Current Correlations in a Majorana Beam Splitter

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We study the current correlation in a T-junction composed of a grounded topological superconductor and of two normal-metal leads which are biased at a voltage V. We show that the existence of an isolated Majorana zero mode in the junction dictates a universal behavior for the cross correlation of the currents through the two normal-metal leads of the junction. The cross correlation is negative and approaches zero at high bias voltages as $1/V$. This behavior survives the presence of disorder and multiple transverse channels, and persists at finite temperatures. We employ numerical transport simulations to corroborate our conclusions.

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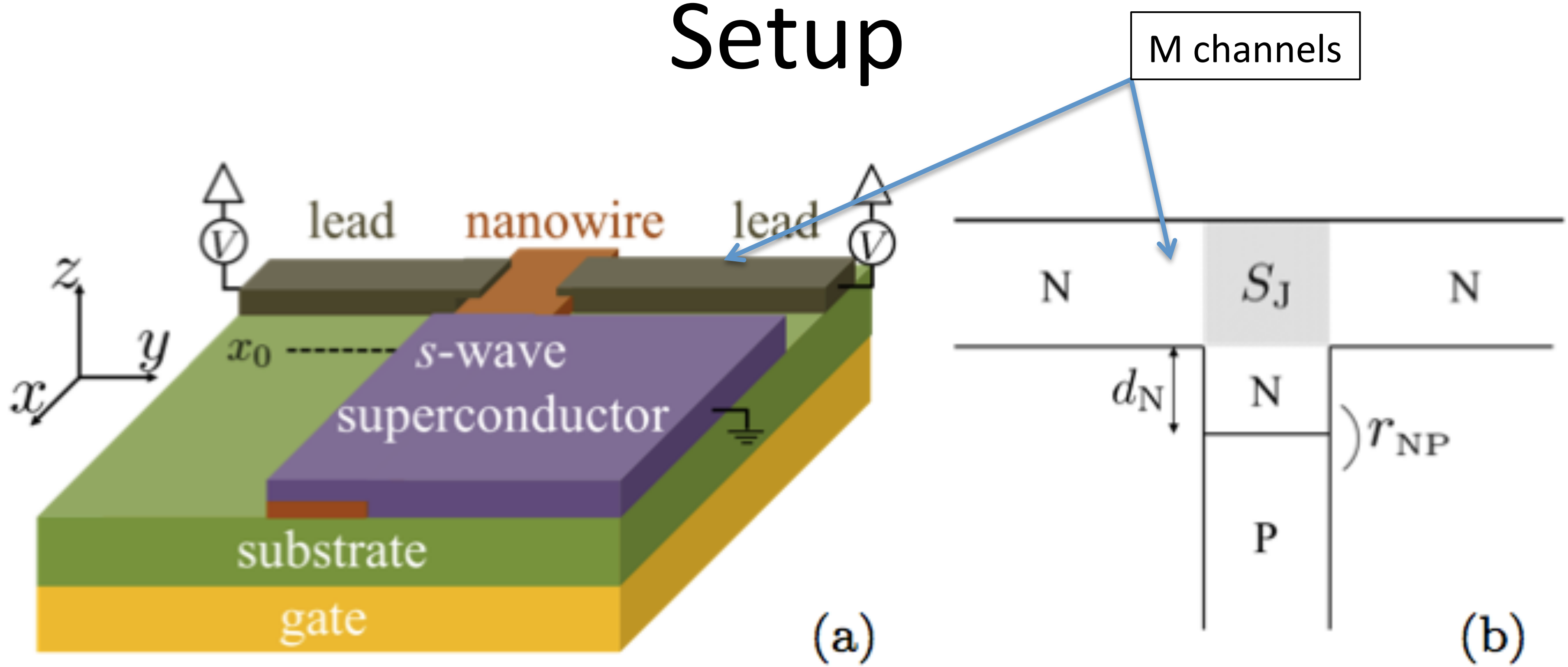
‘Observation’ of Majoranas

