Exchange-interaction of two spin qubits mediated by a superconductor

F. Hassler, G. Catelani, H. Bluhm;arXiv:1509.06380

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Outline

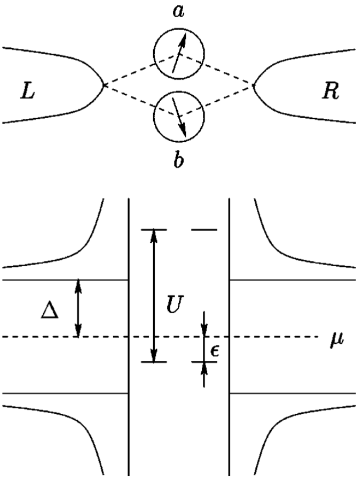
1. Introduction
2. System description
3. Conlusions

Current context

- Single spins as qubits in quantum dots
- Scalable long-range coupling scheme is needed
- Mediators: optical cavities, MW resonators, floating metallic and ferromagnetic couplers, spin chains, SC systems
- Short story for couplings of qubits in QDs by SC: (2000) Choi, Bruder , Loss PRB 62, 13569→1 (2013) Leijnse, Flensberg PRL 111, 060501
- →2 (2015) Hassler, Catelani, Bluhm arXiv:1509.06380

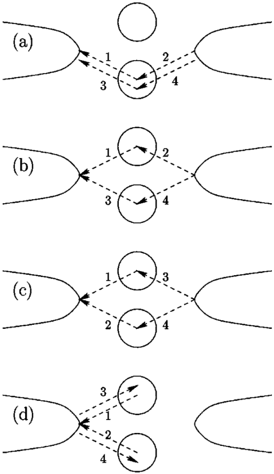
Background I, PRB Choi et al.

- Two dots tunnel-coupled to two SC leads. No direct coupling of QDs
- $H = H_S + H_D + H_T$, H_S-BCS SC. Same μ, Δ in both SC leads
- H_D one level ϵ_i with on-site repulsion U
- $\epsilon_a = \epsilon_b = \epsilon$
- SW transformation in H_T

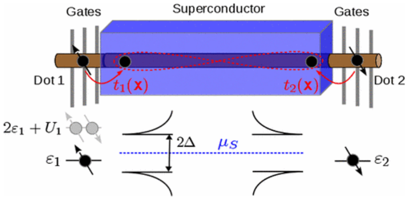


Background II, PRB Choi et al.

- Regime $\Delta, 2\epsilon \ll U$
- Dots are at most singly-occupied
- Some of the relevant processes$\sim t^4$
- Two regimes $\frac{\epsilon}{\Delta} \ll 1, \frac{\epsilon}{\Delta} \gg 1$
- Respective processes a), b) resp. b), c) giving both Josephson type coupling betw. the SC leads and exchange int, betw. spins in QDs
- Just one SC instead of two?$\rightarrow$ d) process analysis in Leijnse et al.

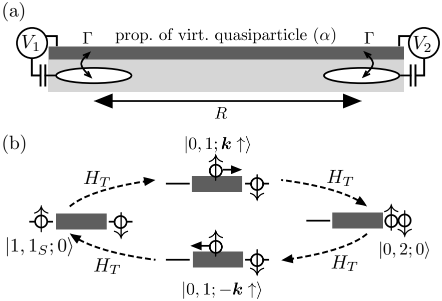


Background III, PRL Leijnse et al.



- Crossed Andreev reflection (CAR) vs. exchange. Intermediate states $|00\rangle$ *vs.* $|02\rangle$.
- Still in the same regime as before $\Delta, 2\epsilon \ll U$.
- $|02\rangle$ blocked by U, CAR is the only relevant process here.
- $J \sim e^{-\frac{x}{\xi_0}}$, ξ_0 SC coherence length (clean Al $\xi_0 = 2.3\mu$m)

Setup



- Ideal operation point: $|11\rangle$, $|02\rangle$ almost degenerate.
- $\varepsilon_{1,1} = \epsilon_1 + \epsilon_2 ; \varepsilon_{0,2} = 2\epsilon_2 + U_2 ; \delta\varepsilon = \varepsilon_{0,2} - \varepsilon_{1,1}$
- Near degeneracy achieved by gating into $\epsilon_1 = \epsilon_2 + U_2 - \delta\varepsilon$

Description of the system

$$H = H_D + H_{\text{BCS}} + H_T; H_D = H_1 + H_2; H_j = \epsilon_j n_j + \frac{1}{2} U_j n_j (n_j - 1)$$

$$H_{\text{BCS}} = \sum_{k\sigma} E_k \beta^\dagger_{k\sigma} \beta_{k\sigma}; E_k = (\xi_k^2 + \Delta^2)^{1/2}; \xi_k = \hbar^2 k^2 / 2m - \mu$$

$$c_{k\uparrow} = u_k \beta_{k\uparrow} + v_k \beta^\dagger_{-k\downarrow}, \quad c^\dagger_{-k\downarrow} = -v_k \beta_{k\uparrow} + u_k \beta^\dagger_{-k\downarrow}$$

$$\begin{aligned} H_T &= -t \sum_\sigma \left[c^\dagger_\sigma(0) d_{1\sigma} + c^\dagger_\sigma(R) d_{2\sigma} \right] + \text{H.c.} \\ &= -\frac{t}{L} \sum_{k\sigma} \left[c^\dagger_{k\sigma} d_{1\sigma} + e^{-ik_x R} c^\dagger_{k\sigma} d_{2\sigma} \right] + \text{H.c.} \end{aligned}$$

