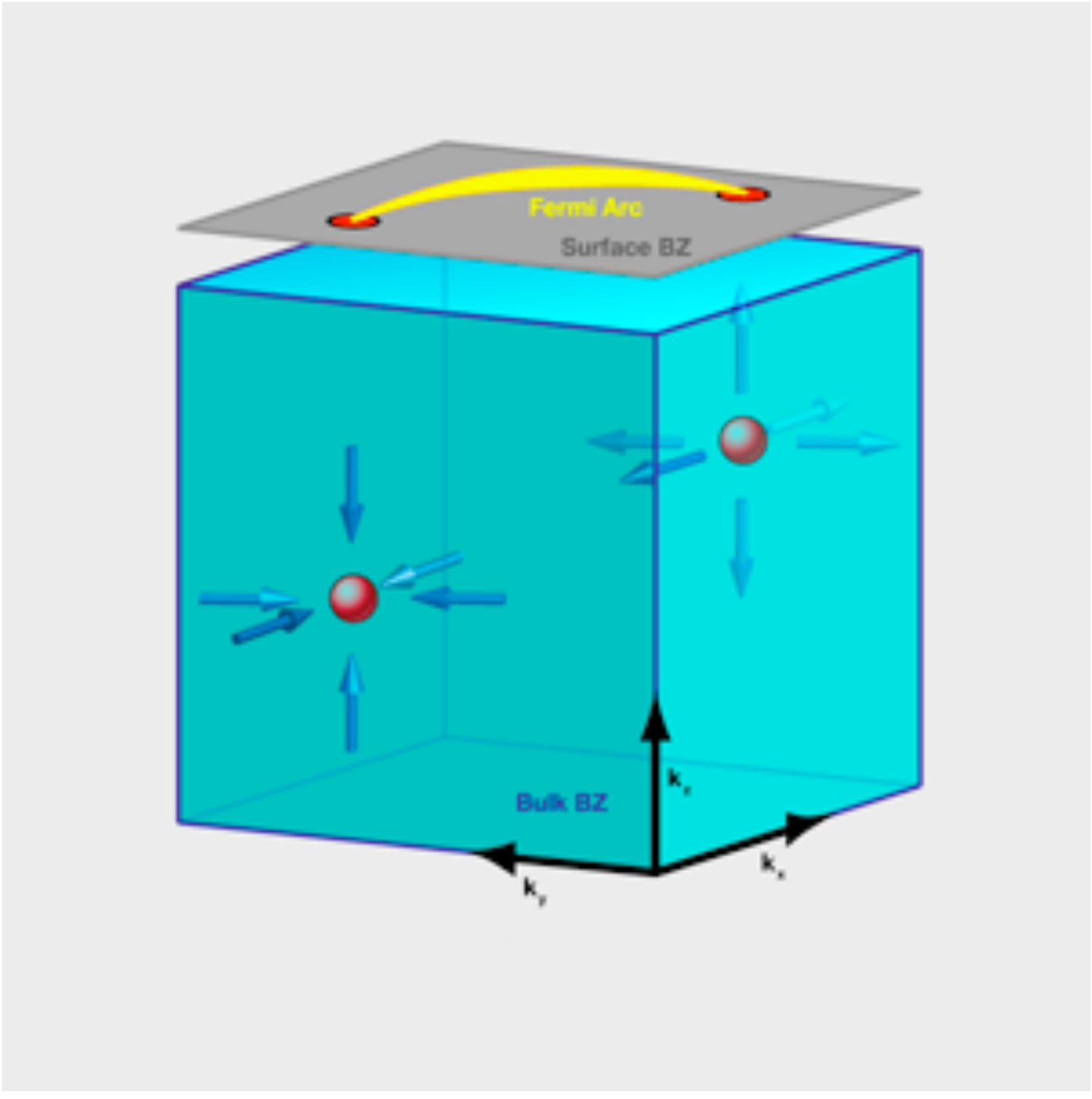
Weak TI -> stacking layers of the 2D quantum spin Hall insulator

spin has to rotate, non-trivial Berry phase

The surface states of a strong TI form 2D topological metal

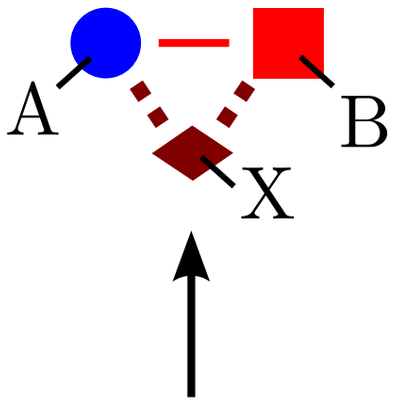
M. Z. Hasan and C. L. Kane: *Colloquium*: Topological insulators