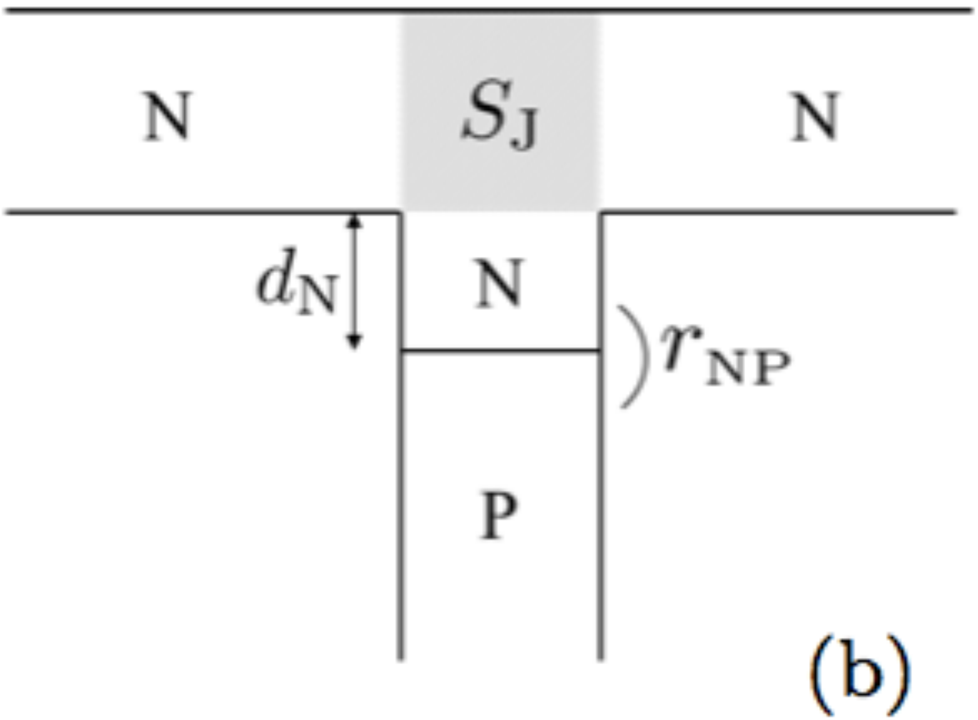
- Differential conductance
- Noise measurements propose exploiting non-locality
- In this proposal, only a **single** MF is needed

Setup



$$P_{\rm RL} = \int_{-\infty}^{\infty} dt \langle \delta \hat{I}_{\rm R}(0) \delta \hat{I}_{\rm L}(t) \rangle$$

Scattering Formulation



$$S_J = \begin{pmatrix} S_e & 0 \\ 0 & S_e^* \end{pmatrix} \; ; \; S_e = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix}$$

r = reflects L to L, R to R, and L (R) to R (L)
t (t') = transmission to (from) middle leg
r' = reflection from middle leg
This a GENERAL unitary matrix

Everything is below gap -> can be described as a reflection matrix

$$r_{\text{tot}} = \begin{pmatrix} r^{ee} & r^{eh} \\ r^{he} & r^{hh} \end{pmatrix}$$

There is a normal (N) region in the transverse direction through which the current from the and right leads interact with the MF

$$r_{\text{NP}}(\varepsilon) = \begin{pmatrix} 0 & a^*(-\varepsilon) \\ a(\varepsilon) & 0 \end{pmatrix}$$

$$a(\varepsilon) = \exp\left[-i \arccos(\varepsilon/\Delta)\right]$$

Topological criterion:

$$Q = \det[r_{\text{NP}}(0)] = -1$$

After some math…

$$\frac{dI}{dV} = \frac{2e^2}{h}\frac{\Gamma^2}{(eV)^2 + \Gamma^2}$$

Differential conductance is peaked at zero bias

$$\Gamma = \Delta D/2\sqrt{1-D}$$

$$D = \sum_{i=1}^{4M} |t|^2$$

$$\Gamma_\eta = \Delta \sum_{i\in\eta} |t_i'|^2/2\sqrt{1-D}$$

$$P_{\mathrm{RL}}(V) = -\frac{2e^2}{h}\Gamma_{\mathrm{R}}\Gamma_{\mathrm{L}}\frac{eV}{(eV)^2 + \Gamma^2}$$