Assumptions

- Interaction mediated by virtual process with cost $\varepsilon_{1,1} - [\varepsilon_{0,1} + E_k]$ parametrized as $M = \Delta - (\epsilon_2 + U_2 - \delta\varepsilon) > 0$
- Originally ϵ_1, ϵ_2 tuned by gates. Now we can tune $M, \delta\varepsilon$. M is in fact energy difference betw. ϵ_1 and gap edge.
- If we begin with triplet states of two electrons, this process is not giving anything (double occup. with same spin forbidden), singlet energy is lowered.
- Effective exchange Hamiltonian $H_{\text{eff}} = \frac{1}{4} J \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$, $J > 0$
- SC in ground state $T_e \ll \Delta$

Exchange mediated by clean SC I

$J = \alpha\Gamma^2/\delta\varepsilon; \Gamma = 2\pi t^2 \rho_0; \rho_0 = m/2\pi\hbar^2$ is DOS in normal state

$$\alpha = \frac{1}{\Gamma^2}\left|\sum_{k\sigma} \frac{\langle 0,2;0|H_T|0,1;k\sigma\rangle\langle 0,1;k\sigma|H_T|1,1_S;0\rangle}{\varepsilon_{1,1} - \varepsilon_{0,1} - E_k}\right|^2$$
$$= \frac{1}{2\pi^2\rho_0^2}\left|g(R;\Delta - M)\right|^2$$

$$g(r;E) = -i\int_0^\infty dt\,\langle 0|c_\sigma(r)e^{i(E-H_{\mathrm{BCS}})t/\hbar}c_\sigma^\dagger(0)|0\rangle$$
$$= \int \frac{d^2k}{(2\pi)^2}\frac{u_k^2 e^{ik\cdot r}}{E - E_k}$$

Exchange mediated by clean SC II

- Into polar coordinates, assuming $M \ll \Delta \ll \mu$ and $k_F r \gg 1$

$$g(r;E) = \rho_0 \left(\frac{\pi\Delta}{Mk_F r}\right)^{1/2} \cos(k_F r + 3\pi/4) e^{-r/2\xi} \quad \xi = \hbar v_F/\sqrt{8\Delta M}$$

- Effective coherence length ξ is $(\Delta/M)^{1/2} \gg 1$ longer than bare $\xi_0 = \hbar v_F/\pi\Delta$

$$\alpha = 2\cos^2(k_F R + 3\pi/4)\alpha_0 \quad \alpha_0 = \frac{\Delta}{4\pi M k_F R} e^{-R/\xi}$$

- $\cos^2$ dependence from tunnel at point, modified in the more realistic case, but relevant only in clean SC.

Exchange mediated by disordered SC I

- $\frac{E+\xi_k}{E^2-E_k^2} \approx \frac{u_k^2}{E-E_k}$ holds in our limit. Thus its useful to go over to Gorkov Greens function

$$G(r;E) = \int \frac{d^2k}{(2\pi)^2} \frac{(E+\xi_k)e^{ik\cdot r}}{E^2-E_k^2}$$

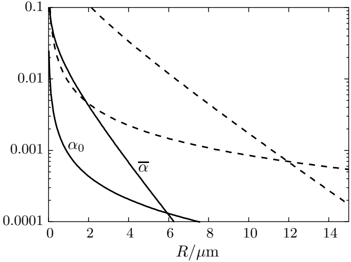
- In the diffusion approximation, with ℓ being mean free path in disordered system, $D = \frac{v_F \ell}{2}$ and for weak disorder $k_F \ell \gg 1$ and $q \ll k_F$

$$\int \frac{d^2k}{(2\pi)^2} \overline{G(k;E)G(k-q;E)} \approx \frac{\pi\rho_0}{2} \frac{\Delta^2}{(\Delta^2-E^2)[(\Delta^2-E^2)^{1/2}+\hbar Dq^2/2]}$$

Exchange mediated by disordered SC II

$$\overline{\alpha} = \frac{\Delta \xi_D^{1/2}}{2(2\pi)^{1/2} M k_F \ell R^{1/2}} e^{-R/\xi_D}$$

- $\xi_D = \sqrt{\ell\xi/2}$ thus the exponential decay is given by different length scale and algebraic decay $\sim R^{-\frac{1}{2}}$ is slower as $\sim R^{-1}$ in clean case. $\frac{M}{\Delta} = 5 \times 10^{-3}$ (solid), $\frac{M}{\Delta} = 5 \times 10^{-4}$ (dashed)



Estimation of the coupling scheme

- Starting from $J_0 \simeq \hbar(10 - 100)\,\mathrm{GHz}$ for the direct exchange, we aim at $\alpha \simeq 10^{-3}$ to end up with $J \simeq \hbar(10 - 100)\,\mathrm{MHz}$.
- For Al $\Delta/k_B = 2.2\,\mathrm{K}$ which corresponds to $\xi_0 = 2.3\,\mu\mathrm{m}$ for a clean sample. Aluminum has $v_F = 2.0 \times 10^6\,\mathrm{m/s}$ that implies $k_F = 17\,\mathrm{nm}^{-1}$.
- To hold a stable detuning over the time of exchange and not to be affected by smearing of SC gap, lowest achievable $M = 0.1\,\mu\mathrm{eV}$.
- That gives $\overline{\alpha}$ larger than 10^{-3} for distances up to $R = 11\,\mu\mathrm{m}$.

Summary

- Long-distance coupling of spin qubits in quantum dots over a distances up to $R = 10\,\mu$m is possible by means of 2d SC film tunnel coupled to the dots.
- Both clean and diffusive dirty SC can provide useful coupling.
- In fact, $R^{-\frac{1}{2}}$ algebraic decay can be eliminated by using 1d SC coupler.
- Everything is controlled by gate voltages.