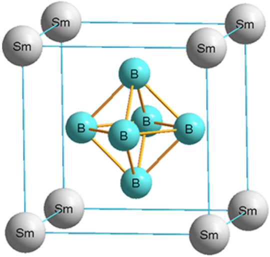
Majorana Fermi Sea in Insulating SmB_6

by Ganapathy Baskaran (arXiv:1507.03477)

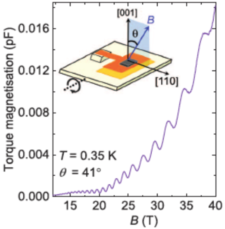


Constantin Schrade

University of Basel

Overview

Unconventional Fermi surface in an insulating state (Science **349**, 287 (2015))



- Quantum Oscillations in the Kondo insulator SmB_6
- The Fermi surface resembles that of metallic LaB_6
- Anomalous temperature-dependence of Quantum Oscillation amplitude

Majorana Fermi Sea in Insulating SmB_6 (arXiv:1507.03477)

$$c^\dagger_{i\uparrow} = \frac{1}{2}(c_{ix} + i c_{iy})$$
$$c^\dagger_{i\downarrow} = \frac{1}{2}(c_{iz} - i c_{i0})$$

- SmB_6 is a "scalar Majorana Fermi liquid"
- Coherent fluctuation of charge of a neutral scalar Majorana fermion can cause Quantum Oscillations

Outline

Review
- Kondo Insulators
- The De Haas-van Alphen effect

Experiments
- Quantum Oscillations in SmB_6

Theory
- Majorana representation of the Kondo lattice at zero H-field
- Majorana representation of the Kondo lattice at finite H-field

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Kondo Insulators

SmB_6, $Ce_3Bi_4Pt_3$, $CeFe_4P_{12}$, $CeRu_4Sn_3$,...

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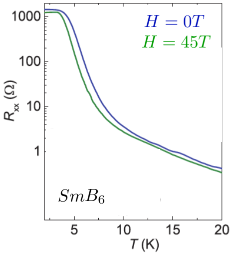
Experimental signature: Change from metallic to insulating behavior in the resistivity measurement with decreasing temperature.



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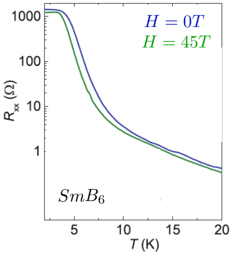
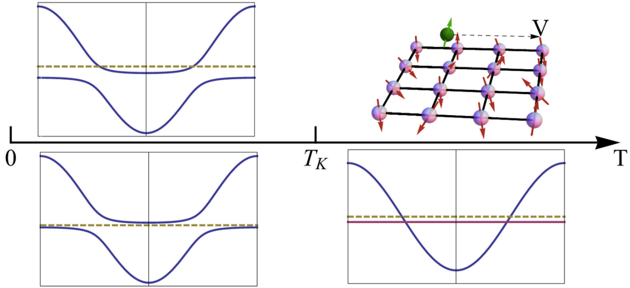


What is the physical origin of this strange behaviour?

Kondo Insulators

Theoretical description: Electrons moving on a lattice of spins.



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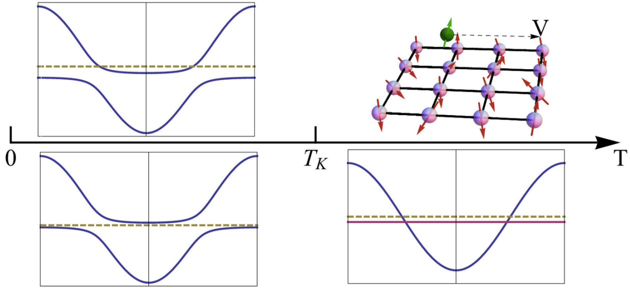
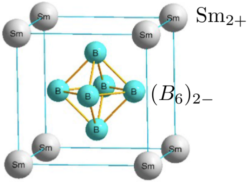
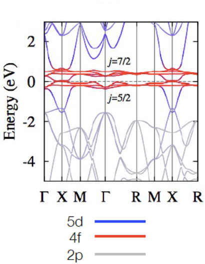


Intuitively the spins act as very strong scatterer for the conduction electrons. This causes the strong increase in the resistivity.

