Helicons in Weyl semimetals

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Outline:

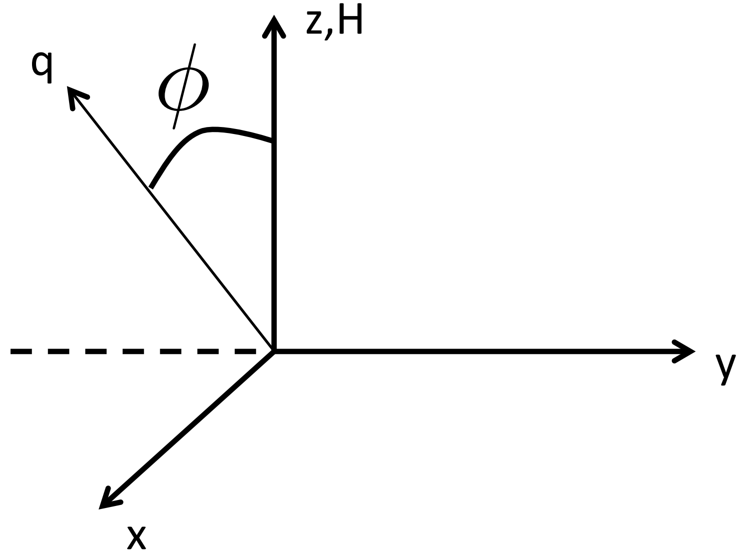
1. Helicons in metals. (Abrikosov, Fundamentals of the Theory of metals. Chapter 9.1) .

2. Maxwell equations in Weyl metals.

3. Evaluation of the conductivity tensor and dispersion of helicons in Weyl metals.

4. Conclusions

Metal in a static magnetic field. Hall conductivity σ_{xy}



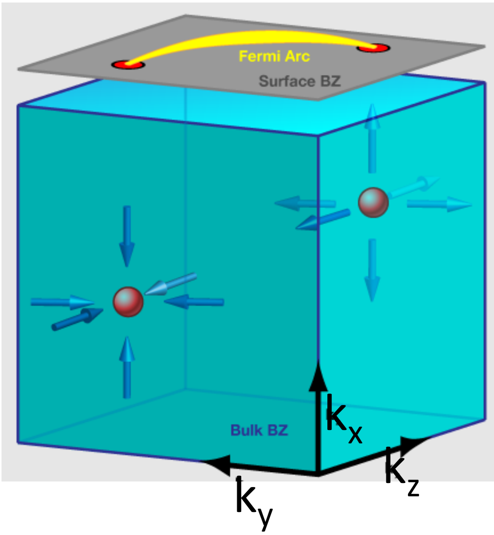
$\omega_c \gg \omega$

$$\omega = \frac{c^2 q^2 \cos\phi}{4\pi|\sigma_{xy}|} \propto q^2 H \cos\phi$$

$$E_x = i\cos\phi E_y \qquad E_z \ll E_x, E_y$$

EM wave of this type is called “helicon”

The conduction and valence bands touch at discrete points,
with a linear dispersion relation in all three momentum space directions
moving away from the Weyl node.



$$\pm v_D \boldsymbol{\sigma} \cdot \mathbf{k}$$

Hamiltonian of the Weyl semimetal with two nodes:

$$H = v_D \tau^z \boldsymbol{\sigma} \cdot (-i\boldsymbol{\nabla} + \tau^z \mathbf{b}) + \tau^z b_0$$

$\mathbf{b}, b_0$ can be eliminated by gauge transformation: $H = v_D \tau^z \boldsymbol{\sigma} \cdot (-i\boldsymbol{\nabla})$

Gauge transformation generates additional term in the Hamiltonian:

$$L_{em} = \frac{1}{8\pi}(\mathbf{E}^2 - \mathbf{B}^2) - \rho\phi + \mathbf{J} \cdot \mathbf{A} - \underline{\frac{\alpha}{4\pi^2}\theta(\mathbf{r}, t)\mathbf{E} \cdot \mathbf{B}}$$

$$\theta(\mathbf{r}, t) = 2(\mathbf{b} \cdot \mathbf{r} - b_0 t)$$

Theta-term modifies two Maxwell equations

$$\boldsymbol{\nabla} \cdot \mathbf{E} = 4\pi(\rho + \frac{\alpha}{2\pi^2}\mathbf{b} \cdot \mathbf{B})$$

$$-\frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \boldsymbol{\nabla} \times \boldsymbol{B} = \frac{4\pi}{c}\left(\boldsymbol{J} - \underline{\frac{\alpha}{2\pi^2}c\boldsymbol{b} \times \boldsymbol{E} + \frac{\alpha}{2\pi^2}b_0\boldsymbol{B}}\right)$$

Wave equation in a Weyl metal

$$-\frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} - \boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \mathbf{E}) = \frac{4\pi}{c^2}\frac{\partial \boldsymbol{J}}{\partial t} - $$

$$-\frac{2\alpha}{\pi c}\boldsymbol{b} \times \frac{\partial \boldsymbol{E}}{\partial t} - \frac{2\alpha}{\pi c}b_0 \boldsymbol{\nabla} \times \boldsymbol{E}$$

We need an expression that relates current to the electric field.