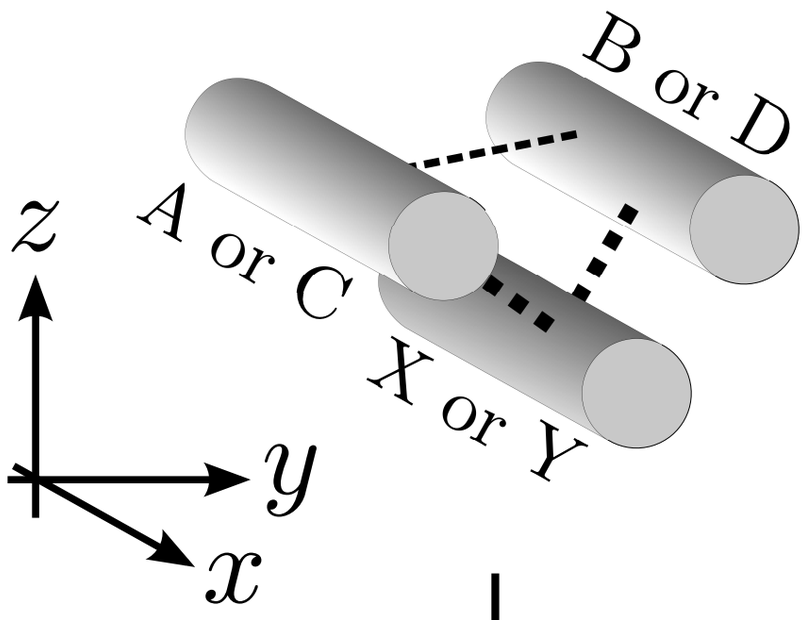
Weyl semimetals

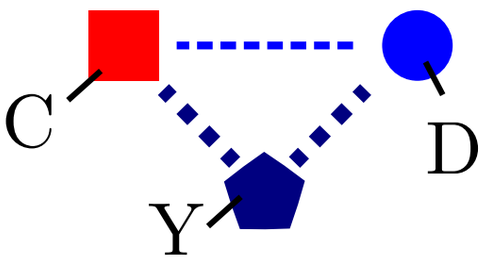


Building blocks [non-interacting case]