Kondo Insulators

Why is SmB_6 a Kondo Insulator?





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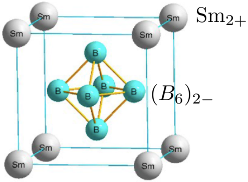

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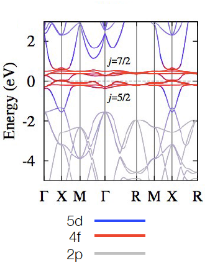

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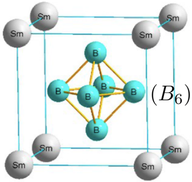

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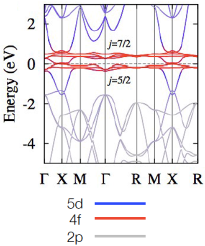

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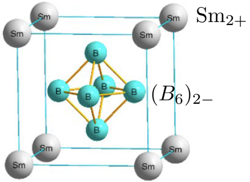

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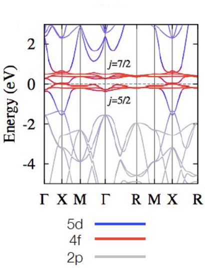

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Transitions between 1) and 2) generate the Kondo coupling

Kondo Insulators

What is the Hamiltonian for SmB_6?

Kondo Insulators

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$$H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}\sigma} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \epsilon_f \sum_{i\sigma} f^\dagger_{i\sigma} f_{i\sigma} + U \sum_i n^f_{i\uparrow} n^f_{i\downarrow} + \sum_{i\mathbf{k}\sigma} V_{\mathbf{k}} (c^\dagger_{\mathbf{k}\sigma} f_{i\sigma} + H.c.)$$

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S_i represents the localized spin of the f electron at i and s_i is the corresponding spin operator of the conduction electron.

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The De Haas-van Alphen effect

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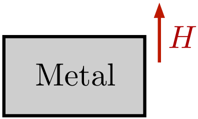
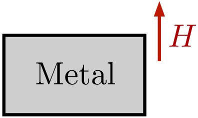


The magnetization M shows oscillations in $\frac{1}{H}$

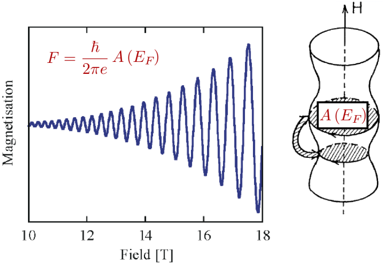
The De Haas-van Alphen effect



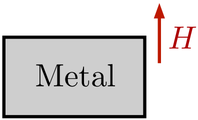
The magnetization M shows oscillations in $\frac{1}{H}$

From these "Quantum oscillations" we can extract two things:

1) The frequency F determines thes Fermi surface cross sectional area



The De Haas-van Alphen effect



The magnetization M shows oscillations in $\frac{1}{H}$

From these "Quantum oscillations" we can extract two things:

2) The quasiparticle effective mass m^* is extracted from the dependence of the oscillation amplitude R on temperature T

$$R(T) = \frac{\frac{2\pi^2 T}{\hbar\omega_c}}{\sinh\left(\frac{2\pi^2 T}{\hbar\omega_c}\right)} \quad \text{with } \omega_c = \frac{eH}{m^* c}$$

This result is attributed to Lifshitz and Kosevich.

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Quantum oscillations in SmB_6

Recent experiments suggest that quantum oscillations can not only be observed in a metal but also in an insulator!

HEAVY FERMIONS

Unconventional Fermi surface in an insulating state

B. S. Tan,[1] Y.-T. Hsu,[1] B. Zeng,[2] M. Ciomaga Hatnean,[3] N. Harrison,[4] Z. Zhu,[4] M. Hartstein,[1] M. Kiourlappou,[1] A. Srivastava,[1] M. D. Johannes,[5] T. P. Murphy,[2] J.-H. Park,[2] L. Balicas,[2] G. G. Lonzarich,[1] G. Balakrishnan,[3] Suchitra E. Sebastian[1*]