Wave vector of EM field: $q \ll R_c^{-1}$

$$R_c = v_d/\omega_c, \quad \omega_c = eB/m_c c, \quad m_c = \epsilon_F/v_D^2$$

Boltzaman equation and equations of motion: $\omega, \omega_c \ll \epsilon_F$

$$\frac{\partial f_g}{\partial t} + \dot{\mathbf{p}} \cdot \boldsymbol{\nabla}_p f_g + \dot{\mathbf{r}} \cdot \boldsymbol{\nabla}_r f_g = -\frac{f_g - f_{eq}}{\tau}$$

intra-node scattering time.

$$\dot{\mathbf{r}} = \mathbf{v}_g(\mathbf{p}) - \dot{\mathbf{p}} \times \boldsymbol{\Omega}_g(\mathbf{p})$$

$$\dot{\mathbf{p}} = -e\mathbf{E} - \dot{\mathbf{r}} \times \boldsymbol{B}e/c$$

$$\mathbf{v}_g(p) = \boldsymbol{\nabla}_p \epsilon_g(\mathbf{p}) \equiv \boldsymbol{\nabla}_p[v_D p(1 - \frac{\gamma e}{c}\boldsymbol{\Omega}_g \cdot \boldsymbol{B})]$$

$\boldsymbol{\Omega}_g = -g\boldsymbol{p}/2p^3$ is the Berry curvature

γ Dimensionless control parameter which they set to 1 at the end

Solution of the Boltzman equation:

1. Set $\boldsymbol{E}$=0 while keeping $\boldsymbol{B}$=$B\boldsymbol{z}$

$$f_g^{(0)}(\mathbf{p}) \equiv \frac{1}{\exp\left(\frac{\epsilon_g(\mathbf{p})-\epsilon_F}{T}\right)+1}$$

2. Solve Boltzman equation up to first order in the amplitude of a homogeneous time-dependent electric field:

$$\mathbf{E} = \tilde{\mathbf{E}}(\omega)e^{-i\omega t}$$

In the collision integral take $f_{eq} = f_g^{(0)}(\mathbf{p})$

and search for the solution in the form:

$$f_g(\mathbf{p},t) = f_g^{(0)}(\mathbf{p}) + \delta f_g(\mathbf{p},t)$$

Second term here is linear in electric field.

Current density carried by electrons at each Weyl node:

$$\mathbf{J}_g(t) = -e\int \frac{d^3\mathbf{p}}{(2\pi)^3}[1 + e\mathbf{\Omega}_g\cdot\mathbf{B}/c]\dot{\mathbf{r}}f_g(\mathbf{p},t)$$

Insert the distribution function

$$f_g^{(0)}(\mathbf{p}) \qquad \delta f_g(\mathbf{p},t)$$

$$\mathbf{J}_g(t) = \overbrace{\gamma\frac{e^3 v_D}{24\pi^2 c\epsilon_F}\mathbf{B}\times\tilde{\mathbf{E}}(\omega)e^{-i\omega t} + g\frac{e^2\epsilon_F}{4\pi^2 c}\mathbf{B}}^{f_g^{(0)}(\mathbf{p})} + \delta\mathbf{J}_g(t)$$

Correction to the Hall conductivity

Has no effect on the total current

They then evaluate the quantity $\delta\mathbf{J}_g(t)$ and obtain the optical conductivity $\omega \ll \omega_c$

$$\sigma_{zz}(\omega) = \frac{\sigma_{\mathrm{D}}}{1-i\omega\tau}\left(1+\frac{1}{5}\frac{\hbar^2\omega_{\mathrm{c}}^2}{\varepsilon_{\mathrm{F}}^2}\right) \qquad \sigma_{xx} = \sigma_{yy} \approx \sigma_{\mathrm{D}}\frac{1-i\omega\tau}{\omega_{\mathrm{c}}^2\tau^2}\left(1-\frac{1}{20}\frac{\hbar^2\omega_{\mathrm{c}}^2}{\varepsilon_{\mathrm{F}}^2}\right)$$

$$\sigma_{xy} = -\sigma_{yx} \approx \frac{\sigma_{\mathrm{D}}}{\omega_{\mathrm{c}}\tau}\left(1+\frac{3}{20}\frac{\hbar^2\omega_{\mathrm{c}}^2}{\varepsilon_{\mathrm{F}}^2}\right)$$