Always negative

Valid for strong disorder as the scattering matrix for the T-junction was left general

After some math…

$$\frac{dI}{dV} = \frac{2e^2}{h}\frac{\Gamma^2}{(eV)^2+\Gamma^2}$$

Differential conductance is peaked at zero bias

$$P_{\mathrm{RL}}(V) = -\frac{2e^2}{h}\Gamma_{\mathrm{R}}\Gamma_{\mathrm{L}}\frac{eV}{(eV)^2+\Gamma^2}$$

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$$D = \sum_{i=1}^{4M} |t|^2$$

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$$\Gamma_\eta = \Delta \sum_{i\in\eta} |t_i'|^2/2\sqrt{1-D}$$

The result Eq. (30) can be understood from simple considerations based on the properties of MBSs. At low bias voltage $V \ll \Gamma$ and at zero temperature the conductance through the MBS is quantized to $2e^2/h$, resulting in an overall noiseless current [50]. Upon splitting the current into the two parts I_{R} and I_{L}, the total noise P is related to the cross correlation via $P = P_{\mathrm{R}} + P_{\mathrm{L}} + 2P_{\mathrm{RL}}$, where P_η is the current noise through the η lead. Since $P \to 0$ at low voltage, while P_{R} and P_{L} are positive by definition, we must have $P_{\mathrm{RL}} \leq 0$.

After some math…

$$\frac{dI}{dV} = \frac{2e^2}{h}\frac{\Gamma^2}{(eV)^2+\Gamma^2}$$

$$P_{\mathrm{RL}}(V) = -\frac{2e^2}{h}\Gamma_{\mathrm{R}}\Gamma_{\mathrm{L}}\frac{eV}{(eV)^2+\Gamma^2}$$