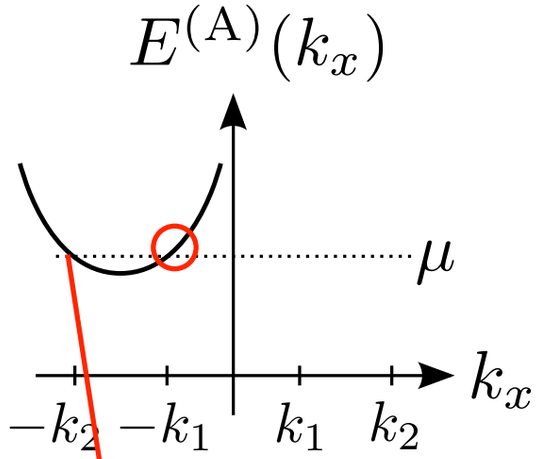
$k_1=0$
$k_2=k_3$

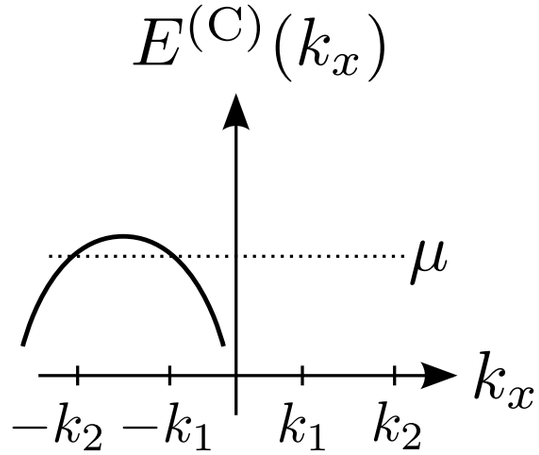


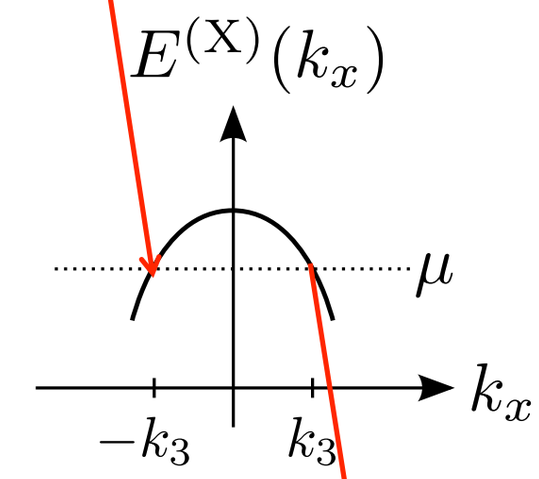


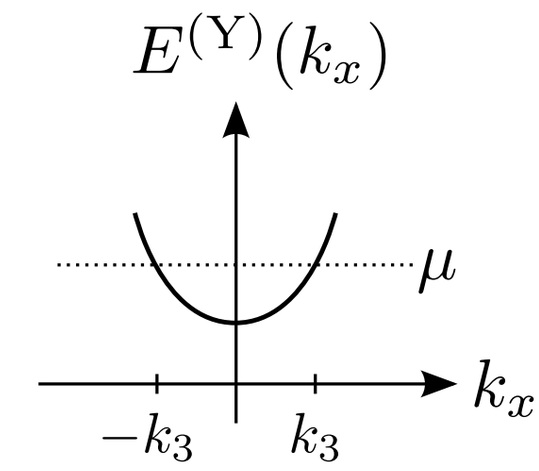


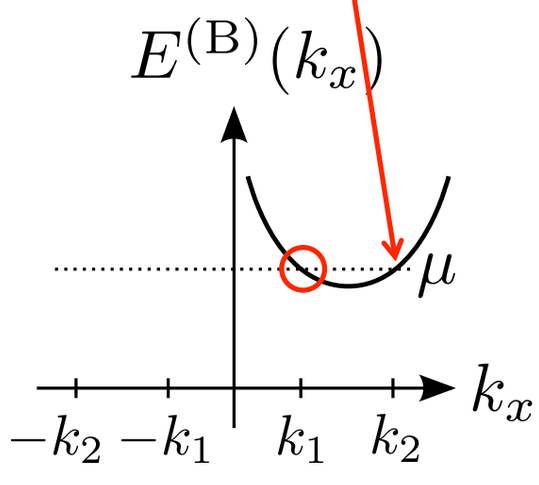
SOI + magnetic field

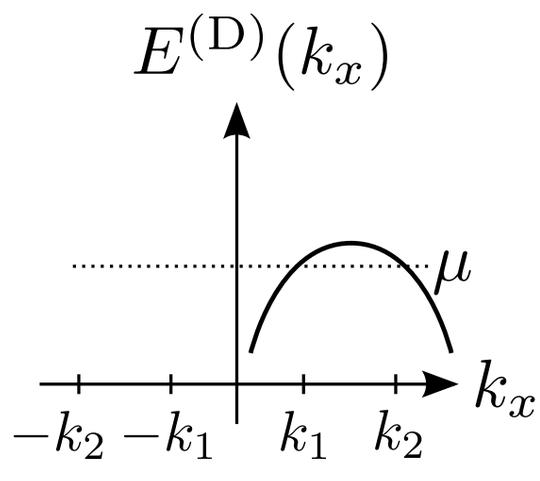








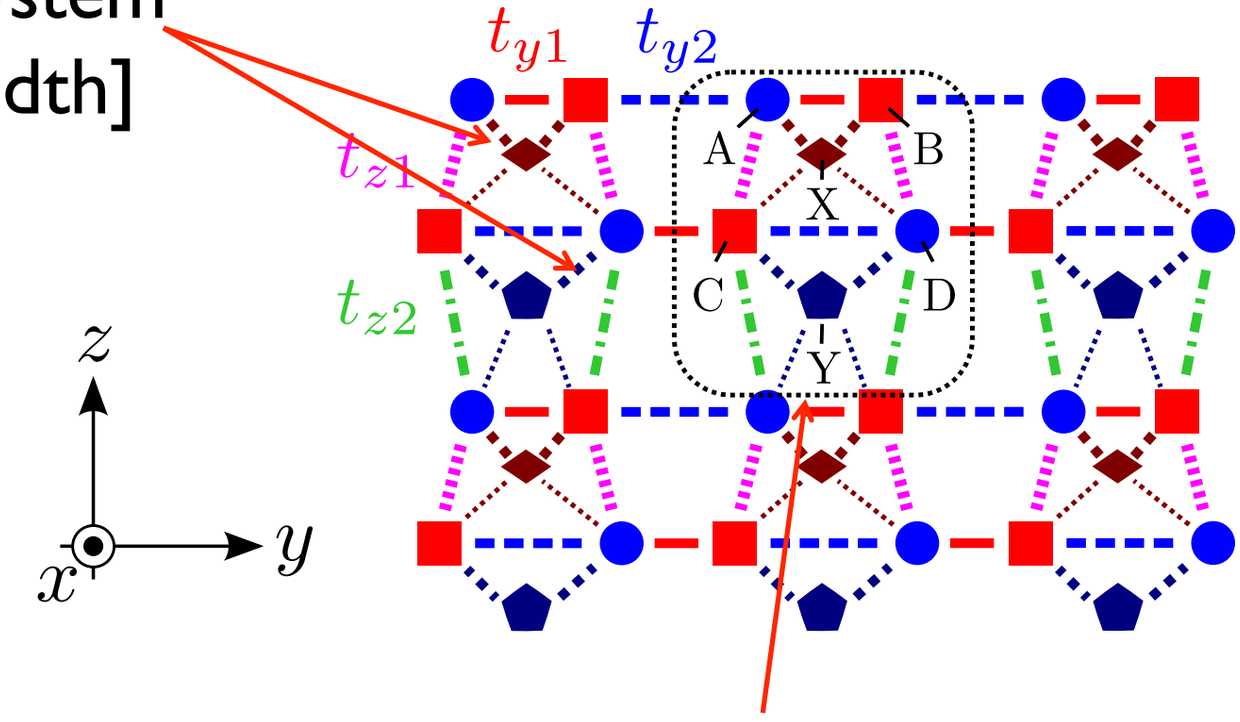




S. L. Sondhi and K. Yang, Phys. Rev. B 63, 054430 (2001)

3D system of non-interacting wires

largest energy scale in the system
[together with wire bandwidth]



$$\Psi_{k_x pq} = (c^{(\mathrm{A})}_{k_x pq}, c^{(\mathrm{B})}_{k_x pq}, c^{(\mathrm{C})}_{k_x pq}, c^{(\mathrm{D})}_{k_x pq})^T$$

$$H = \sum_{k_x} \sum_{p,q,p'q'} \Psi^\dagger_{k_x pq} \begin{pmatrix} \mathcal{H}_{11}\delta_{q,q'} & \mathcal{H}_{12}\delta_{p,p'} \\ \mathcal{H}_{21}\delta_{p,p'} & \mathcal{H}_{22}\delta_{q,q'} \end{pmatrix} \Psi_{k_x p'q'} \,, \quad (1)$$

unit cell has 4 modes

