

arXiv:1508.02292

# Revising the musical equal temperament

Haye Hinrichsen

---

arXiv:1508.01223

# Reduced sensitivity to charge noise in semiconductor spin qubits via symmetric operation

M. D. Reed *et al.*

# RECALL

arXiv:1203.5101

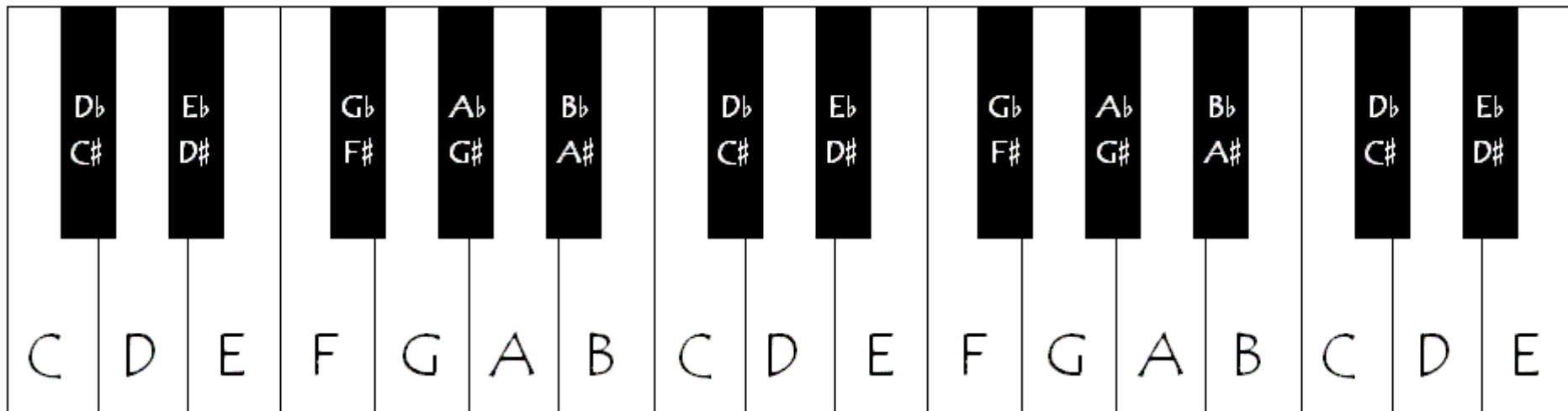
Rev. Bras. Ens. Fis. **34**, 2301 (2012)

## **Entropy-based Tuning of Musical Instruments**

Haye Hinrichsen

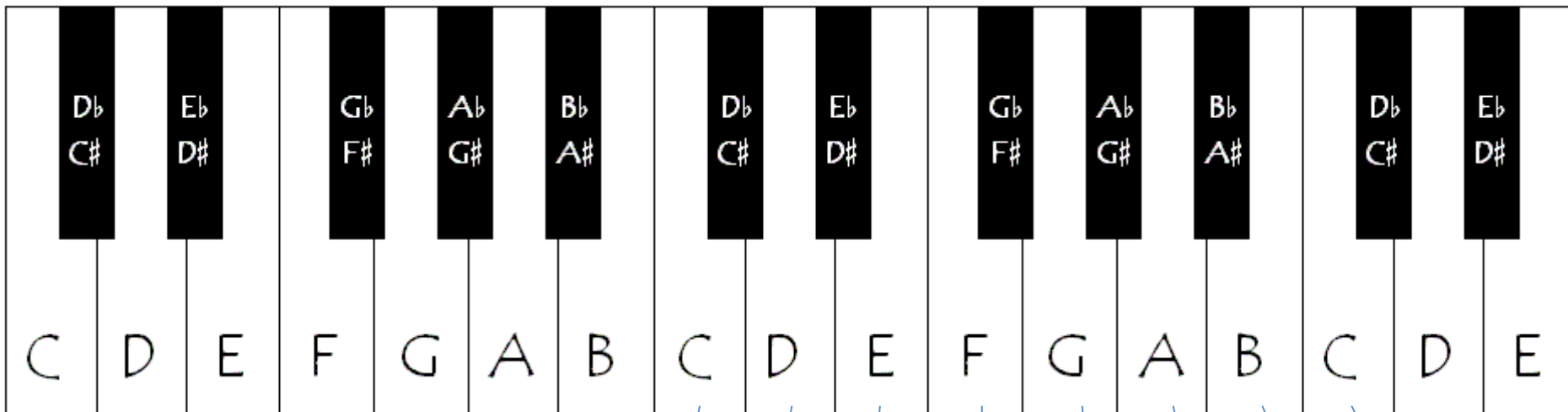
*Fakultät für Physik und Astronomie, Universität Würzburg, Germany*

# Tuning Systems



Crucial for the sound of  
chords and melodies:  
**Frequency ratios!**

# Tuning Systems



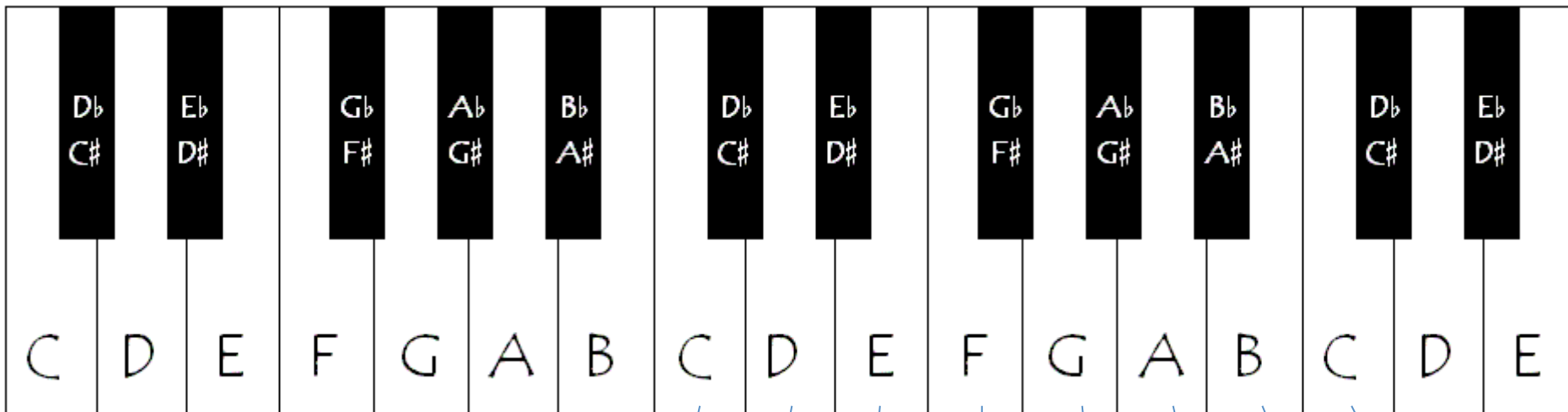
Crucial for the sound of chords and melodies:  
**Frequency ratios!**

**“Just Intonation”:**

Here: common example for C-Major scale

1     $\frac{9}{8}$      $\frac{5}{4}$      $\frac{4}{3}$      $\frac{3}{2}$      $\frac{5}{3}$      $\frac{15}{8}$     2

# Tuning Systems



Crucial for the sound of chords and melodies:  
**Frequency ratios!**

**“Just Intonation”:**

Here: common example for C-Major scale

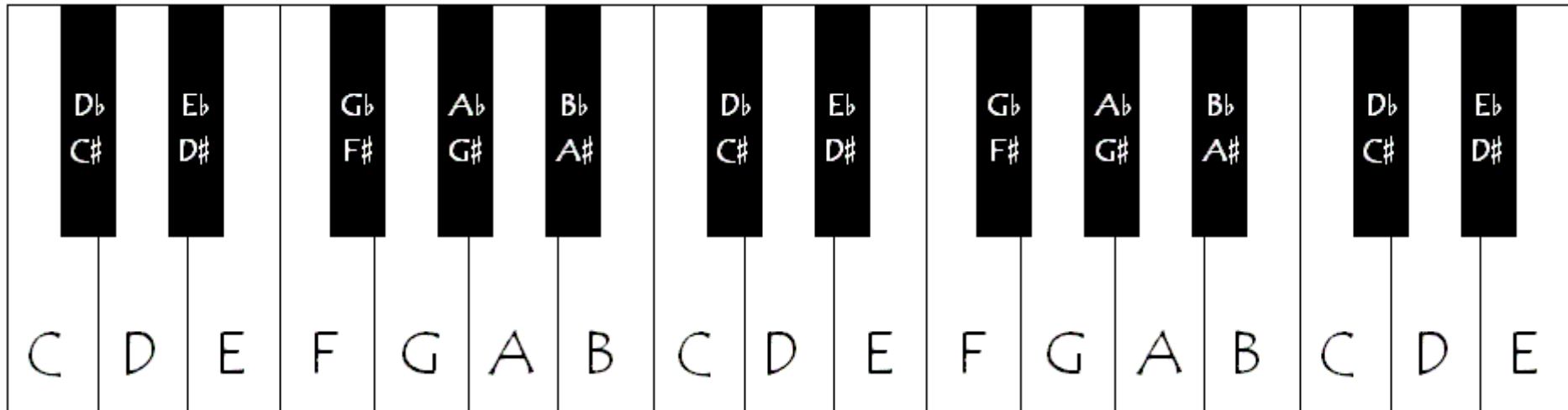
1    $\frac{9}{8}$     $\frac{5}{4}$     $\frac{4}{3}$     $\frac{3}{2}$     $\frac{5}{3}$     $\frac{15}{8}$    2

**“Equal Temperament”:**

Same ratio ( $2^{1/12}$ ) between adjacent notes

1    $2^{2/12}$     $2^{4/12}$     $2^{5/12}$     $2^{7/12}$     $2^{9/12}$     $2^{11/12}$    2

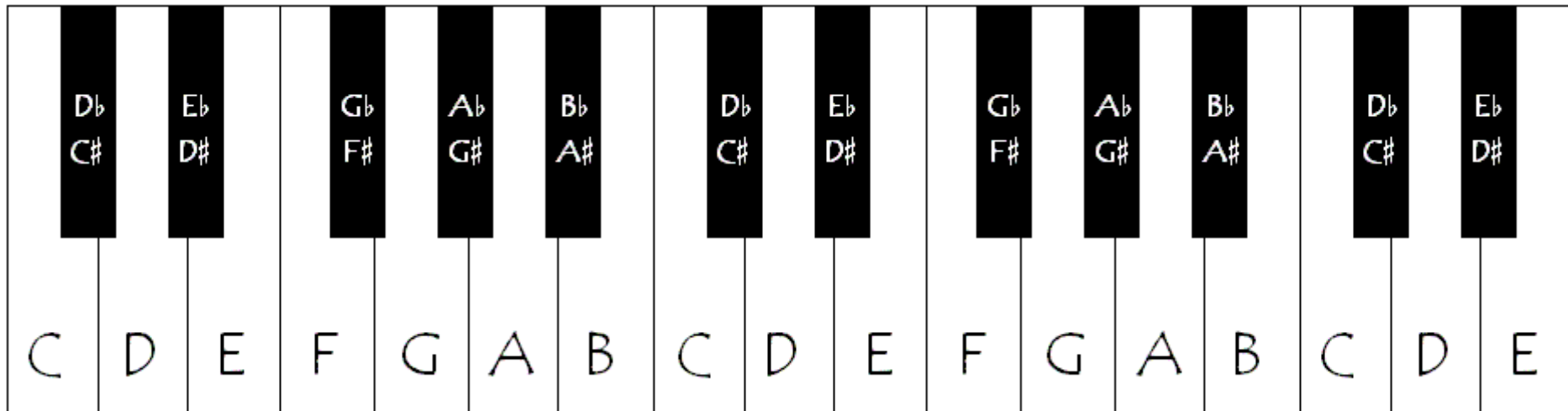
# Equal Temperament



Since ~ 19<sup>th</sup> century, Western music is based on **Equal Temperament**

Adjacent notes differ by factor  $2^{1/12}$  in frequency → Translational invariance

# Equal Temperament



Since ~ 19<sup>th</sup> century, Western music is based on **Equal Temperament**

Adjacent notes differ by factor  $2^{1/12}$  in frequency → Translational invariance



# Professional Piano Tuning: Aural



*Picture from  
Wikipedia, by  
Henry Heatly*

**Why can't we tune it ourselves?**

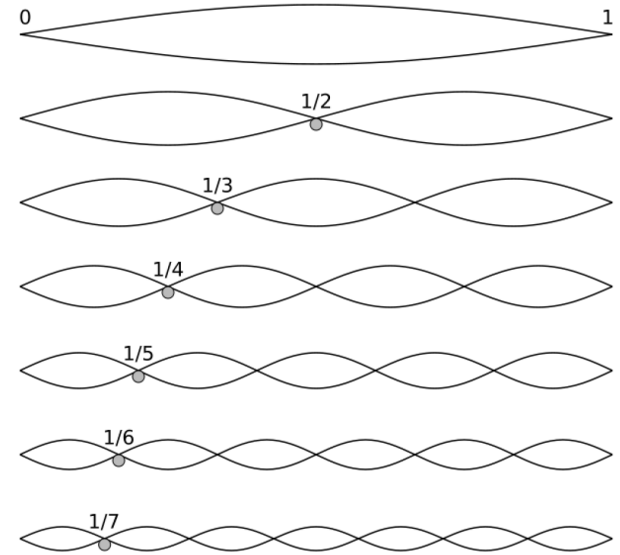


# Overtones & Stiffness of Strings

Besides its **fundamental mode** (frequency  $f_1$ ), a string features several **overtones** of frequencies  $f_n$