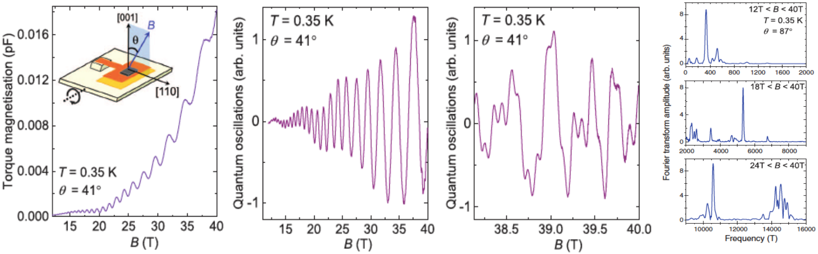
Insulators occur in more than one guise; a recent finding was a class of topological insulators, which host a conducting surface juxtaposed with an insulating bulk. Here, we report the observation of an unusual insulating state with an electrically insulating bulk that simultaneously yields bulk quantum oscillations with characteristics of an unconventional Fermi liquid. We present quantum oscillation measurements of magnetic torque in high-purity single crystals of the Kondo insulator SmB_6, which reveal quantum oscillation frequencies characteristic of a large three-dimensional conduction electron Fermi surface similar to the metallic rare earth hexaborides such as PrB_6 and LaB_6. The quantum oscillation amplitude strongly increases at low temperatures, appearing strikingly at variance with conventional metallic behavior.

Science **349**, 287 (2015)

Quantum oscillations in SmB_6

Experimental results:

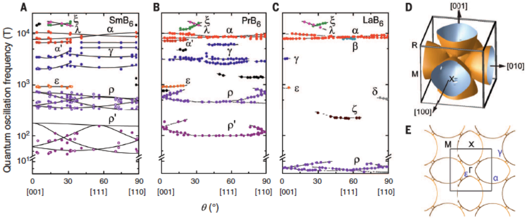
1. Quantum oscillations in measurements of the magnetic torque $\vec{\tau} = V\vec{M} \times \vec{B}$ with V the sample volume.



Quantum oscillations in SmB_6

Experimental results:

2. The Fermi surface resembles the conduction electron Fermi surface in metallic rare earth hexaborides (="SmB_6 without spin lattice")

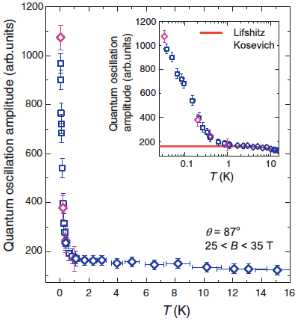


(A-C) High α frequncies reveal large prolate spheroids centered at X
(D-E) SmB_6 Fermi surface when the Fermi energy is shifted from the insulating gap into the conduction or valence band.

Quantum oscillations in SmB_6

Experimental results:

3. The temperature dependence of the quantum oscillations follows Lifshitz-Kosevich for $2K < T < 25K$ with small effective mass $m^* = 0.18 m_e$ but deviates dramatically for $T < 2K$!



Experimental puzzles

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1. Why do we observe quantum oscillations in a bulk insulator?
2. What is the physical origin of the Fermi surface that occupies half of the Brillouin zone?
3. Why does the temperature dependence of the quantum oscillation amplitudes not follow the Lifshitz-Kosevich theory?

We will adress the first two questions in this journal club.

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Majorana representation of the Kondo lattice

Conduction electrons hopping on a bipartite cubic lattice at half filling ($\mu = 0$) and interacting with spins via on-site Kondo coupling:

$$H = -t \sum_\sigma \sum_{<\mathrm{ij}>} (c^\dagger_{i\sigma} c_{j\sigma} + \mathrm{H.c.}) + J \sum_i \vec{s}_i \cdot \vec{S}_i$$

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Goal:

1. $J = 0$: The free spinful fermi sea can be rewritten as one spinless scalar "Majorana fermi sea" and three spinful "Majorana fermi sea".

2. $J \neq 0$: The scalar Majorana fermi sea is unaffected by interactions. The spectrum of vector Majorana fermions gets gapped out.