$$\mathcal{H}_{11} = \begin{pmatrix} v_F k_x \delta_{p,p'} & t_{y1}\delta_{p,p'} + t_{y2}\delta_{p,p'+1} \\ t_{y1}\delta_{p,p'} + t_{y2}\delta_{p,p'-1} & -v_F k_x \delta_{p,p'} \end{pmatrix}, \quad (2)$$

$$\mathcal{H}_{12} = (t_{z1}\delta_{q,q'} + t_{z2}\delta_{q,q'-1})\, \mathbb{1}_{2\times 2} \,, \quad (3)$$

$$\mathcal{H}_{21} = (t_{z1}\delta_{q,q'} + t_{z2}\delta_{q,q'+1})\, \mathbb{1}_{2\times 2} \,, \quad (4)$$

$$\mathcal{H}_{22} = \begin{pmatrix} -v_F k_x \delta_{p,p'} & t_{y2}\delta_{p,p'} + t_{y1}\delta_{p,p'+1} \\ t_{y2}\delta_{p,p'} + t_{y1}\delta_{p,p'-1} & v_F k_x \delta_{p,p'} \end{pmatrix}, \quad (5)$$

->Fourier transform

$$c_{\mathbf{k}}^{(\mathrm{B})} \rightarrow e^{ik_y a_y/2}\, c_{\mathbf{k}}^{(\mathrm{B})} \qquad c_{\mathbf{k}}^{(\mathrm{C})} \rightarrow e^{-ik_z a_z/2}\, c_{\mathbf{k}}^{(\mathrm{C})},\; c_{\mathbf{k}}^{(\mathrm{D})} \rightarrow e^{ik_y a_y/2}\, e^{-ik_z a_z/2}\, c_{\mathbf{k}}^{(\mathrm{D})}$$