Ideal string:  $\ddot{y} \propto -y'' \quad f \propto |k|$

$\longrightarrow f_n = n f_1$



$n = 1, 2, \dots$

# Overtones & Stiffness of Strings

Besides its **fundamental mode** (frequency  $f_1$ ), a string features several **overtones** of frequencies  $f_n$

Ideal string:  $\ddot{y} \propto -y'' \quad f \propto |k|$

$$\longrightarrow f_n = n f_1$$

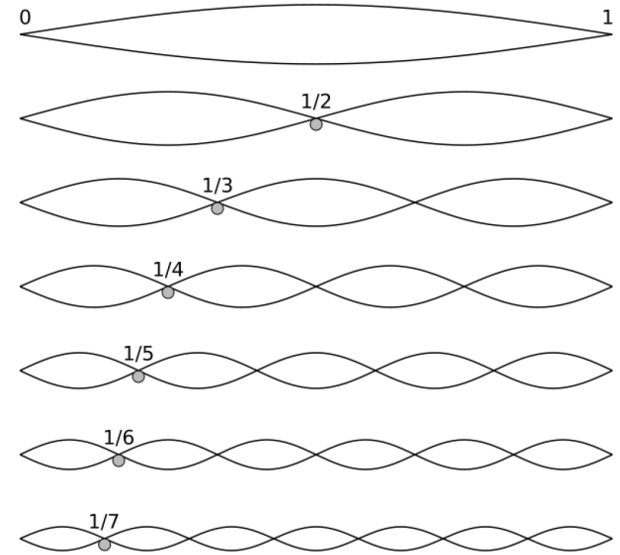
Stiff bar:  $\ddot{y} \propto -y'''' \quad f \propto k^2$

Realistic string:  $\ddot{y} \propto -y'' - \epsilon y'''' \quad f^2 \propto k^2 + \epsilon k^4$

$$\longrightarrow f_n \propto n f_1 \sqrt{1 + B n^2}$$

**B**: Inharmonicity coefficient

$n = 1, 2, \dots$



# Overtones & Stiffness of Strings

## Further complications:

- Inharmonicity coefficient is different for each string (depends on length, diameter, tension, material properties, ...)
- For each string, the amplitudes of the overtones are different (depending on position of hammer, ...)

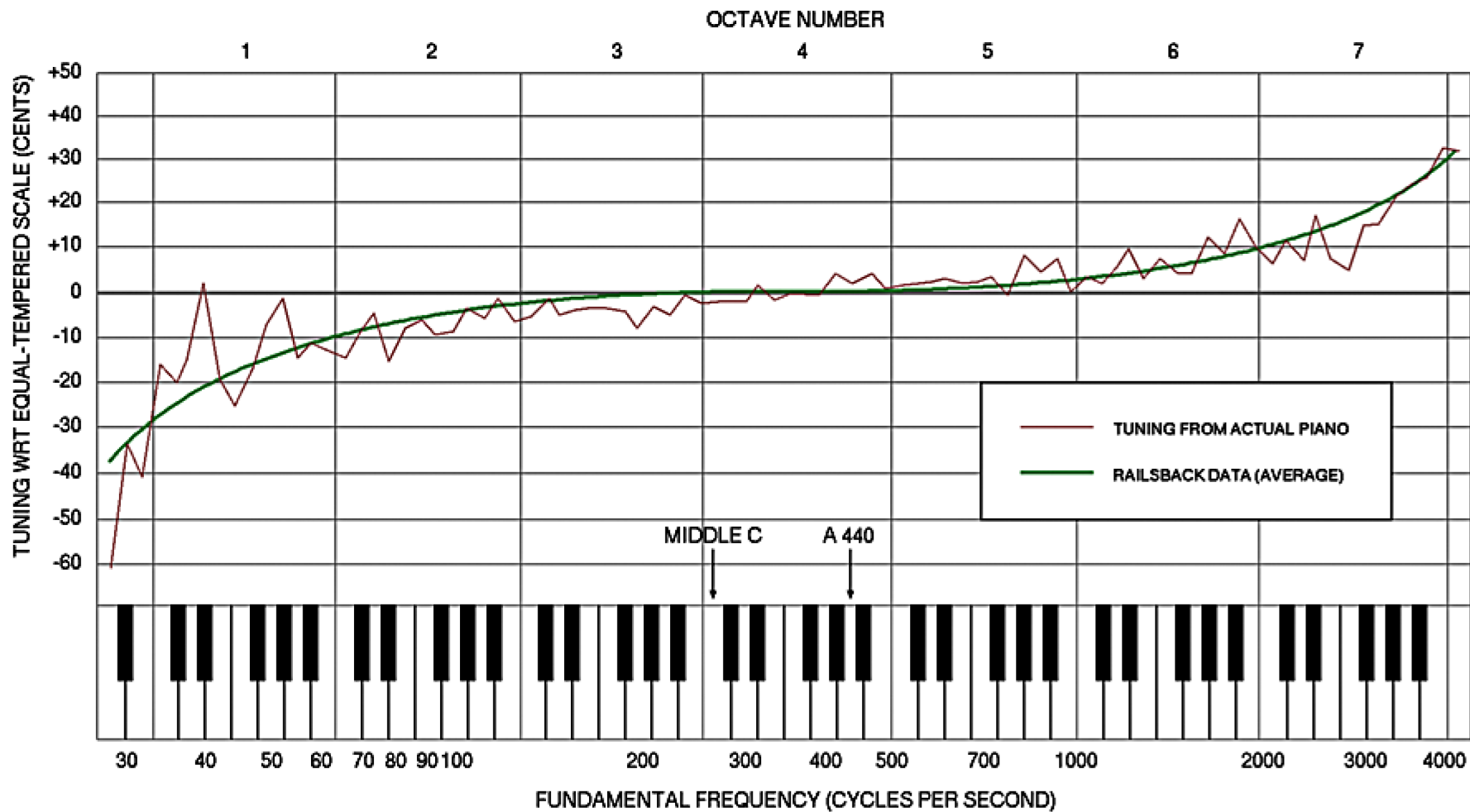
Realistic string:  $\ddot{y} \propto -y'' - \epsilon y''''$        $f^2 \propto k^2 + \epsilon k^4$

$$\longrightarrow f_n \propto n f_1 \sqrt{1 + B n^2}$$

**B:** *Inharmonicity coefficient*

$$n = 1, 2, \dots$$

# Tuning Curve of High-Quality Aural Tuning



Green: Average

Red: Individual Piano

# Tuning via Entropy

---

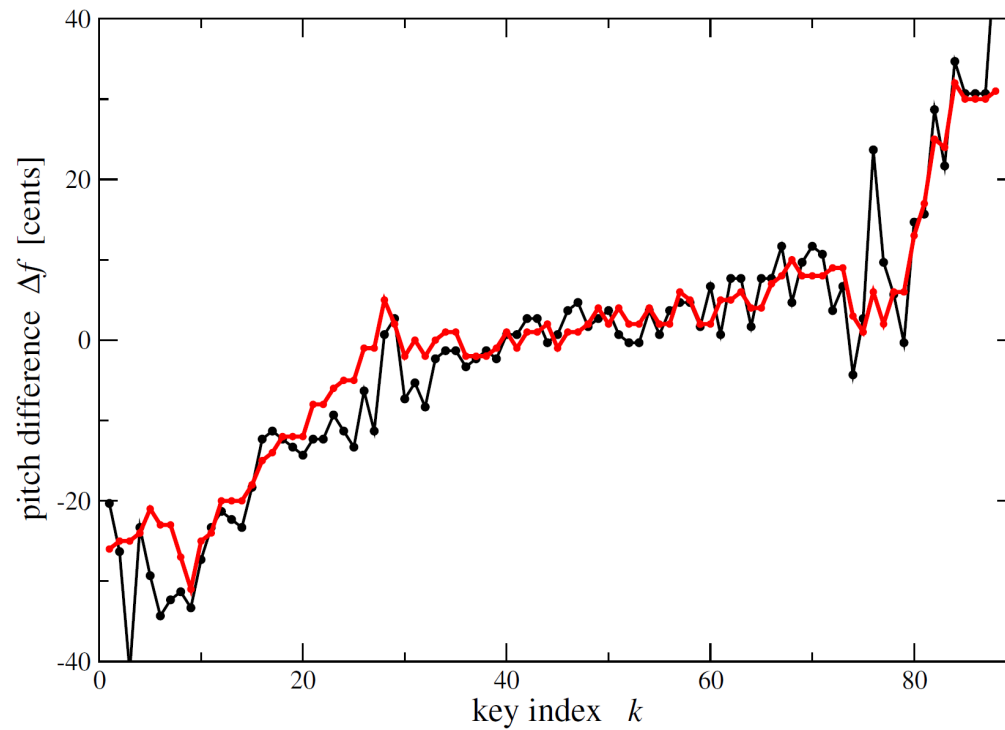
## Idea of the paper:

Human brain perceives sounds as “pleasant” (“in tune”) when there is some kind of order

Entropy is a measure of disorder

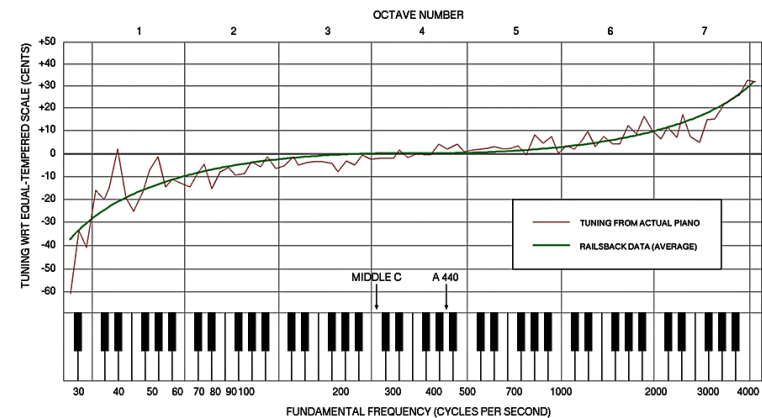
→ **Find tuning curve via entropy minimization**

# Results

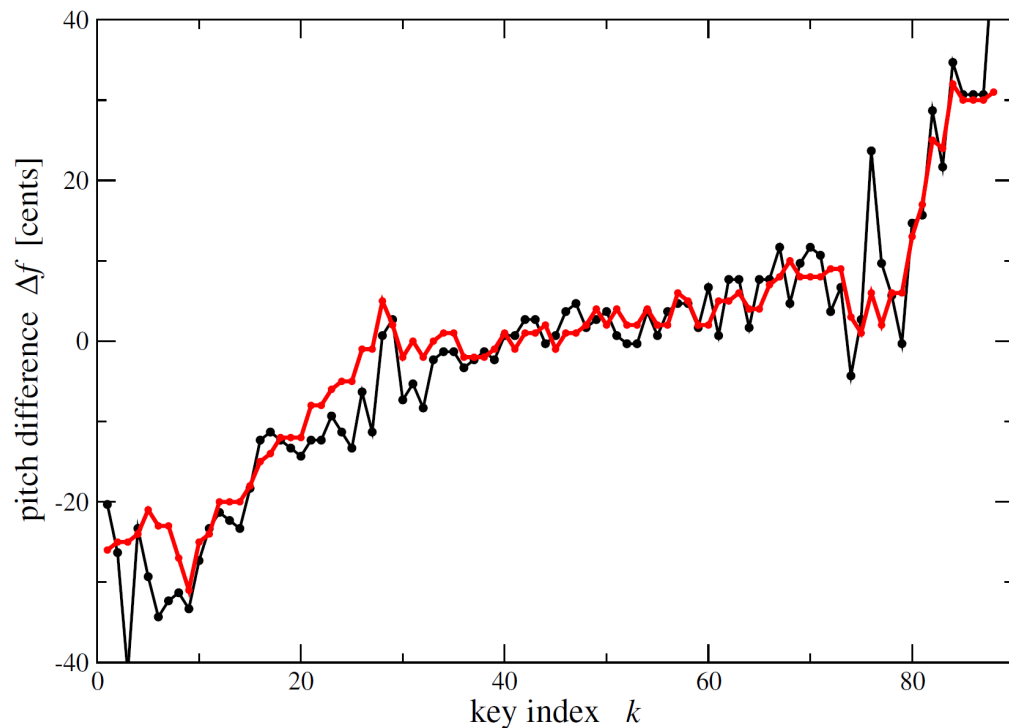


Red: Theoretical result

Black: Aural tuning



# Results

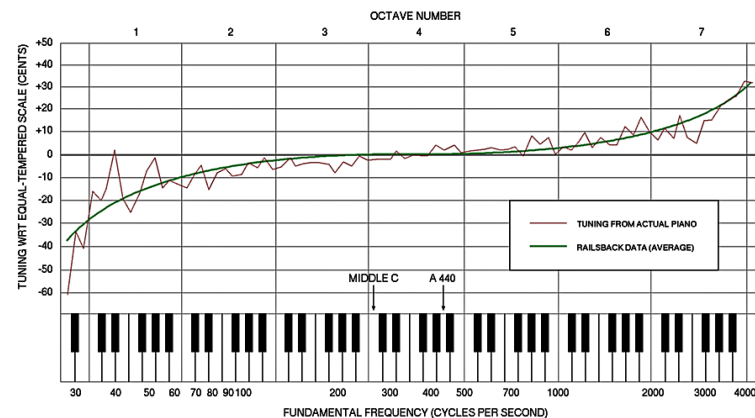


Red: Theoretical result

Black: Aural tuning

**Method reproduces the stretch curve**

**Fluctuations are correlated (!), especially in the treble and the bass**



# Conclusions

---

**Author:** Several open questions and remaining tasks

- Method tested on only one piano so far
- Apparently there are many local minima, and the present algorithm gives similar but not reproducible results
- ... (see article)