Majorana representation of the Kondo lattice

Non-interacting case (J=0): $H_0 = -t \sum_\sigma \sum_{<\mathrm{ij}>} c^\dagger_{i\sigma} c_{j\sigma} + \mathrm{H.c.}$

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Step 1: Transform operators on one of the two sublattices $c^\dagger_{i\sigma} \to i c^\dagger_{i\sigma}$.

$$H_0 \to -it \sum_\sigma \sum_{<\mathrm{ij}>} (c^\dagger_{i\sigma} c_{j\sigma} - H.c.)$$

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Step 2: Introduce scalar Majorana fermion c_{0i} and vector Majorana fermion $\vec{c}_i = (c_{ix}, c_{iy}, c_{iz})$ defined by $c_{i\uparrow}^\dagger = \frac{1}{2}(c_{ix} + i c_{iy})$ and $c_{i\downarrow}^\dagger = \frac{1}{2}(c_{iz} - i c_{i0})$.

$$H_0 = -it \sum_{<\mathrm{ij}>} c_{i0} c_{j0} + \vec{c}_i \cdot \vec{c}_j$$

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$$c_{i\alpha} = \frac{1}{\sqrt{N}} \sum_{1/2BZ} (a_{\vec{k}\alpha} e^{i\vec{k}\cdot\vec{R}_i} + a^\dagger_{\vec{k}\alpha} e^{-i\vec{k}\cdot\vec{R}_i})$$

with operators $a_{\vec{k}\alpha}, a^\dagger_{\vec{k}\alpha} \equiv a_{-\vec{k}\alpha}$ with $\{a_{\vec{k}\alpha}, a^\dagger_{\vec{k}'\alpha}\} = \delta_{\vec{k},\vec{k}'}$.

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with $\epsilon_{\vec{k}} = 2t \sum_{\vec{R}_i} \sin(\vec{R}_i \cdot \vec{k})$ and $E_0 = \sum_{\epsilon_{\vec{k}}<0} \epsilon_{\vec{k}}$

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Scalar and vector Majorana fermions form a "Majorana Fermi sea" characterized by

1. A zero energy Fermi surface in k-space.
2. Only particle-like positive energy excitations
3. Absence of hole-like excitations

Majorana representation of the Kondo lattice

Interacting case (J$\neq$ 0) : H=$H_0 + J\sum_i \vec{s}_i \cdot \vec{S}_i$

Majorana representation of the Kondo lattice

Interacting case ($J \neq 0$) : $H = H_0 + J \sum_i \vec{s}_i \cdot \vec{S}_i$

Step 1: Rewrite the Kondo term in terms of Majorana fermions
$\vec{S}_i = -i(\vec{\eta}_i \times \vec{\eta}_i)/2$.

$$H = H_0 + \frac{J}{2} \sum_i \left[c_{i0} \vec{c}_i \cdot (\vec{\eta}_i \times \vec{\eta}_i) - \frac{1}{2} (\vec{c}_i \cdot \vec{\eta}_i)^2 \right] + \text{const.}$$

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$\dim(\mathcal{H}_{Majorana}) = 2^{3N/2}$ with 3 Majoranas at each of the N sites
$\dim(\mathcal{H}_{Majorana})/\dim(\mathcal{H}_{phys}) = 2^{N/2}$

So there are $2^{N/2}$ extra gauge copies of the Hilbert space.

Majorana representation of the Kondo lattice

$$H = H_0 + \frac{J}{2}\sum_i \left[c_{i0}\vec{c}_i \cdot (\vec{\eta}_i \times \vec{\eta}_i) - \frac{1}{2}(\vec{c}_i \cdot \vec{\eta}_i)^2 \right] + \text{const.}$$

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Step 2: Mean field approximation

1. $\langle c_{i0}\vec{c}_i \rangle = 0$, $\langle \vec{S}_i \rangle = 0$ ("non-magnetic solutions")
2. $$(\vec{c}_i \cdot \vec{\eta}_i)^2 = (\vec{c}_i \cdot \vec{\eta}_i - \langle \vec{c}_i \cdot \vec{\eta}_i \rangle)^2 + 2(\vec{c}_i \cdot \vec{\eta}_i)\langle \vec{c}_i \cdot \vec{\eta}_i \rangle - \langle \vec{c}_i \cdot \vec{\eta}_i \rangle^2$$
   $$\approx 2(\vec{c}_i \cdot \vec{\eta}_i)\langle \vec{c}_i \cdot \vec{\eta}_i \rangle - \langle \vec{c}_i \cdot \vec{\eta}_i \rangle^2$$

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$$H = H_0 - J_0\chi_0 \sum_i \vec{c}_i \cdot \vec{\eta}_i + \text{const.} \quad , \quad \chi_0 = \langle \vec{c}_i \cdot \vec{\eta}_i\rangle$$

Notice that c_{i0} drops out of the interaction!