Differential conductance is peaked at

$$D = \sum_{i=1}^{4M} |t|^2$$

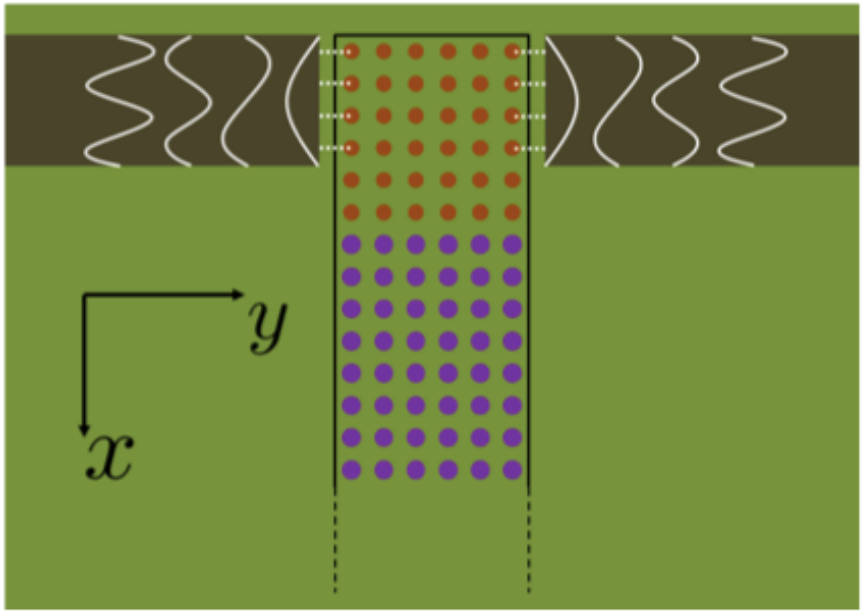
$$\Gamma = \Delta D/2\sqrt{1-D}$$

$$\Gamma_\eta = \Delta \sum_{i\in\eta} |t_i'|^2/2\sqrt{1-D}$$

At high bias voltages $V \gg \Gamma$ (i.e., off resonance) transport can be described classically by sequential tunneling events of charges. In the weak coupling limit, $\Gamma \ll \Delta$, the mechanism by which current is conducted involves a splitting of a Cooper pair, such that one electron flips the bound state (either from empty to occupied or vice versa), while the other electron is transmitted into one of the leads. Importantly, for a MBS the probabilities for being transmitted to the right and to the left do not depend on the occupation of the bound state. This is because all local properties of these two states are identical. Based on this, it can be shown [46] that $P_{\mathrm{RL}} \propto -1/V$ in aggreement with Eq. (30).

Numerical Results

(b)



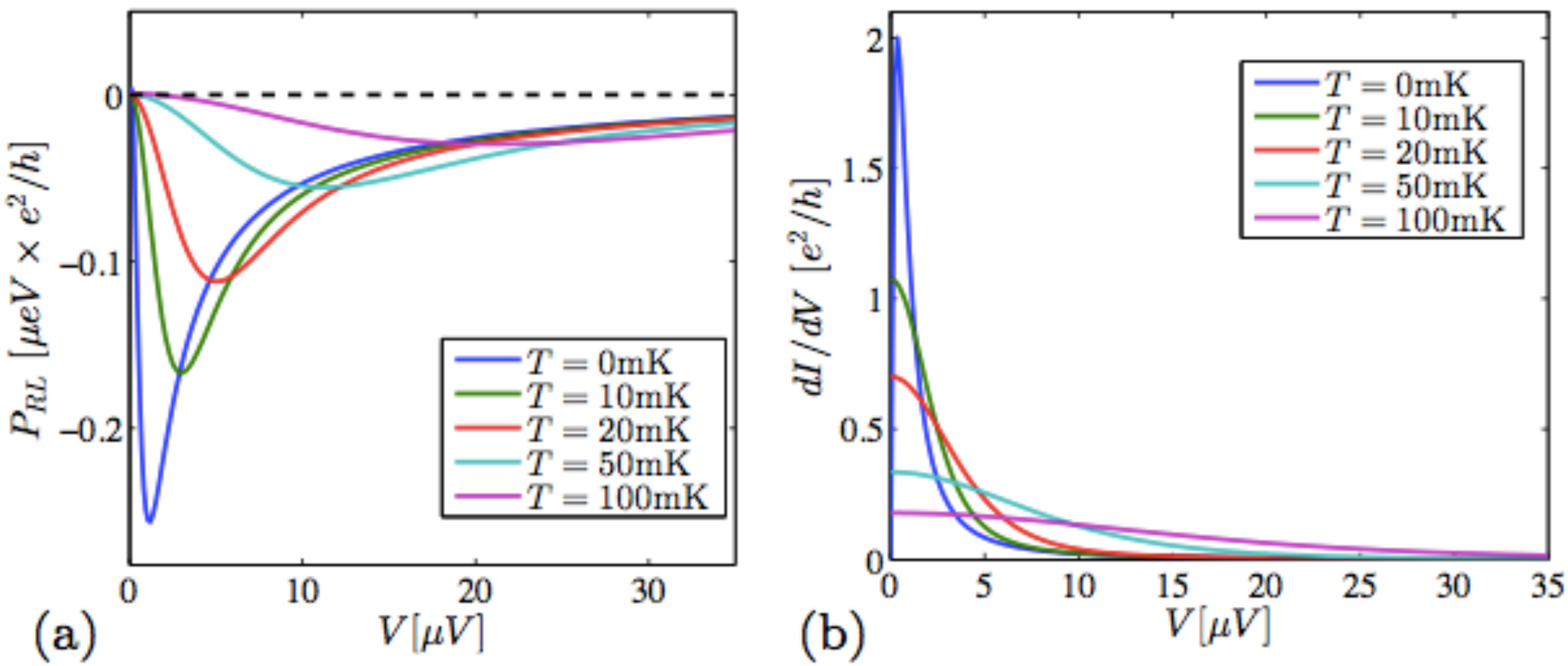
Find the Green’s functions, from which the reflection matrix can be determined