The fluctuations on top of the smooth stretch curve are not random, but to some extent essential for the good results as achieved by professional, aural tuning

Whether or not the presented idea based on entropy minimization can be used to improve existing electronic tuning methods remains to be seen



# Conclusions

---

**Author:** Several open questions and remaining tasks

- Method tested on only one piano so far
- Apparently there are many local minima, and the present algorithm gives similar but not reproducible results
- ... (see article)

The fluctuations on top of the smooth stretch curve are not random, but to some extent essential for the good results as achieved by professional, aural tuning

Whether or not the presented idea based on entropy minimization can be used to improve existing electronic tuning methods remains to be seen

**MIT Technology Review:** *“Algorithm Spells the End for Professional Musical Instrument Tuners”*

**Wall Street Journal:** *“Are the Days of Human Piano-Tuners Numbered?”*

# What Happened Since 2012?

---

**Article reprinted in “Europiano” (2012/4)**

**Open-source software:**

**<http://piano-tuner.org>**

(Current version: 1.1.2)

arXiv:1508.02292

# Revising the Musical Equal Temperament

Haye Hinrichsen

*Fakultät für Physik und Astronomie, Universität Würzburg, Germany*

# Important Difference to Study from 2012

---

## Assumption in this paper

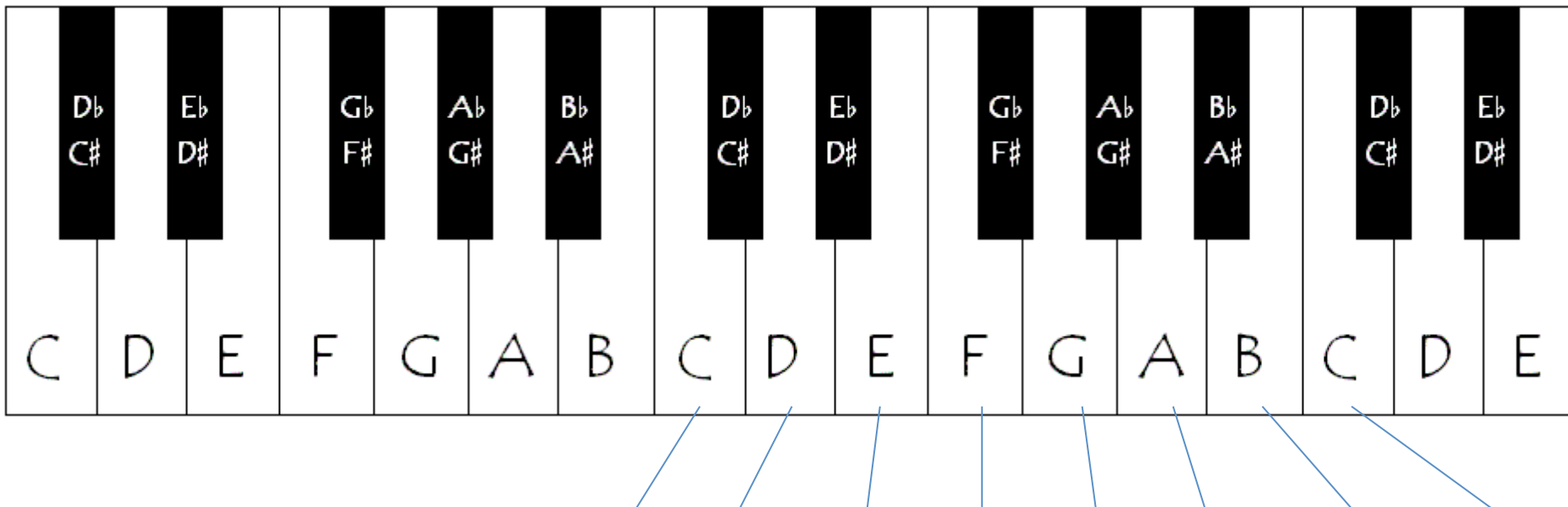
For each note, the frequencies  $f_n$  of overtones are integer multiples of the fundamental frequency  $f_1$

$$\longrightarrow f_n = n f_1$$

(Ideal strings are assumed)

**$\longrightarrow$  A discussion of the equal temperament itself**

# Equal Temperament



**“Equal Temperament”:**

1    $2^{2/12}$     $2^{4/12}$     $2^{5/12}$     $2^{7/12}$     $2^{9/12}$     $2^{11/12}$    2

Same ratio ( $2^{1/12}$ ) between adjacent notes

Since ~ 19<sup>th</sup> century, Western music is based on **Equal Temperament**

Adjacent notes differ by factor  $2^{1/12}$  in frequency



Translational invariance  
(Invariance under key changes)

# Stretched Equal Temperaments

---

Some musicians have repeatedly expressed their discomfort with the harmonicity of certain intervals. Are improvements possible?

# Stretched Equal Temperaments

Some musicians have repeatedly expressed their discomfort with the harmonicity of certain intervals. Are improvements possible?

Proposals: Stretched equal temperaments

$$f^{(k)} = f_{\text{ref}} 2^{\frac{(1+\epsilon/100)}{12} (k-k_{\text{ref}})}$$

Stretch parameter

Reference tone  
(usually A4, 440 Hz)

The diagram shows the formula for stretched equal temperament. The stretch parameter  $\epsilon$  is highlighted in blue, with an arrow pointing to it from the label 'Stretch parameter'. The reference frequency  $f_{\text{ref}}$  and the reference index  $k_{\text{ref}}$  are also highlighted in blue, with an arrow pointing to them from the label 'Reference tone (usually A4, 440 Hz)'.

# Stretched Equal Temperaments

Some musicians have repeatedly expressed their discomfort with the harmonicity of certain intervals. Are improvements possible?

Proposals: Stretched equal temperaments

$$f^{(k)} = f_{\text{ref}} 2^{\frac{(1+\epsilon/100)}{12} (k-k_{\text{ref}})}$$

Stretch parameter

Reference tone  
(usually A4, 440 Hz)

**Stopper Tuning:**

$$\epsilon = 0.103$$

The duodecime is pure, not the octave

B. Stopper, 1988

**Cordier Tuning:**

$$\epsilon = 0.279$$

The fifth is pure, not the octave

S. Cordier, 1995

**Circular Harmonic System:**

$$\epsilon = 0.038$$

No interval is pure

A. Capurso, 2009



# Entropy Minimization

---

**Idea of the paper, similar to the author's work from 2012:**

Human brain perceives sounds as “pleasant” (“in tune”) when there is some kind of order

Entropy is a measure of disorder

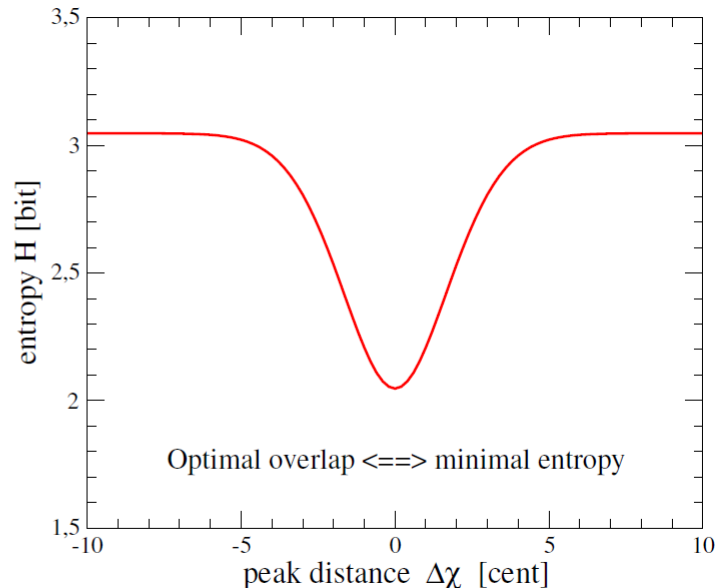
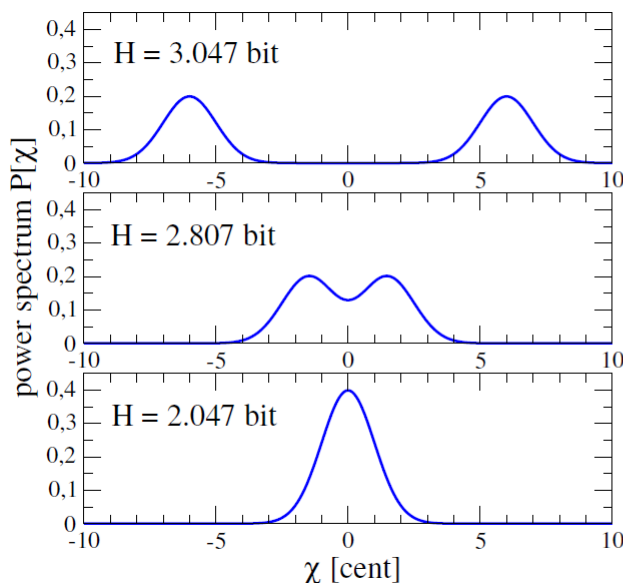
**→ Find stretch parameter via entropy minimization**

# Entropy Minimization

Example: Two Gaussian peaks of width  $\sigma$ , separated by a distance  $\Delta\chi$

$$p(\chi) = \frac{1}{2} \left[ p_\sigma(\chi + \Delta\chi/2) + p_\sigma(\chi - \Delta\chi/2) \right]$$
$$= \frac{1}{2\sigma\sqrt{2\pi}} \left( e^{-\frac{(\chi + \Delta\chi/2)^2}{2\sigma^2}} + e^{-\frac{(\chi - \Delta\chi/2)^2}{2\sigma^2}} \right)$$

$$\text{Entropy: } H = - \int d\chi p(\chi) \log_2 p(\chi)$$



# Model

---

- Probability density for the entropy is a normalized power spectrum

# Model

- Probability density for the entropy is a normalized power spectrum
- All (over)tones are Gaussians with a width  $\sigma$

Typical value for  $\sigma$ :

2 to 16 cents

*(Deviations of Equal Temperament from pure tuning, may be considered as a reasonable tolerance for human hearing)*

“Cent”:  
1/100 of a step in  
the chromatic scale

# Model

- Probability density for the entropy is a normalized power spectrum
- All (over)tones are Gaussians with a width  $\sigma$

Typical value for  $\sigma$ :

2 to 16 cents

*(Deviations of Equal Temperament from pure tuning, may be considered as a reasonable tolerance for human hearing)*

“Cent”:  
1/100 of a step in  
the chromatic scale

- The power of overtones decays exponentially and  $\lambda$  is a decay parameter

$$P_n^{(k)} := P_1^{(k)} e^{-(n-1)/\lambda} \propto e^{-n/\lambda}$$

# Model

- Probability density for the entropy is a normalized power spectrum
- All (over)tones are Gaussians with a width  $\sigma$

Typical value for  $\sigma$ :

2 to 16 cents

*(Deviations of Equal Temperament from pure tuning, may be considered as a reasonable tolerance for human hearing)*

“Cent”:  
1/100 of a step in  
the chromatic scale

- The power of overtones decays exponentially and  $\lambda$  is a decay parameter

$$P_n^{(k)} := P_1^{(k)} e^{-(n-1)/\lambda} \propto e^{-n/\lambda}$$

- The number of included notes is  $K$ , each note is weighted equally

Standard piano:

$K = 88$ , reference tone A4 (440 Hz)

# Model

- Probability density for the entropy is a normalized power spectrum
- All (over)tones are Gaussians with a width  $\sigma$

Typical value for  $\sigma$ :

2 to 16 cents

*(Deviations of Equal Temperament from pure tuning, may be considered as a reasonable tolerance for human hearing)*

“Cent”:  
1/100 of a step in  
the chromatic scale

- The power of overtones decays exponentially and  $\lambda$  is a decay parameter

$$P_n^{(k)} := P_1^{(k)} e^{-(n-1)/\lambda} \propto e^{-n/\lambda}$$

- The number of included notes is  $K$ , each note is weighted equally

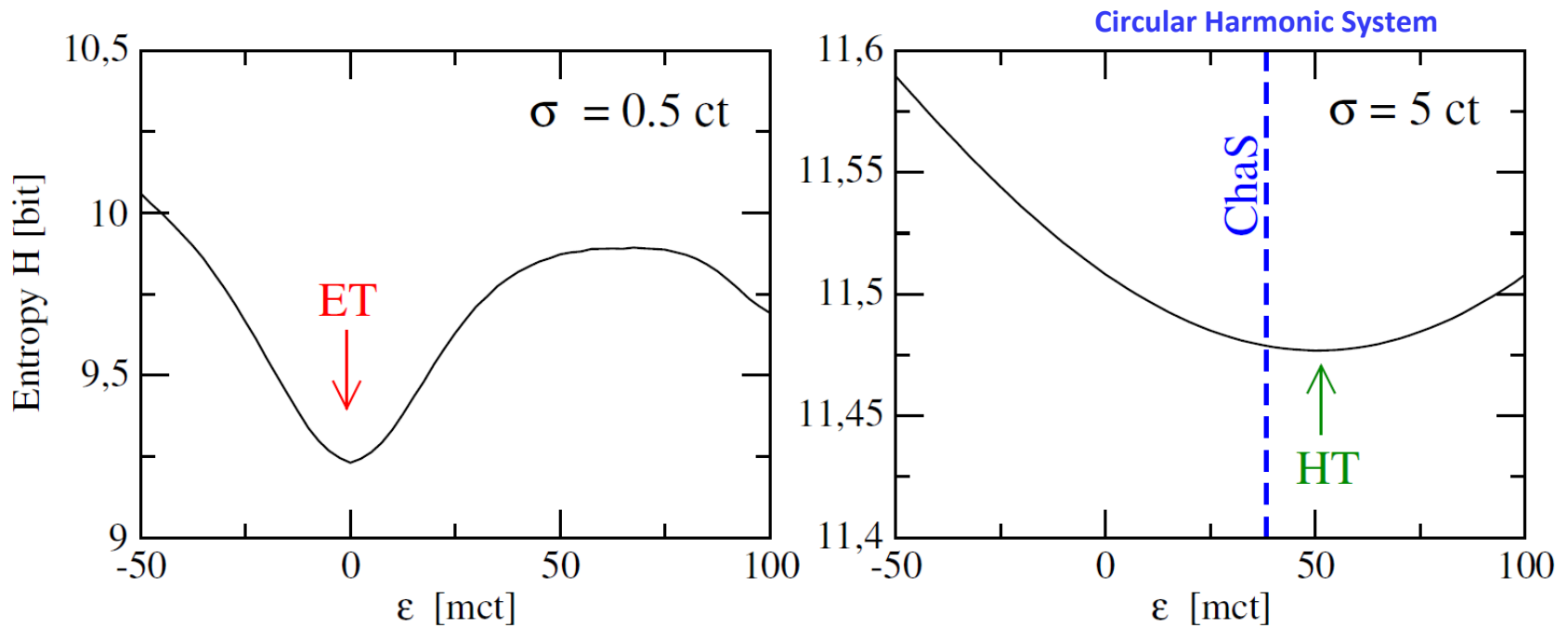
Standard piano:

$K = 88$ , reference tone A4 (440 Hz)

**The integral for the entropy is calculated with C++, with a step size of 0.001 cents**

# Results

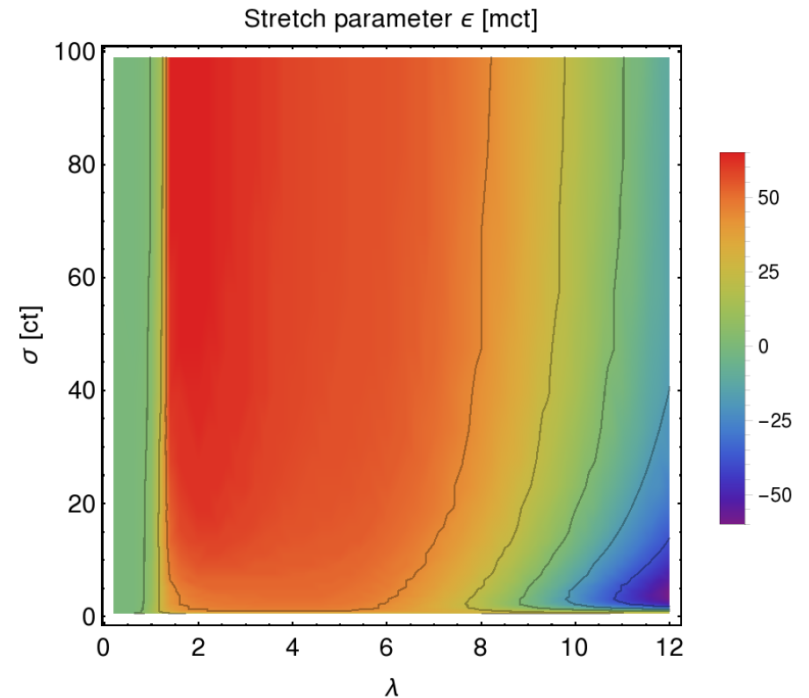
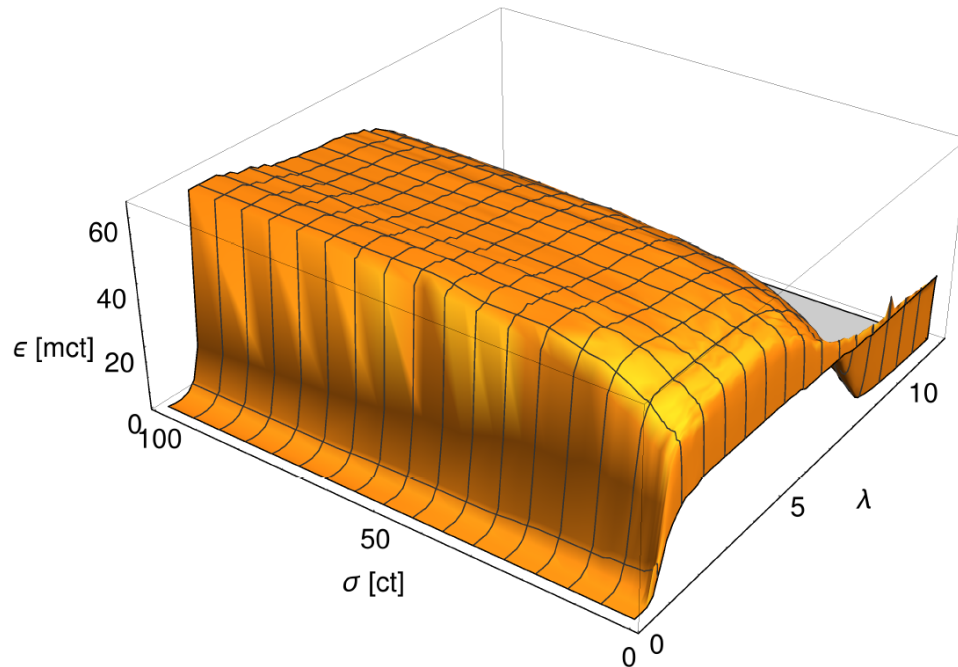
Example for  $K = 88$  and  $\lambda = 10$





# Results

Example for  $K = 88$



For a wide range of parameters, the model yields  $0.035 < \epsilon < 0.065$ , with a plateau near  $\epsilon = 0.052$

# Conclusions

---

The model based on entropy minimization suggests that the equal temperament should be replaced by a stretched equal temperament

$$f^{(k)} = f_{\text{ref}} 2^{\frac{(1+\epsilon/100)}{12}(k-k_{\text{ref}})}$$

with  $\epsilon$  around 0.05.

The calculated value for  $\epsilon$  is similar to that of the Circular Harmonic System ( $\epsilon = 0.038$ ), but smaller than those of the Stopper ( $\epsilon = 0.103$ ) and Cordier ( $\epsilon = 0.279$ ) tunings.

# Conclusions

The model based on entropy minimization suggests that the equal temperament should be replaced by a stretched equal temperament

$$f^{(k)} = f_{\text{ref}} 2^{\frac{(1+\epsilon/100)}{12}(k-k_{\text{ref}})}$$

with  $\epsilon$  around 0.05.

The calculated value for  $\epsilon$  is similar to that of the Circular Harmonic System ( $\epsilon = 0.038$ ), but smaller than those of the Stopper ( $\epsilon = 0.103$ ) and Cordier ( $\epsilon = 0.279$ ) tunings.

The proposed corrections to the temperament itself are rather small compared with those of typical tuning curves for pianos (resulting from the stiffness of the strings), around 5–10% for the lowest/highest keys.

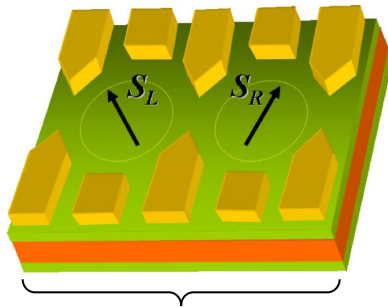
arXiv:1508.01223

# Reduced sensitivity to charge noise in semiconductor spin qubits via symmetric operation

M. D. Reed, B. M. Maune, R. W. Andrews, M. G. Borselli, K. Eng,  
M. P. Jura, A. A. Kiselev, T. D. Ladd, S. T. Merkel, I. Milosavljevic,  
E. J. Pritchett, M. T. Rakher, R. S. Ross, A. E. Schmitz, A. Smith,  
J. A. Wright, M. F. Gyure, and A. T. Hunter

*HRL Laboratories, LLC, 3011 Malibu Canyon Road, Malibu, CA 90265, USA*

# Spin Qubits in Quantum Dots

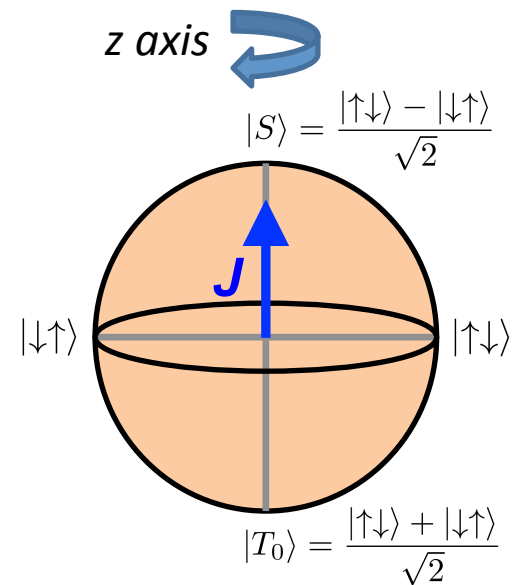


two single-spin qubits  
or  
one singlet-triplet qubit

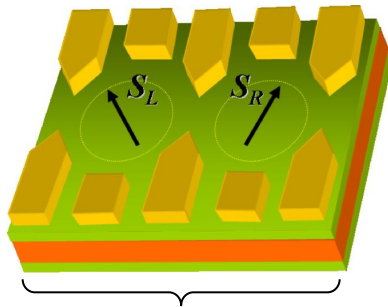
Loss/DiVincenzo, Phys. Rev. A **57**, 120 (1998)

Common approach:  
Singlet-triplet qubits  
in double quantum dots

Levy, PRL (2002)  
Petta *et al.*, Science (2005)  
Shulman *et al.*, Science (2012)  
Klinovaja *et al.*, PRB (2012)



# Spin Qubits in Quantum Dots



two single-spin qubits  
or  
one singlet-triplet qubit

Loss/DiVincenzo, Phys. Rev. A **57**, 120 (1998)

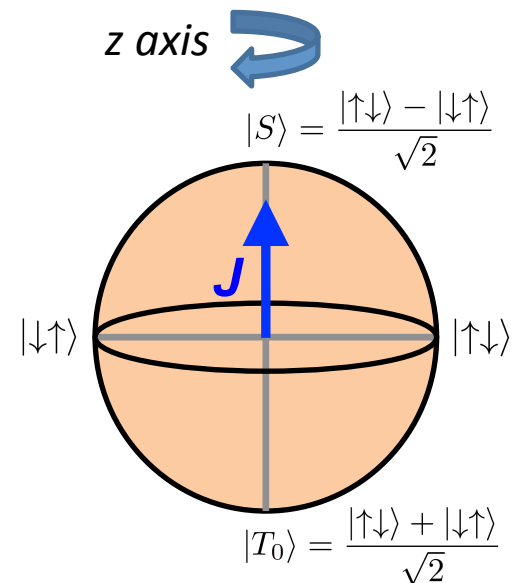
Common approach:  
Singlet-triplet qubits  
in double quantum dots

Levy, PRL (2002)  
Petta *et al.*, Science (2005)  
Shulman *et al.*, Science (2012)  
Klinovaja *et al.*, PRB (2012)

## Exchange splitting $J$

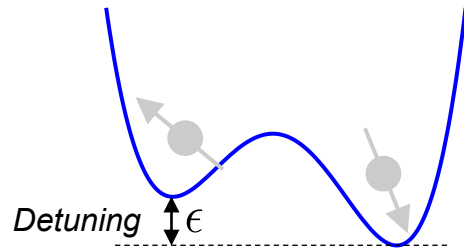
Two-qubit gate for  
single-spin qubits

Single-qubit gate for  
singlet-triplet qubits

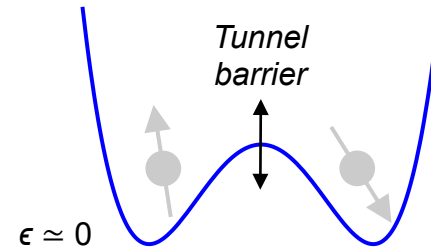


# Operation Schemes

## Control via detuning

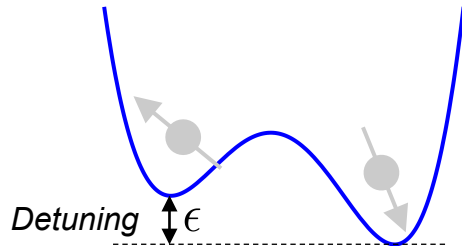


## Control via tunnel barrier



# Operation Schemes

## Control via detuning



Approach chosen in almost all experiments

Petta *et al.*, Science (2005)

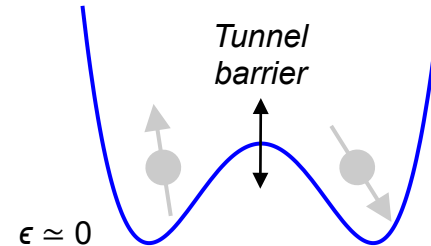
Shulman *et al.*, Science (2012)

Dial *et al.*, PRL (2013)

...

Measured decoherence times for exchange-based gates are rather short

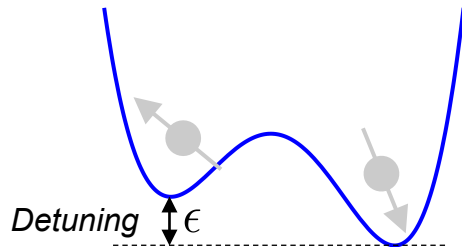
## Control via tunnel barrier





# Operation Schemes

## Control via detuning



Approach chosen in almost all experiments

Petta *et al.*, Science (2005)

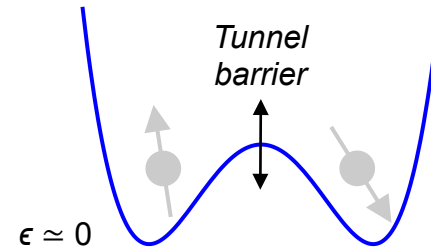
Shulman *et al.*, Science (2012)

Dial *et al.*, PRL (2013)

...