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$$H = \sum_{1/2BZ} \epsilon_{\vec{k}} a^{\dagger}_{\vec{k}0} a_{\vec{k}0} + \sum_{BZ} \varepsilon_{\vec{k}} \vec{A}^{\dagger}_{\vec{k}0} \vec{A}_{\vec{k}0} \quad , \quad \varepsilon_{\vec{k}} = \frac{\epsilon_{\vec{k}}}{2} \pm \sqrt{\left(\frac{\epsilon_{\vec{k}}}{2}\right) + (J\chi_0)^2}$$

Scalar Majorana Fermi sea: Unaffected by Kondo interactions!

Vector Majorana Fermi sea: Gapped with neutral fermionic excitations

Outline

Review

Kondo Insulators

The De Haas-van Alphen effect

Experiments

Quantum Oscillations in SmB_6

Theory

Majorana representation of the Kondo lattice at zero H-field

Majorana representation of the Kondo lattice at finite H-field

Majorana representation of the Kondo lattice

$$H = -t \sum_\sigma \sum_{<\mathrm{ij}>} (\exp\left(\frac{ie}{\hbar c} \int_i^j \vec{A} \cdot \mathrm{d}\vec{\ell}\right) c_{i\sigma}^\dagger c_{j\sigma} + \mathrm{H.c.}) + J \sum_i \vec{s}_i \cdot \vec{S}_i$$

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Result:

Majorana fermions can exhibit Quantum oscillations
- even though they are spinless and charge neutral.

Intuition:
Majorana fermions are superpositions of charged particles and holes. So while their average charge vanishes there can be quantum fluctuations that couple to magnetic fields!

Majorana representation of the Kondo lattice

J=0: $H_0 = -t \sum_\sigma \sum_{<\mathrm{ij}>} \exp\left(\frac{ie}{\hbar c} \int_i^j \vec{A} \cdot \mathrm{d}\vec{\ell}\right) c^\dagger_{i\sigma} c_{j\sigma} + \mathrm{H.c.}$

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Step 1: Formally solve the problem:

$$H_0 = \sum_\sigma \sum_\alpha \epsilon_\alpha c^\dagger_{\alpha\sigma} c_{\alpha\sigma} + E_0(\vec{H})$$

with $\vec{H} = \vec{\nabla} \times \vec{A}$ and $E_0(\vec{H}) = 2\sum_{\epsilon_\alpha<0} \epsilon_\alpha$.

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Step 3: Express Majoranas as positive energy complex fermions:

$$H_0 = \sum_{\epsilon_\alpha>0} \epsilon_\alpha (a^\dagger_{\alpha 0} a_{\alpha 0} + \vec{a}^\dagger_\alpha \vec{a}_\alpha) + 4\left(\frac{1}{2}\sum_{\epsilon_\alpha<0} \epsilon_\alpha\right)$$

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Results:

1) $E(\vec{H})$ changes with strength and direction of the magnetic field due to non-spherical Fermi surfaces.

$\rightarrow$ Quantum oscillations!

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Results:

1) $E(\vec{H})$ changes with strength and direction of the magnetic field due to non-spherical Fermi surfaces.

   $\rightarrow$ Quantum oscillations!

2) All Majoranas contribute equally by 1/4 to the Quantum oscillations in the non-interacting case!

   Even though they are charge neutral!

Majorana representation of the Kondo lattice

Interacting case (J$\neq$ 0) : H=$\mathrm{H}_0 + J\sum_i \vec{s}_i \cdot \vec{S}_i$

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Vacuum energy $E_v(\vec{H}, \chi)$ of vector Majoranas has weak H-field dependence. But the scalar Majoranas still contribute 1/4 of the free fermi gas value at the same H-field. Hence one can still observe quantum oscillations!