In the present work we use parameters consistent with an InAs nanowire, namely $E_{\rm so} = m_e\lambda_R^2/2 = 75\mu\text{eV}$, $l_{\rm so} = 1/(m_e\lambda_R) = 130\text{nm}$, and $g = 20$ [20]. The induced pair potential is taken to be $\Delta_0 = 150\mu\text{eV}$. The length of the wire is $L_x = 2\mu\text{m}$, with the section not covered by the superconductor being $x_0 = 200\text{nm}$ in length, and the width of the wire is $W_y = 130\text{nm}$.

Include:

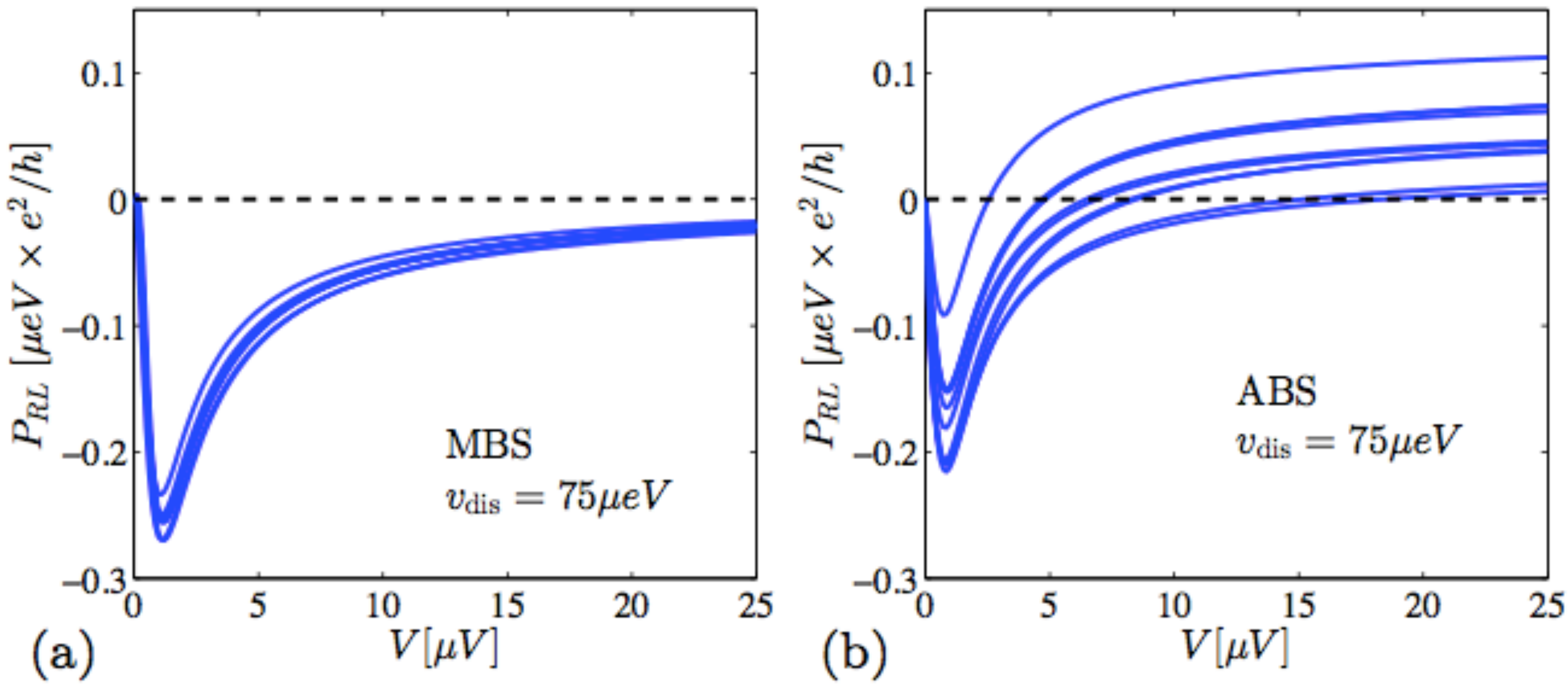
- Hopping between sites(kinetic)
- Magetic field (along x axis)
- Spin orbit interaction (z axis)
- SC pairing (purple sites)
- Disorder (Gaussian distributed potential), characterized by $v_{\rm dis}^2$