σ (within the building-block modes)
τ (between the building blocks)

$$H = \sum_{\mathbf{k}} \Psi_{\mathbf{k}}^{\dagger}\, \mathrm{diag}(\mathcal{H}_{\mathbf{k},+}, \mathcal{H}_{\mathbf{k},-})\, \Psi_{\mathbf{k}}$$

$$\mathcal{H}_{\mathbf{k},\pm} = v_F k_x \sigma_z + M_{\pm}(\mathbf{k})\sigma_x - t_{y-} \sin(k_y a_y/2)\sigma_y$$

$$M_{\pm}(\mathbf{k}) = \pm\sqrt{t_{z1}^2 + t_{z2}^2 + 2t_{z1}t_{z2}\cos(k_z a_z)} + t_{y+}\cos(k_y a_y/2)$$

$$t_{y\pm} = t_{y1} \pm t_{y2}$$

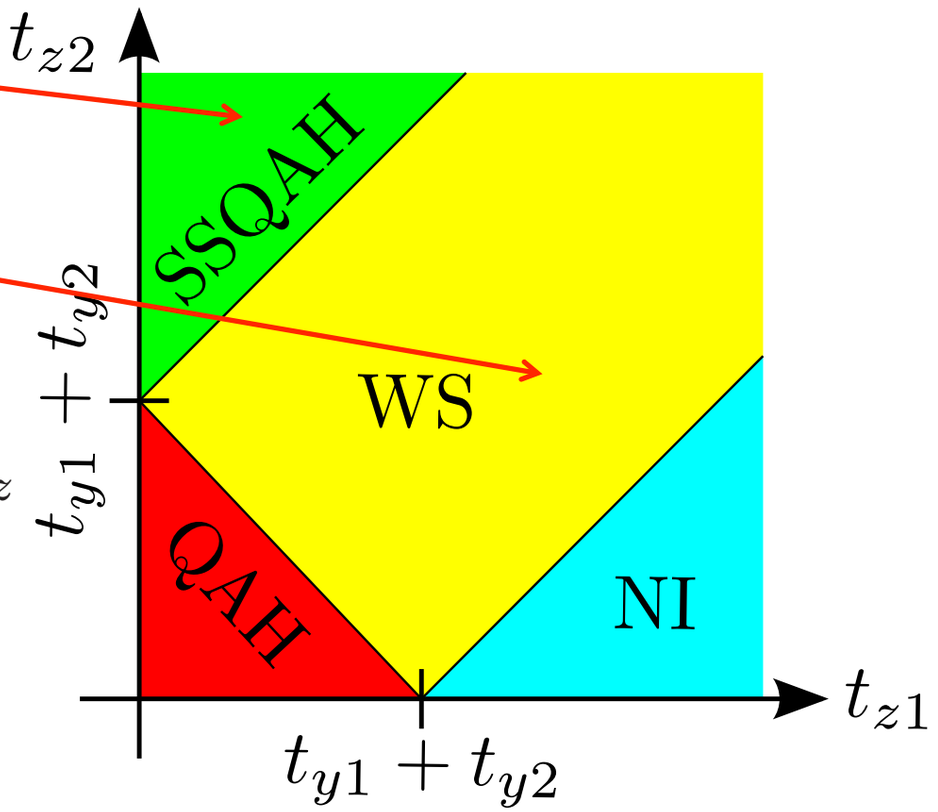
A. A. Burkov and L. Balents, Phys. Rev. Lett. 107, 127205 (2011).

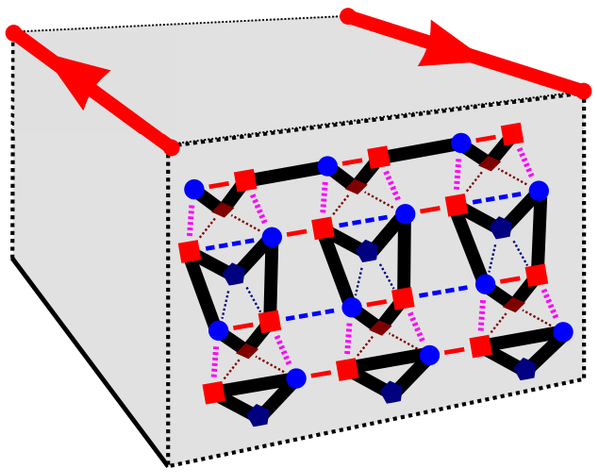
Phase diagram of the non-interacting model