Measured decoherence times for exchange-based gates are rather short

## Control via tunnel barrier



Suggested approach

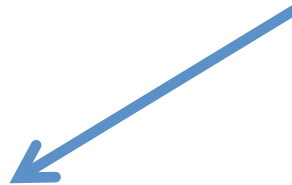
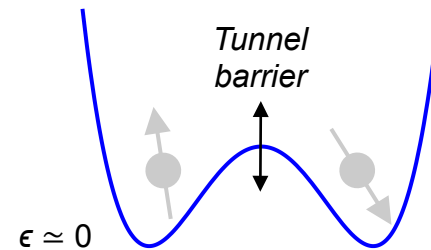
Loss/DiVincenzo, PRA (1998)

- Qubit protected against charge noise as  $dJ/d\epsilon \approx 0 \approx \langle dJ/d\epsilon \rangle$   
Burkard/Loss/DiVincenzo, PRB (1999)
- Dephasing via phonons suppressed  
Kornich/Kloeffel/Loss, PRB (2014)

Long decoherence times expected

# Operation Schemes

## Control via tunnel barrier



## Recent experiments:

Marcus Group, GaAs

Improvement of decoherence time by orders of magnitude!

Reed et al., Si

*This Paper*

## Suggested approach

Loss/DiVincenzo, PRA (1998)

- Qubit protected against charge noise as  $dJ/d\epsilon \approx 0 \approx \langle dJ/d\epsilon \rangle$   
Burkard/Loss/DiVincenzo, PRB (1999)
- Dephasing via phonons suppressed  
Kornich/Kloeffel/Loss, PRB (2014)

Long decoherence times expected

# Setup

The authors study several samples, all of which are similar (but not exactly identical)

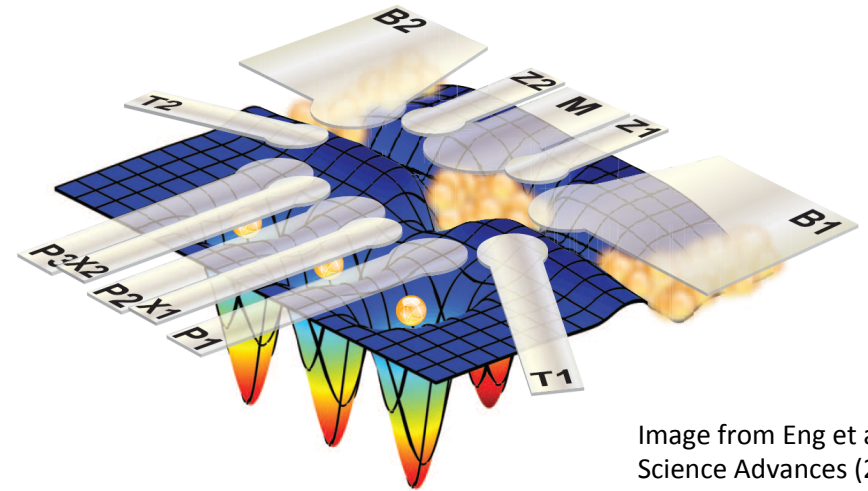
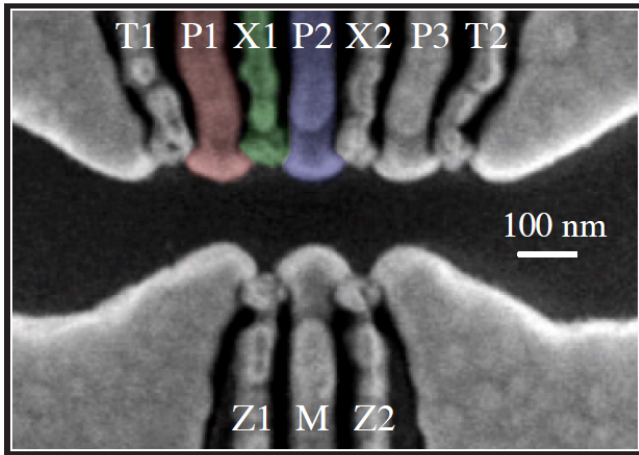
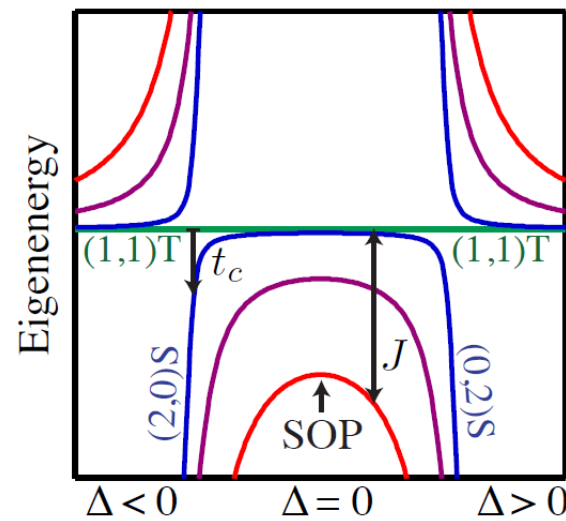
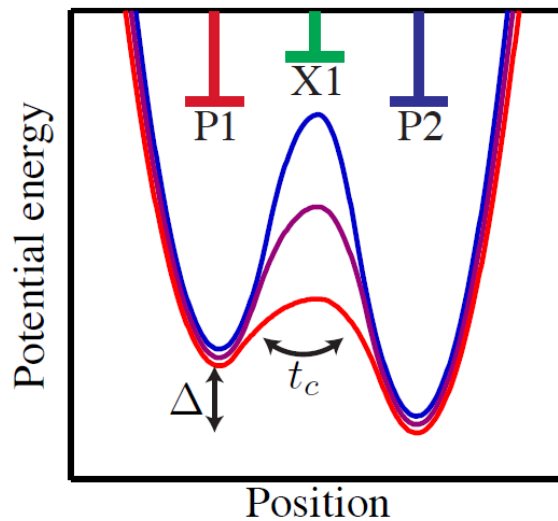


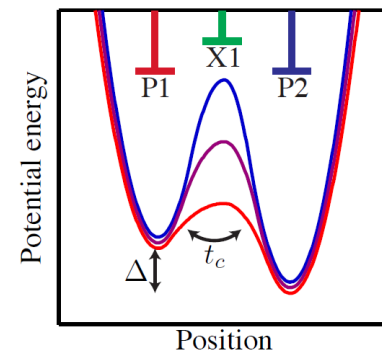
Image from Eng et al.,  
Science Advances (2015)

Heterostructure: Si/SiGe



# Basic Experiment

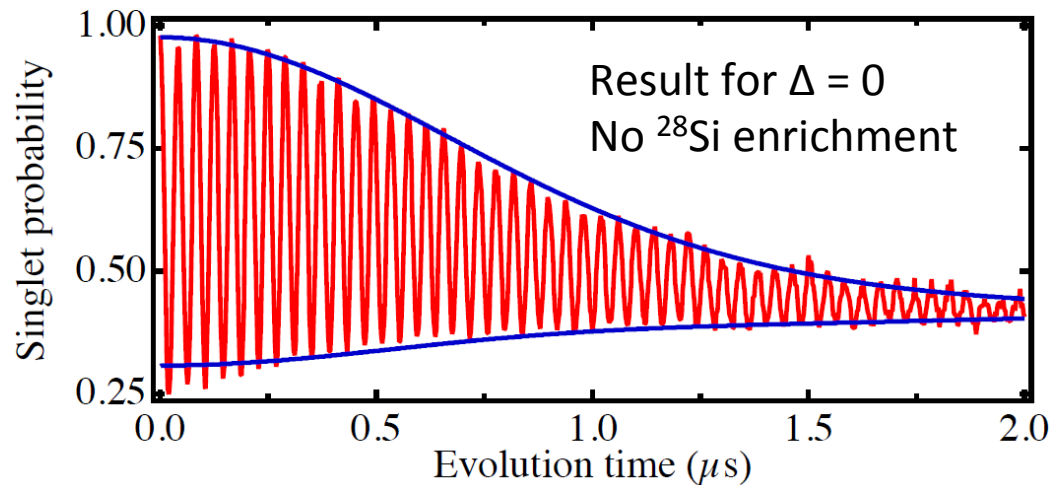
- Prepare system in  $(1,0,2)$  charge configuration, spin singlet in the right dot
- Change to  $(1,1,1)$  configuration and let the system evolve for the desired evolution time, with a given tunnel coupling and detuning between the left and middle dot
- Move to  $(1,1,1)-(1,0,2)$  transition and read out the singlet probability
  - One expects oscillations between 100% and 25%



# Basic Experiment

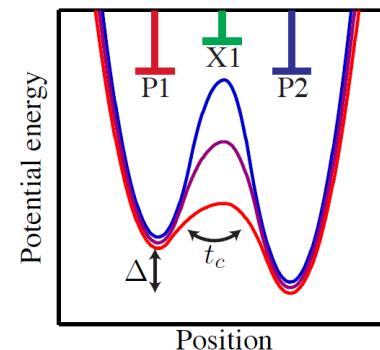
- Prepare system in (1,0,2) charge configuration, spin singlet in the right dot
- Change to (1,1,1) configuration and let the system evolve for the desired evolution time, with a given tunnel coupling and detuning between the left and middle dot
- Move to (1,1,1)-(1,0,2) transition and read out the singlet probability

→ One expects oscillations between 100% and 25%



Fit:  
Double Gaussian decay with  $1/e$  decay time of  $1.0 \mu\text{s}$  for hyperfine interactions and  $1.5 \mu\text{s}$  due to charge noise

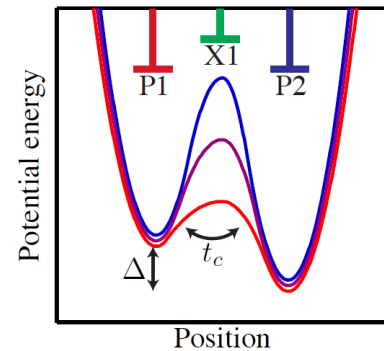
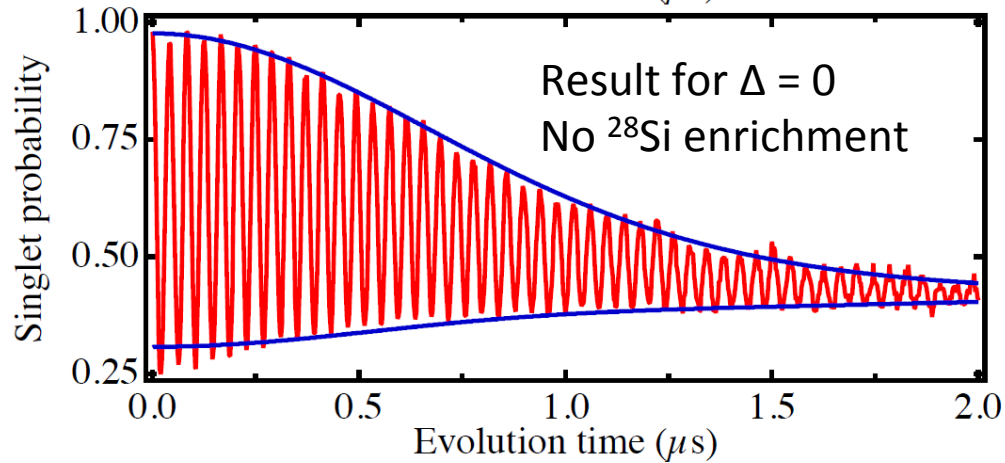
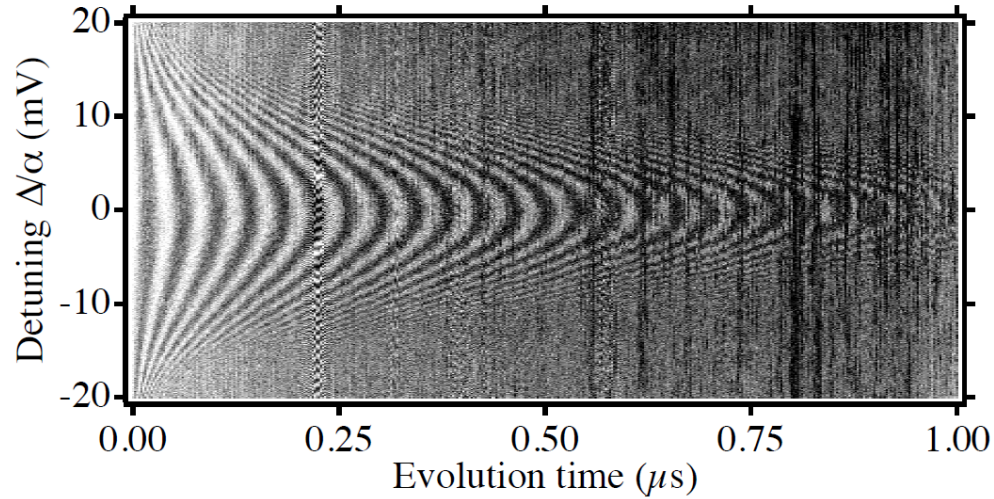
Model for hyperfine-induced decay:  
T. D. Ladd, PRB (2012)