As a function of temperature



- Chemical potential set to zero
- Magnetic field tuned to topological nontrivial regime

Effect of disorder

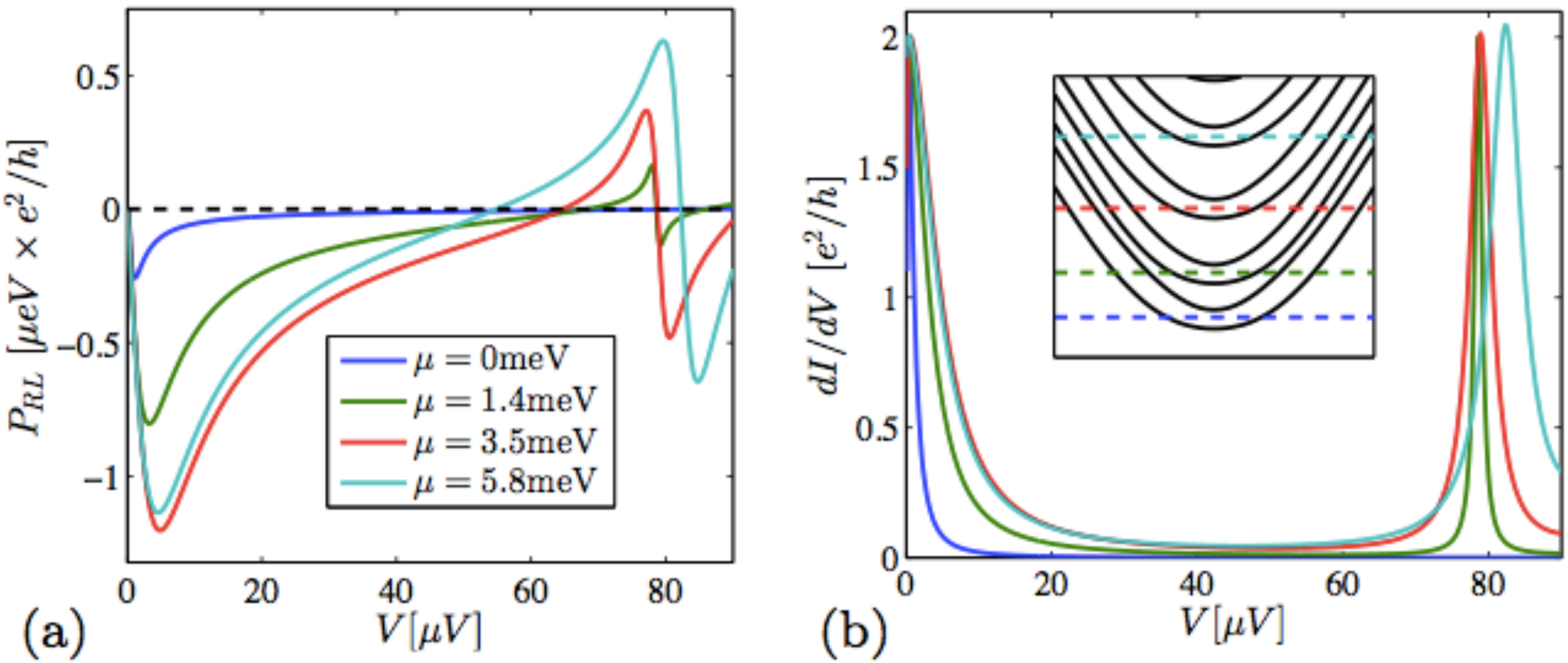


- For ABS, magnetic field tuned so a single ABS state is at zero energy but the system remains topological trivial
- Large voltage, the cross correlation of the ABS state goes above zero