new phase

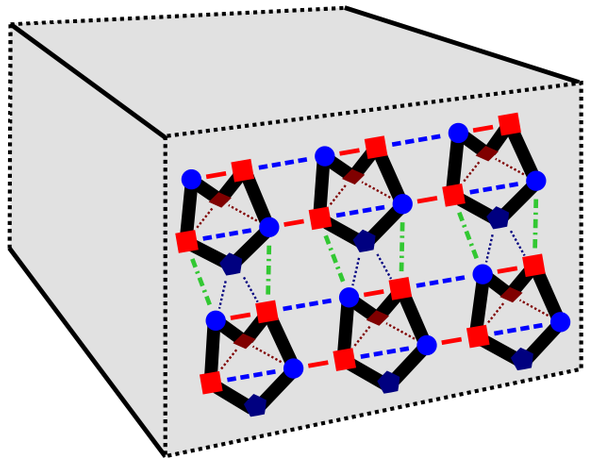
$$\mathbf{k}_{\pm} = (0, 0, \pi/a_z \pm k_{z0})^T$$

$$k_{z0} = \arccos(1 - [(t_{y1}+t_{y2})^2 - (t_{z1}-t_{z2})^2]/2t_{z1}t_{z2})/a_z$$

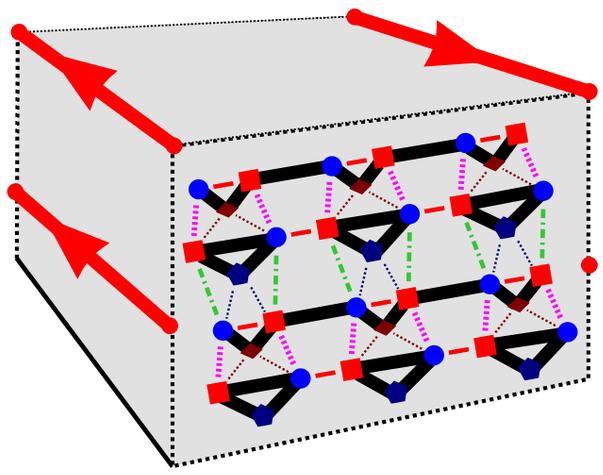




SS(F)QAH



NI



(F)QAH

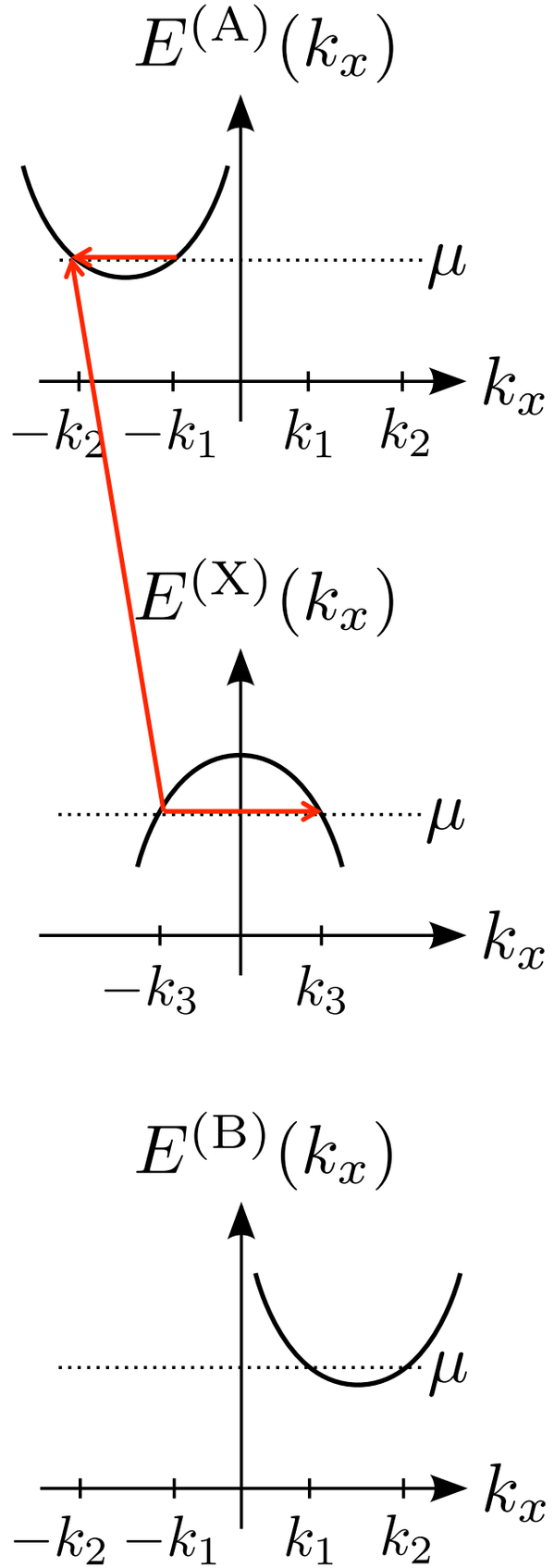
Electron-electron interaction and fractionalization