# Basic Experiment

Dependence on detuning:

$$\Delta = \alpha(V_{P1} - V_{P2})$$



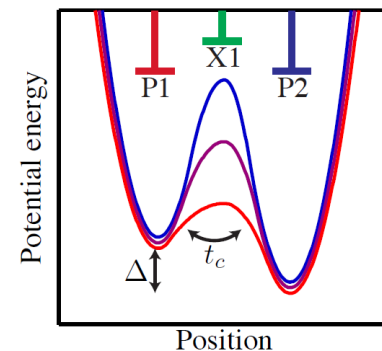
# Model and Insensitivity

Model for the decay of the amplitude due to charge noise:

$$\exp\left(-\sigma_V^2 \sum_j |dJ/dV_j|^2 t^2 / \hbar^2\right)$$

Variance of the noise

*Details:  
See supplementary  
information*



# Model and Insensitivity

Model for the decay of the amplitude due to charge noise:

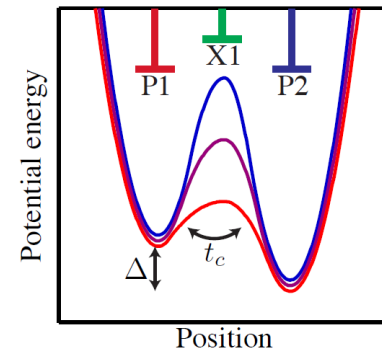
*Details:  
See supplementary  
information*

$$\exp\left(-\sigma_V^2 \sum_j |dJ/dV_j|^2 t^2 / \hbar^2\right)$$

Variance of the noise

“Insensitivity”:

$$\mathcal{I} = J / \sqrt{\sum_j |dJ/dV_j|^2}$$





# Model and Insensitivity

Model for the decay of the amplitude due to charge noise:

*Details:  
See supplementary information*

$$\exp(-\sigma_V^2 \sum_j |dJ/dV_j|^2 t^2 / \hbar^2)$$

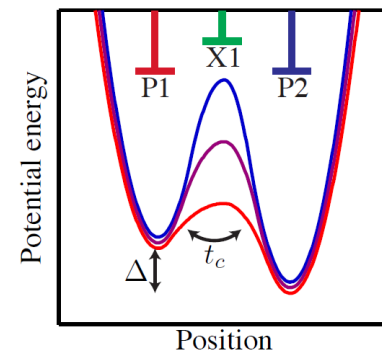
Variance of the noise

“Insensitivity”:

$$\mathcal{I} = J / \sqrt{\sum_j |dJ/dV_j|^2}$$

Number  $N_{\text{Rabi}}$  of Rabi oscillations before the amplitude decays by a factor  $1/e$ :

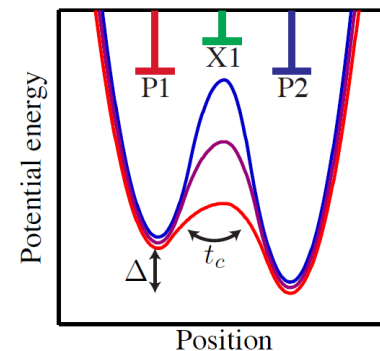
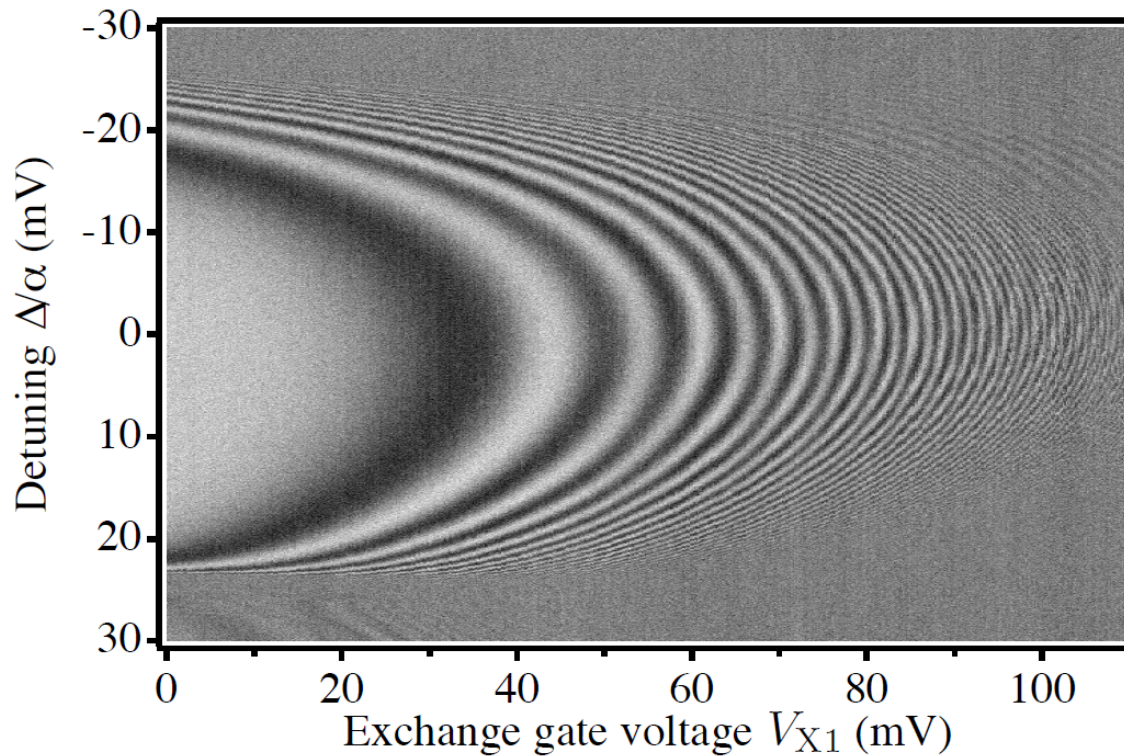
$$\mathcal{I} / (2\pi\sigma_V)$$



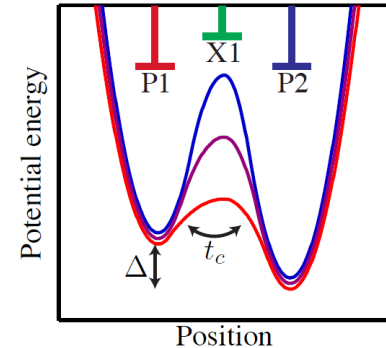
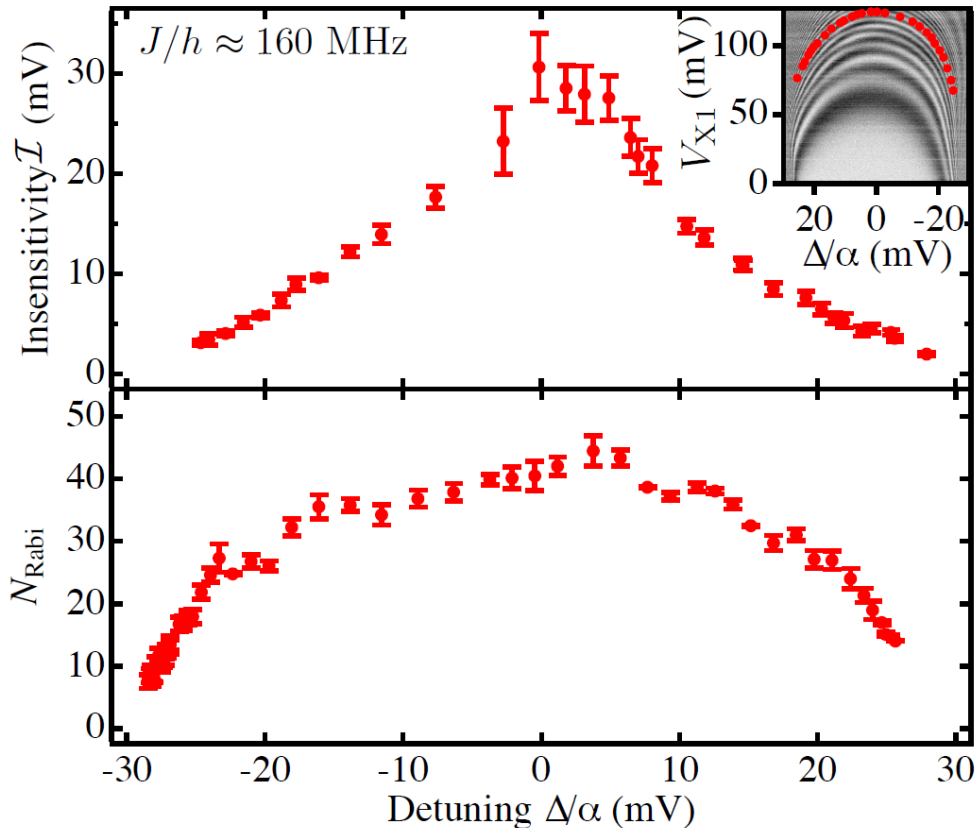
# Experiments with Isotopically Purified Si

Example for sample with 800 ppm enriched  $^{28}\text{Si}$   
and an additional screening gate

**Here: Evolution time fixed!**



# Experiments with Isotopically Purified Si



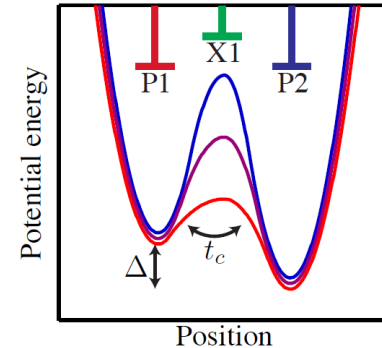
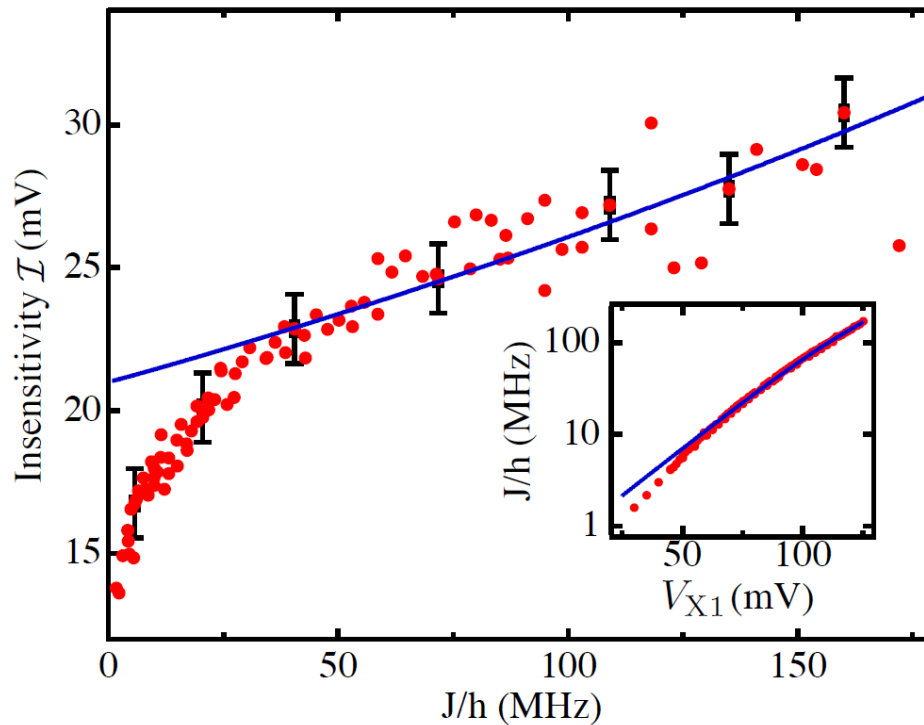
$N_{\text{Rabi}}$  is maximal/minimal when the insensitivity is maximal/minimal

The direct proportionality expected from the model is not observed

The best results are achieved at zero detuning

# Inensitivity at Zero Detuning

At zero detuning, the dominant derivative is  $dJ/dV_{X1}$



Model (blue line):  
1D Wentzel-Kramers-Brillouin  
(WKB) approximation appropriate  
for shallow barrier tunneling

**The largest insensitivity is achieved at large  $J$**

# Conclusions

---

Experimental confirmation with Si quantum dots that qubits are less sensitive to charge noise when the detuning is zero (number of observable Rabi oscillations is maximal)

# Conclusions

---

Experimental confirmation with Si quantum dots that qubits are less sensitive to charge noise when the detuning is zero (number of observable Rabi oscillations is maximal)

The insensitivity at zero detuning increases with increasing  $J$

# Conclusions

---

Experimental confirmation with Si quantum dots that qubits are less sensitive to charge noise when the detuning is zero (number of observable Rabi oscillations is maximal)

The insensitivity at zero detuning increases with increasing  $J$

The simple model does not fully reproduce the experimental data

# Conclusions

---

Experimental confirmation with Si quantum dots that qubits are less sensitive to charge noise when the detuning is zero (number of observable Rabi oscillations is maximal)

The insensitivity at zero detuning increases with increasing  $J$

The simple model does not fully reproduce the experimental data

Some samples show additional oscillations (other samples do not!), which probably results from excited states. Whether this is due to orbital or valley excited states is unclear and will be subject of future investigations.



Experimental confirmation with Si quantum dots that qubits are less sensitive to charge noise when the detuning is zero (number of observable Rabi oscillations is maximal)

The insensitivity at zero detuning increases with increasing  $J$

The simple model does not fully reproduce the experimental data

Some samples show additional oscillations (other samples do not!), which probably results from excited states. Whether this is due to orbital or valley excited states is unclear and will be subject of future investigations.