Conclusions

- Measuring current correlations between left and right legs through a p-wave island can distinguish MBS from other bound states
- This is because local properties of occupied and unoccupied MBS cannot be distinguished
- Best measured away from MBS resonance

Multiple channels



- Additional conductance channels can support ABS
- ABS Result in positive uptick of the correlation

Explicit form for reflection matrices

We can now concatenate S_J with r_NP to obtain the overall reflection matrix r_tot of Eq. (2). This results in [46]

$$r^{ee} = r + \frac{a(\varepsilon)^2 r t^\dagger t}{1 + |r'|^2 a(\varepsilon)^2} \;;\; r^{he} = \frac{a(\varepsilon) t'^* t}{1 + |r'|^2 a(\varepsilon)^2} \,. \quad (5)$$

with $r^{eh}(\varepsilon) = [r^{he}(-\varepsilon)]^*$ and $r^{hh}(\varepsilon) = [r^{ee}(-\varepsilon)]^*$ in compliance with particle-hole symmetry [47].

Formulas for current and correlation

At zero temperature the currents in the leads and their cross correlation are given by [29, 48]

$$\langle \hat{I}_\eta \rangle = \frac{2e}{h} \sum_{i \in \eta} \int_0^{eV} d\varepsilon |\mathcal{R}_{ii}^{he}(\varepsilon)|^2 \,,$$
$$P_{\mathrm{RL}} = \frac{e^2}{h} \sum_{i \in \mathrm{R}, j \in \mathrm{L}} \int_0^{eV} d\varepsilon \mathcal{P}_{ij}(\varepsilon) \,, \qquad (6)$$
$$\mathcal{P}_{ij} = |\mathcal{R}_{ij}^{he}|^2 + |\mathcal{R}_{ij}^{eh}|^2 - |\mathcal{R}_{ij}^{ee}|^2 - |\mathcal{R}_{ij}^{hh}|^2 \,,$$

where $\mathcal{R}^{\alpha\beta} = r^{\alpha e} r^{\beta e \dagger}$, $\eta = \mathrm{R}, \mathrm{L}$, and i, j label the channels in the right and left lead, respectively, as defined below Eq. (2).

Tight binding Hamiltonian

tight-binding Hamiltonian

$$H = \sum_{\mathbf{r}} \sum_{s,s'} \{ [V_{\mathbf{r}} \delta_{ss'} + B \sigma^x_{ss'}] c^\dagger_{\mathbf{r},s} c_{\mathbf{r},s'} - \sum_{\mathbf{d}=\hat{x},\hat{y}} [(t\delta_{ss'} + iu(\boldsymbol{\sigma}_{ss'} \times \mathbf{d}) \cdot \hat{z}) c^\dagger_{\mathbf{r},s} c_{\mathbf{r}+a\mathbf{d},s'} + \text{h.c.}] \} + \sum_{\mathbf{r}\cdot\hat{x} > x_0} [\Delta_0 c^\dagger_{\mathbf{r},\uparrow} c^\dagger_{\mathbf{r},\downarrow} + \text{H.c.}], \tag{10}$$

where $\mathbf{r}$ runs over the sites of an N_x by N_y square lattice with spacing a. Here $t = 1/2m_e a^2$, $u = \lambda_R/2a$, $V_{\mathbf{r}} = \mu - 4t + V^{\text{dis}}_{\mathbf{r}}$, μ is the chemical potential, and $V^{\text{dis}}_{\mathbf{r}}$ is a Gaussian-distributed disorder potential with zero average and correlations $\overline{V^{\text{dis}}_{\mathbf{r}} V^{\text{dis}}_{\mathbf{r}'}} = v^2_{\text{dis}} \delta_{\mathbf{r}\mathbf{r}'}$.