$\omega\tau \gg 1$ Dielectric tensor:

$$\varepsilon_{\ell m} = \delta_{\ell m}\epsilon_{\rm b} + \frac{4\pi i}{\omega}\left[\sigma_{\ell m} - \epsilon_{\ell m n}\frac{\alpha c}{2\pi^2}\left(b_n - q_n\frac{b_0}{\omega}\right)\right]$$

Zeroes of determinant

$$Det[c^2(q^2\delta_{\ell m} - q_\ell q_m) - \omega^2\epsilon_{\ell m}] = 0$$

corresponds to the modes of a Weyl metal

Gapless mode: q is parallel to the static magnetic field

$$\Omega_{\rm h}(q\to 0) = \frac{2\alpha b_0 c q/\pi + c^2q^2}{\omega_{\rm p}^2/\omega_{\rm c} + 2\alpha c b_z/\pi}$$

$$\omega_{\rm p}^2 = 4\pi n_{\rm e}e^2/m_{\rm c}$$

$$m_{\rm c} = \varepsilon_{\rm F}/v_{\rm D}^2$$

$$\Omega_{\rm h}(q) = \frac{\hbar q^2}{2m_{\rm h}} \ll \omega_{\rm c} \qquad m_{\rm h} = \hbar\omega_{\rm p}^2/(2\omega_{\rm c}c^2) \propto 1/B$$

Main conclusions:

Theory of helicons propagating through a 3D Weyl semimetal is presented.

The optical conductivity tensor is calculated from Boltzmann transport theory, with the inclusion of Berry curvature corrections.

It is demonstrated that the axion term characterizing the electromagnetic response of Weyl semimetals alters the helicon dispersion with respect to that in non-topological metals.

find that the gapped modes are given by: $\Omega_{\mathrm{p},1}(q=0) = \omega_-$, $\Omega_{\mathrm{p},2}(q=0) = \omega_{\mathrm{p}}/\sqrt{\epsilon_{\mathrm{b}}}$, and $\Omega_{\mathrm{p},3}(q=0) = \omega_+$, where $\omega_\pm = \sqrt{(\alpha c b)^2/(\pi\epsilon_{\mathrm{b}})^2 + \omega_{\mathrm{p}}^2/\epsilon_{\mathrm{b}}} \pm \alpha c b/(\pi\epsilon_{\mathrm{b}})$, with $b = |\boldsymbol{b}|$.

$$\omega_{\mathbf{p}}^{\mathbf{2}} = 4\pi n_{\mathbf{e}} e^{\mathbf{2}}/m_{\mathbf{c}}$$

$$m_{\mathbf{c}} = \varepsilon_{\mathbf{F}}/v_{\mathbf{D}}^{\mathbf{2}}$$

$$\delta f_g(\boldsymbol{p},t) = -\frac{\partial f_g^{(0)}}{\partial \varepsilon_g}\left(X_- e^{i\varphi} + X_+ e^{-i\varphi} + X_0\right) e^{-i\omega t}$$

$$X_\pm = e v_{\mathrm{D}} \delta \frac{1 - \gamma g \frac{e}{\hbar c}\frac{p_z}{p^3} B}{1 - g \frac{e}{2\hbar c}\frac{p_z}{p^3} B} \frac{\sqrt{p^2 - p_z^2}}{2p} \frac{\tilde{E}_x \pm i \tilde{E}_y}{i(\bar{\omega} \pm \omega_{\mathrm{c}}^\star)}$$

$$X_0 = e v_{\mathrm{D}} \left\{ (\gamma - 1) g \frac{e}{2\hbar c}\frac{B}{p^2} + \delta \frac{\left[1 - \gamma g \frac{e}{\hbar c}\frac{p_z}{p^3} B + (2\gamma - 1)\left(\frac{e}{2\hbar c}\right)^2 \frac{B^2}{p^4}\right]}{1 - g \frac{e}{2\hbar c}\frac{p_z}{p^3} B} \frac{p_z}{p} \right\} \frac{\tilde{E}_z}{i\bar{\omega}}$$

$$\bar{\omega} = \omega + i/\tau$$

$$\omega_{\mathrm{c}}^\star = \omega_{\mathrm{c}}^\star(p, p_z) \equiv \omega_{\mathrm{c}} \frac{1 - \gamma g \frac{e}{\hbar c}\frac{p_z}{p^3} B}{1 - g \frac{e}{2\hbar c}\frac{p_z}{p^3} B}$$