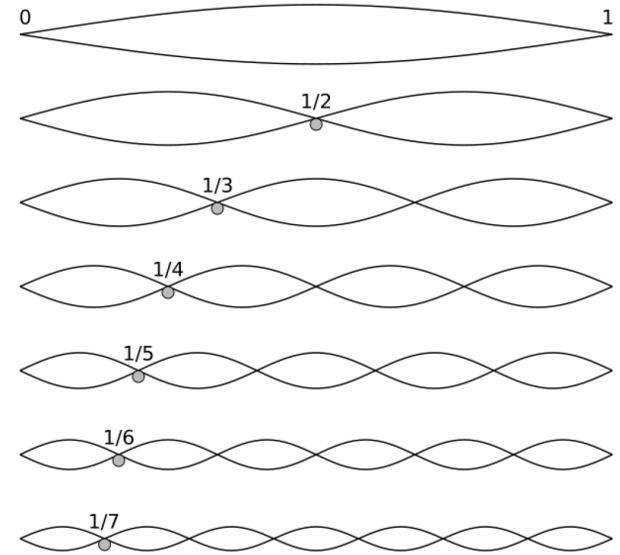
# **APPENDIX**

# Overtones & Stiffness of Strings

Besides its **fundamental mode** (frequency  $f_1$ ), a string features several **overtones** of frequencies  $f_n$

Ideal string:  $\ddot{y} \propto -y'' \quad f \propto |k|$

$\longrightarrow f_n = n f_1$



$n = 1, 2, \dots$

# Overtones & Stiffness of Strings

Besides its **fundamental mode** (frequency  $f_1$ ), a string features several **overtones** of frequencies  $f_n$

Ideal string:  $\ddot{y} \propto -y'' \quad f \propto |k|$

$\longrightarrow f_n = n f_1$

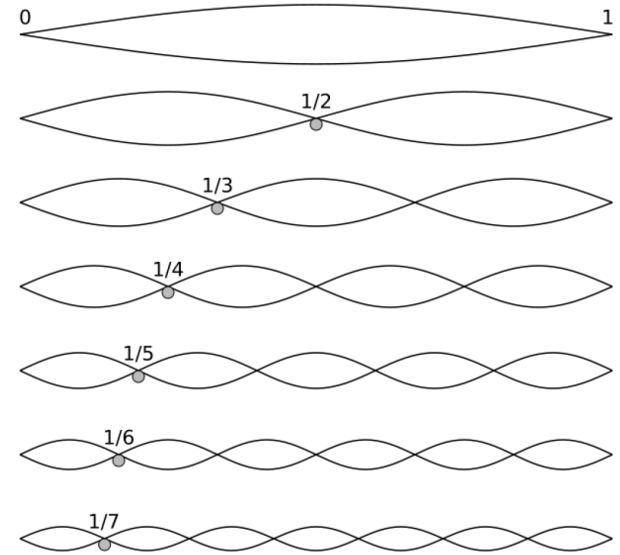
Stiff bar:  $\ddot{y} \propto -y'''' \quad f \propto k^2$

Realistic string:  $\ddot{y} \propto -y'' - \epsilon y'''' \quad f^2 \propto k^2 + \epsilon k^4$

$\longrightarrow f_n \propto n f_1 \sqrt{1 + B n^2}$

**B**: Inharmonicity coefficient

$n = 1, 2, \dots$



# Overtones & Stiffness of Strings

## Further complications:

- Inharmonicity coefficient is different for each string (depends on length, diameter, tension, material properties, ...)
- For each string, the amplitudes of the overtones are different (depending on position of hammer, ...)

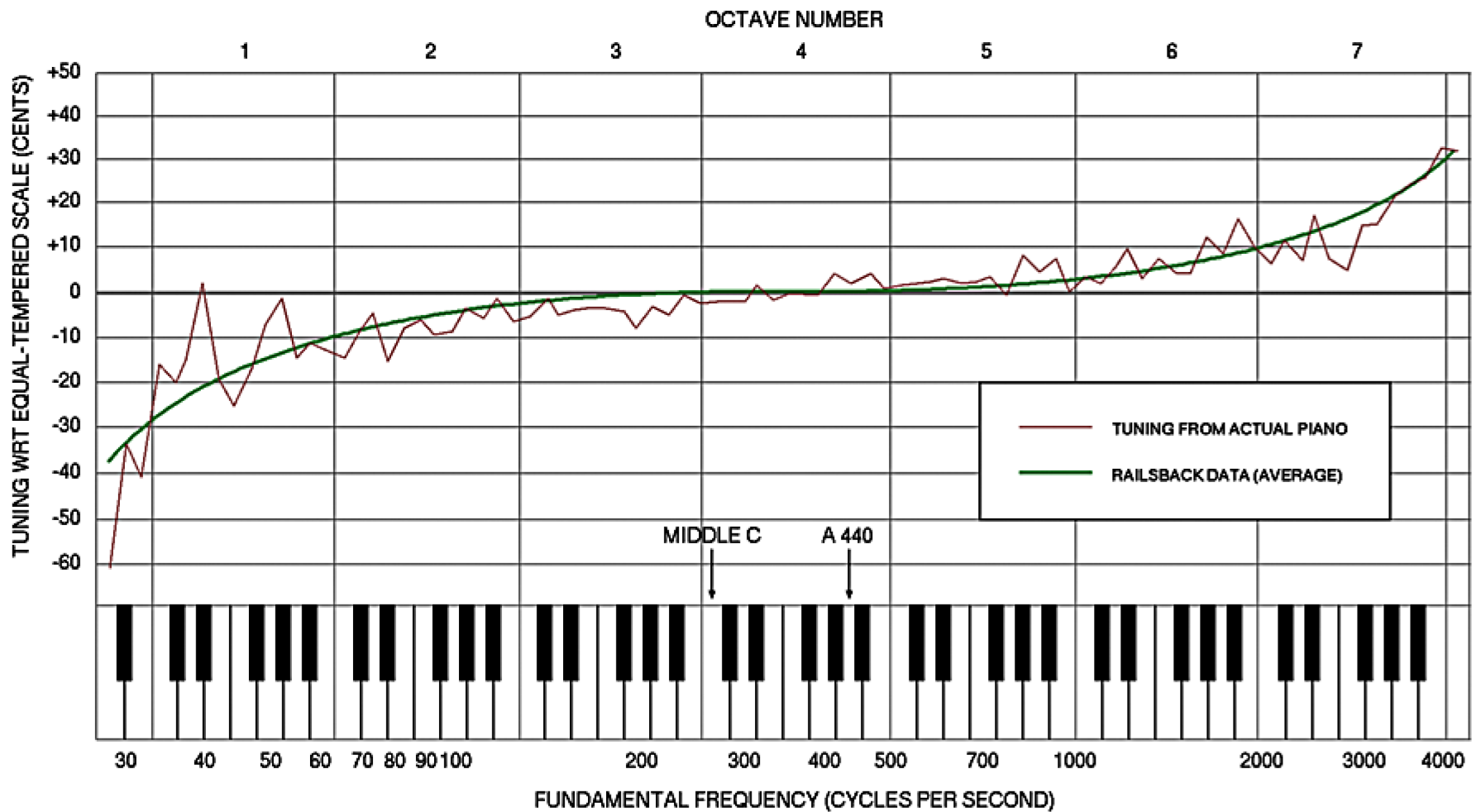
Realistic string:  $\ddot{y} \propto -y'' - \epsilon y''''$        $f^2 \propto k^2 + \epsilon k^4$

$$\longrightarrow f_n \propto n f_1 \sqrt{1 + Bn^2}$$

**B:** *Inharmonicity coefficient*

$$n = 1, 2, \dots$$

# Tuning Curve of High-Quality Aural Tuning



Green: Average

Red: Individual piano

# Tuning via Entropy

---

## Idea of the paper:

Human brain perceives sounds as “pleasant” (“in tune”) when there is some kind of order

Entropy is a measure of disorder

→ **Find tuning curve via entropy minimization**

# Entropy-Based Tuning: Preparation

---

Step 1: Play and record each of the keys



# Entropy-Based Tuning: Preparation

Step 1: Play and record each of the keys

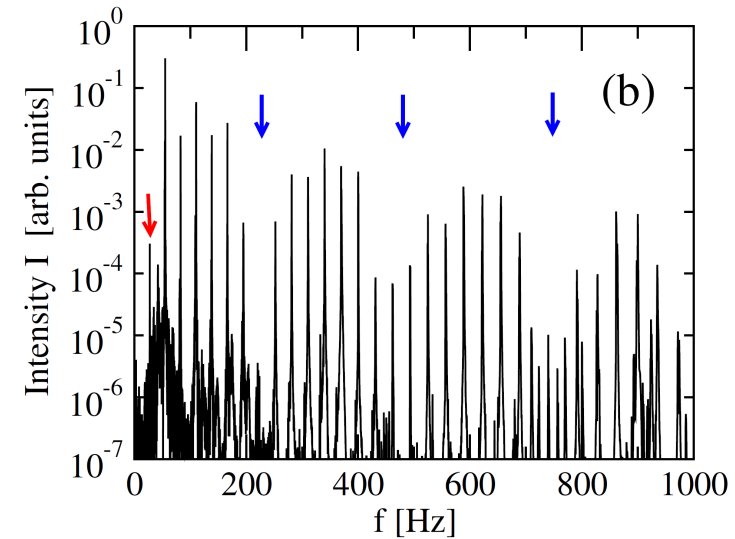
Step 2: Calculate power spectrum

$$I(f) = |\text{Fourier transform}|^2$$

Power spectrum for the lowest key (out of 88)

*Red arrow: Fundamental mode*

*Blue: Suppressed overtones (position of hammer, ...)*



# Entropy-Based Tuning: Preparation

Step 1: Play and record each of the keys

Step 2: Calculate power spectrum

$$I(f) = |\text{Fourier transform}|^2$$

Step 3: Calculate **A-weighted sound pressure level**  $L_A(f)$  (in dBA)

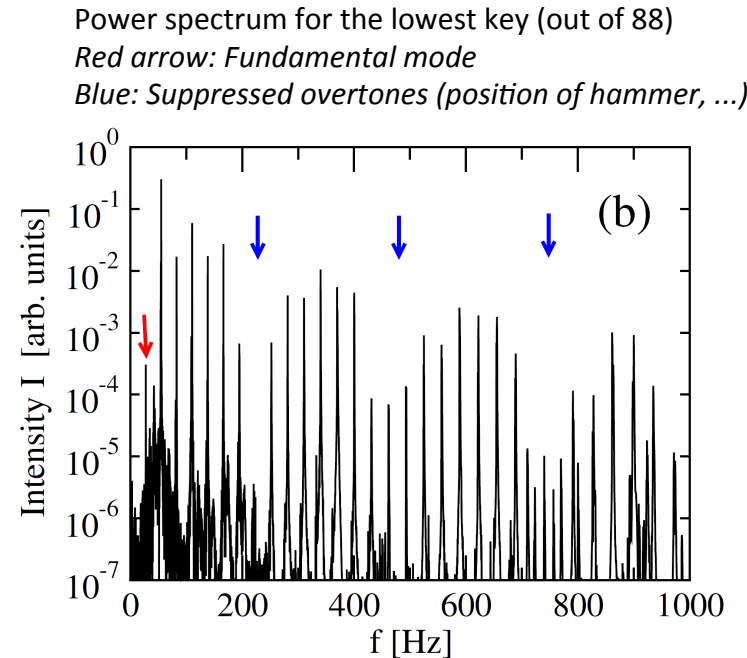
*Can be considered a rough measure of frequency-dependent energy deposition in the inner ear (cochlea)*

$$L_A(f) = \left( 2.0 + 20 \log_{10} R_A(f) \right) L(f)$$

$$L(f) = 10 \log_{10} \left( \frac{I(f)}{I_0} \right)$$

$$R_A(f) = \frac{12200^2 f^4}{(f^2 + 20.6^2)(f^2 + 12200^2) \sqrt{(f^2 + 107.7^2)(f^2 + 737.9^2)}}$$

← Filter function:  
Outer → Inner ear



# Entropy-Based Tuning: Algorithm (Start)

## Start configuration:

- Quantize frequency, ranging from 10 Hz to 10 kHz, in steps of cents:

$$f_m = 2^{m/1200} \cdot 10 \text{ Hz} \quad 0 \leq m \leq 12000$$

- For each of the 88 keys  $k$ , map the A-leveled sound pressure level  $L_A(f)$  onto  $f_m$  to obtain  $L_m^{(k)}$
- Shift  $L_m^{(k)}$  such that the fundamental modes of the keys correspond exactly to that of an equal temperament (with A4 = 440 Hz)
- Compute the sum  $p_m$  over all keys:  $p_m = \sum_{k=1}^{88} L_m^{(k)}$
- Normalize:  $\sum_m p_m = 1$

# Entropy-Based Tuning: Algorithm (Start)

## Start configuration:

- Quantize frequency, ranging from 10 Hz to 10 kHz, in steps of cents:

$$f_m = 2^{m/1200} \cdot 10 \text{ Hz} \quad 0 \leq m \leq 12000$$

- For each of the 88 keys  $k$ , map the A-leveled sound pressure level  $L_A(f)$  onto  $f_m$  to obtain  $L_m^{(k)}$
- Shift  $L_m^{(k)}$  such that the fundamental modes of the keys correspond exactly to that of an equal temperament (with A4 = 440 Hz)
- Compute the sum  $p_m$  over all keys:  $p_m = \sum_{k=1}^{88} L_m^{(k)}$
- Normalize:  $\sum_m p_m = 1$