$$k_1 = mk_3$$

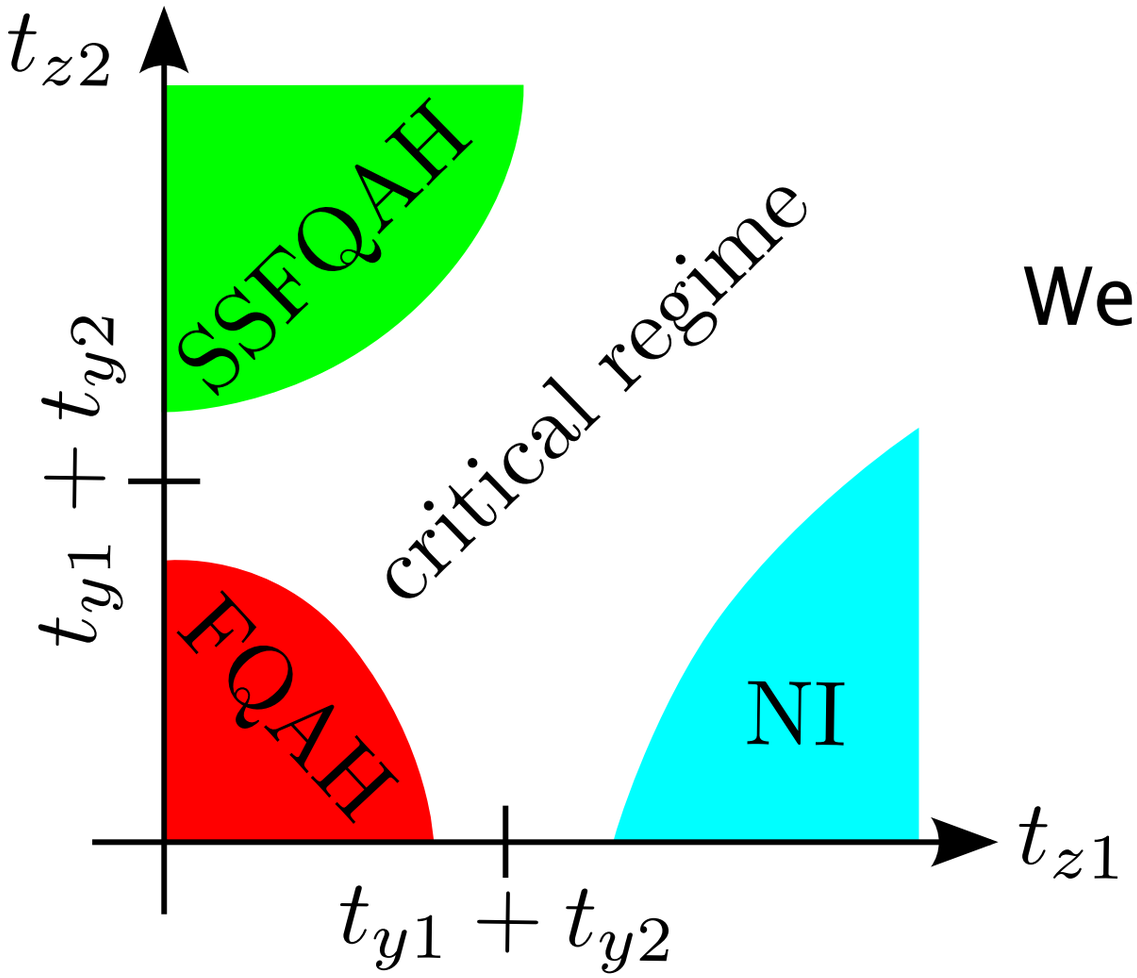
$$k_2 = (m+1)k_3$$

$$R^{\dagger}_{A(D)}{}^{m} R^{\dagger}_{X(Y)}{}^{m+1} L_{X(Y)}{}^{m} L_{A(D)}{}^{m+1} + \text{H.c.} \ ,$$
$$R^{\dagger}_{X(Y)}{}^{m} R^{\dagger}_{B(C)}{}^{m+1} L_{B(C)}{}^{m} L_{X(Y)}{}^{m+1} + \text{H.c.} \ .$$

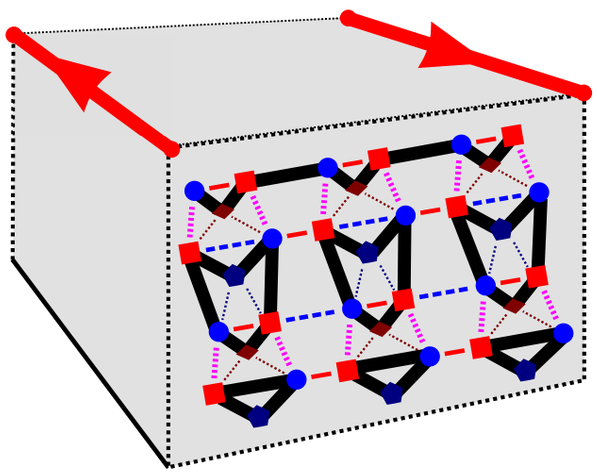
$$R^{\dagger}_{B}{}^{m} R^{\dagger}_{A}{}^{m+1} L_{A}{}^{m} L_{B}{}^{m+1} + \text{H.c.} \ ,$$
$$R^{\dagger}_{C}{}^{m} R^{\dagger}_{D}{}^{m+1} L_{D}{}^{m} L_{C}{}^{m+1} + \text{H.c.} \ ,$$
$$R^{\dagger}_{C}{}^{m} R^{\dagger}_{A}{}^{m+1} L_{A}{}^{m} L_{C}{}^{m+1} + \text{H.c.} \ ,$$
$$R^{\dagger}_{B}{}^{m} R^{\dagger}_{D}{}^{m+1} L_{D}{}^{m} L_{B}{}^{m+1} + \text{H.c.} \ .$$



Electron-electron interaction and fractionalization



Weyl semimetal
-> extended critical phase



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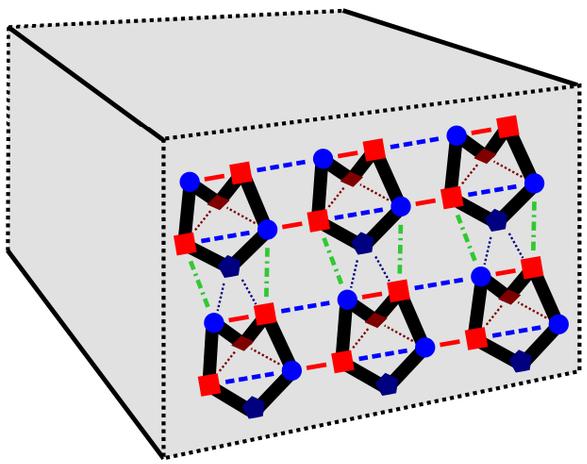

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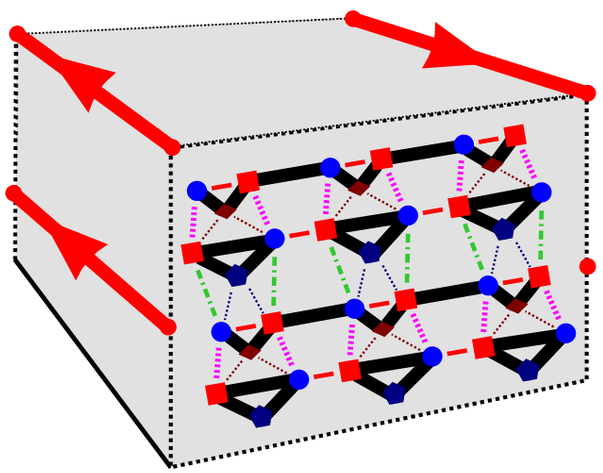

(F)QAH

Conclusions

- Non-interacting modes encompasses normal insulating phase, a quantum anomalous Hall, a Weyl semimetal and signle surface quantum anomalous Hall phase

- Phase transitions are controlled by ratio of tunnel couplings

- With e-e interactions there are analogous phases compose of closed loops and open planes of fractional quantum Hall states

- non-interacting Weyl semimetal probably evolves into an exotic critical phase

- all topological phase are related to weak topological insulators

THE END