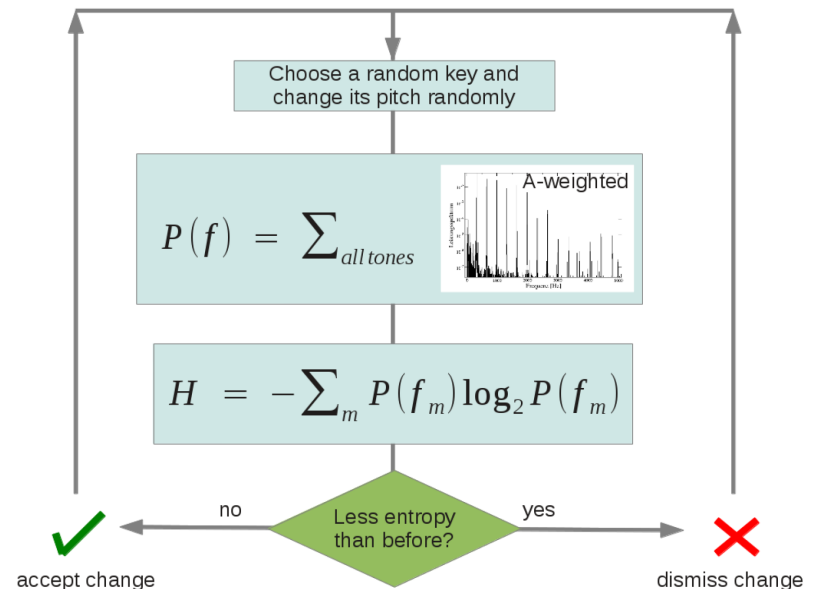
**Start configuration is a quantized (cents) probability distribution based on the power spectrum generated in the inner ear when the piano is exactly tuned to equal temperament**

# Entropy-Based Tuning: Algorithm (Dynamics)

Entropy: 
$$H = - \sum_m p_m \ln p_m$$

## Monte-Carlo dynamics:

- Randomly shift one of the keys by  $\pm 1$  cent
- Compute again the sum  $p_m$  over all keys:  $p_m = \sum_{k=1}^{88} L_m^{(k)}$
- Normalize:  $\sum_m p_m = 1$
- Compute the entropy
- If entropy decreased, keep the change, otherwise undo it



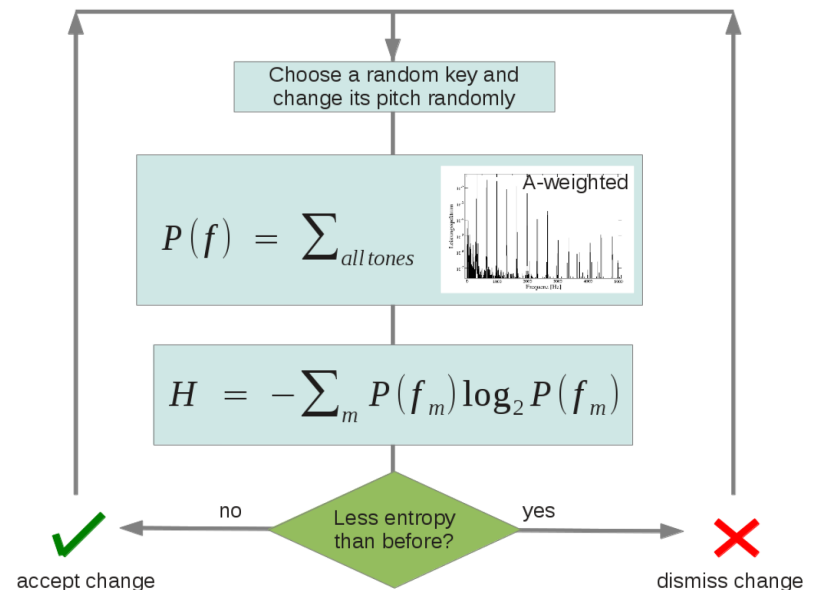
# Entropy-Based Tuning: Algorithm (Dynamics)

Entropy: 
$$H = - \sum_m p_m \ln p_m$$

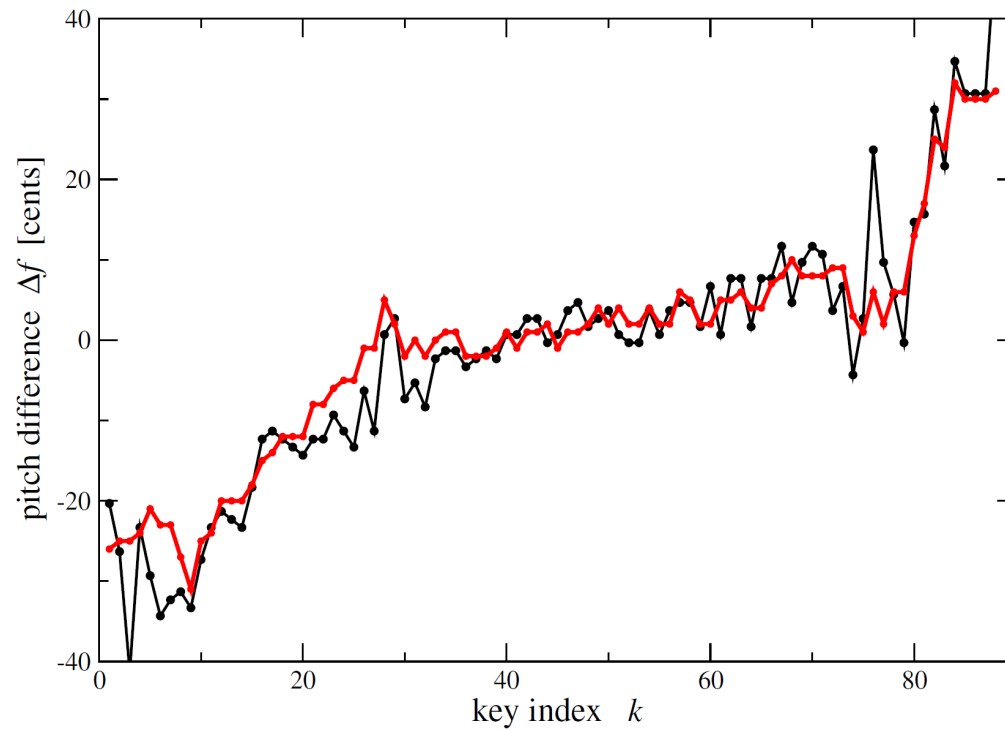
## Monte-Carlo dynamics:

- Randomly shift one of the keys by  $\pm 1$  cent
- Compute again the sum  $p_m$  over all keys:  $p_m = \sum_{k=1}^{88} L_m^{(k)}$
- Normalize:  $\sum_m p_m = 1$
- Compute the entropy
- If entropy decreased, keep the change, otherwise undo it

→ **Find minimum and return the tuning curve**

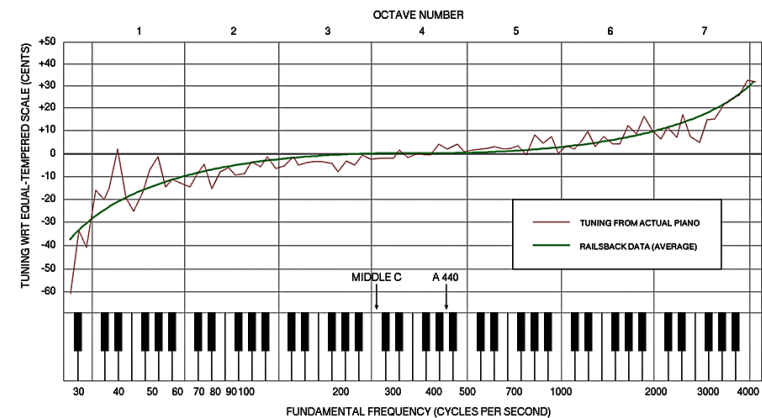


# Results

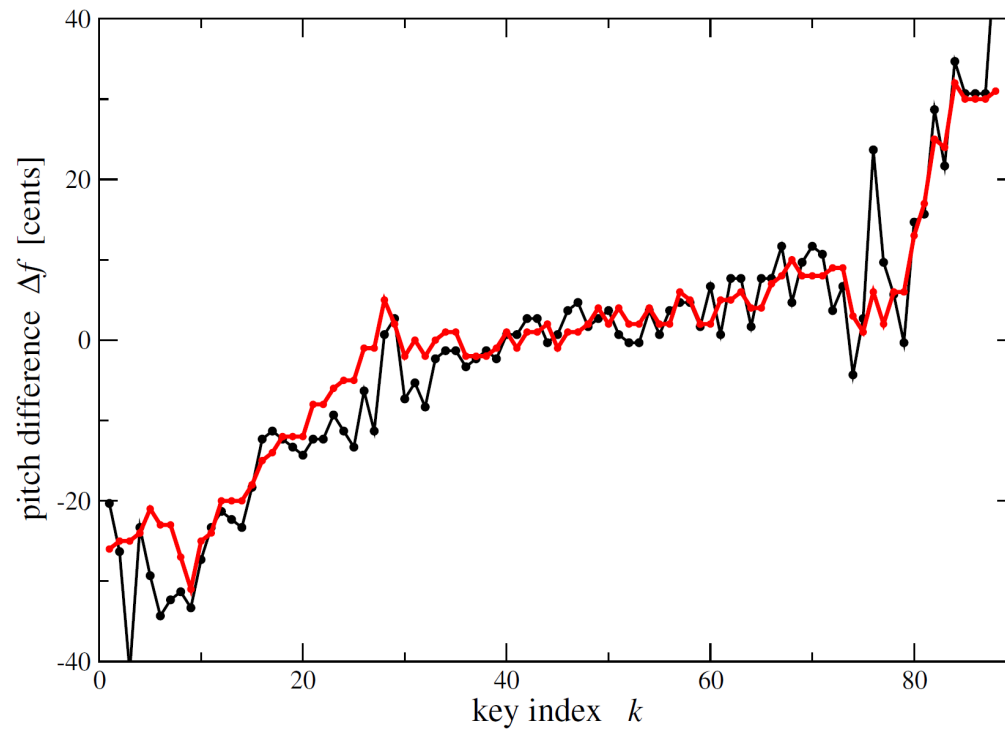


Red: Theoretical result

Black: Aural tuning



# Results

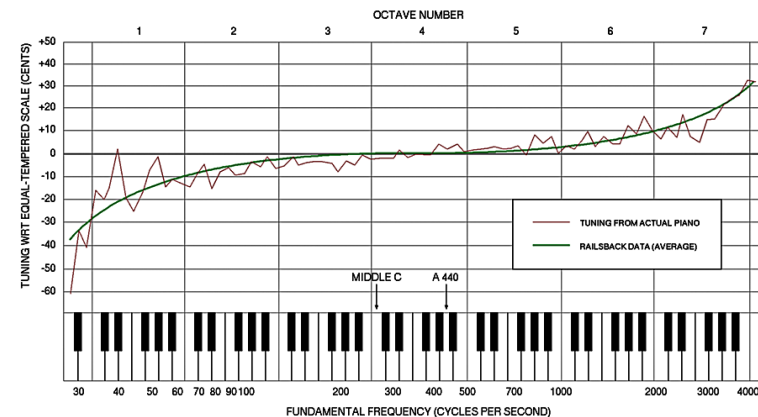


Red: Theoretical result

Black: Aural tuning

**Method reproduces the stretch curve**

**Fluctuations are correlated (!),  
especially in the treble and the bass**





# Media Interest: Articles, Blogs, ...

---

## English

IOP PhysicsWorld.com  
MIT Technology Review  
The Wall Street Journal  
Daily Mail – Mail Online  
Discover Magazine  
Pano News Archiv  
Microsoft Future Tech  
Physics4me  
The Week behind  
Quantummaniac  
33rd Square  
Piano Tuner Technicians Forum  
Tune a Piano Yourself Blog  
Editorial RBEF  
...

## German

Heise Newsticker  
Technology Review Heise Online  
Deutschlandradio Kultur  
Pressestelle Uni Würzburg  
showmedia.de  
Nürnberger Zeitung (NZ)  
Wiley Interscience pro-physik  
Codex Flores: Viel Aufregung...  
Medizin&Technik: Wir wollen Spaß  
Neurosociology & Neuromarketing  
Interview Klassikradio  
Interview BR2  
Mainpost  
...

# Conclusions

---

**Author:** Several open questions and remaining tasks

- Method tested on only one piano so far
- Apparently there are many local minima, and the present algorithm gives similar but not reproducible results
- Step-size of one cent is smaller than the resolution of the ear
- When additional filter function for “inner ear → brain” (“loudness”) are included, one obtains unreasonable stretches in the bass
- ... (see article)

# Conclusions

---

**Author:** Several open questions and remaining tasks

- Method tested on only one piano so far
- Apparently there are many local minima, and the present algorithm gives similar but not reproducible results
- Step-size of one cent is smaller than the resolution of the ear
- When additional filter function for “inner ear → brain” (“loudness”) are included, one obtains unreasonable stretches in the bass
- ... (see article)

The fluctuations on top of the smooth stretch curve are not random, but to some extent essential for the good results as achieved by professional, aural tuning

# Conclusions

---

**Author:** Several open questions and remaining tasks

- Method tested on only one piano so far
- Apparently there are many local minima, and the present algorithm gives similar but not reproducible results
- Step-size of one cent is smaller than the resolution of the ear
- When additional filter function for “inner ear → brain” (“loudness”) are included, one obtains unreasonable stretches in the bass
- ... (see article)

The fluctuations on top of the smooth stretch curve are not random, but to some extent essential for the good results as achieved by professional, aural tuning

Whether or not the presented idea based on entropy minimization can be used to improve existing electronic tuning methods remains to be seen

# Conclusions

**Author:** Several open questions and remaining tasks

- Method tested on only one piano so far
- Apparently there are many local minima, and the present algorithm gives similar but not reproducible results
- Step-size of one cent is smaller than the resolution of the ear
- When additional filter function for “inner ear → brain” (“loudness”) are included, one obtains unreasonable stretches in the bass
- ... (see article)

The fluctuations on top of the smooth stretch curve are not random, but to some extent essential for the good results as achieved by professional, aural tuning

Whether or not the presented idea based on entropy minimization can be used to improve existing electronic tuning methods remains to be seen

**MIT Technology Review, ... :** *“Algorithm Spells the End for Professional Musical Instrument Tuners”*