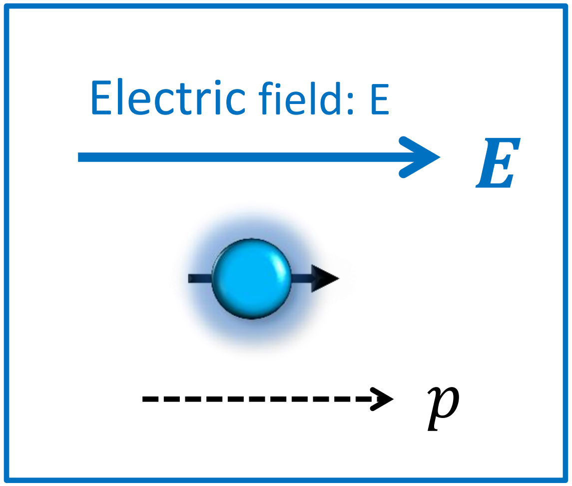
JC on 20 Oct. 2015 by Kouki Nakata

[Phys. Rev. D **92**, 085011 (2015)]

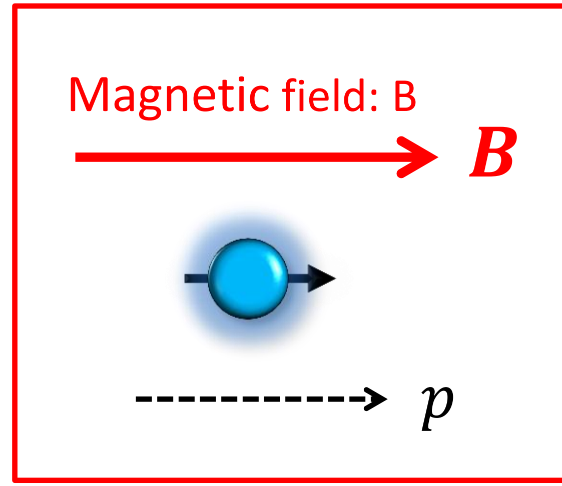
Generalized Bloch theorem
and
chiral transport phenomena

→ **Another view of ``Chiral Magnetic Effect (CME)''**

QUESTION



$$j_e \propto \boldsymbol{E}$$

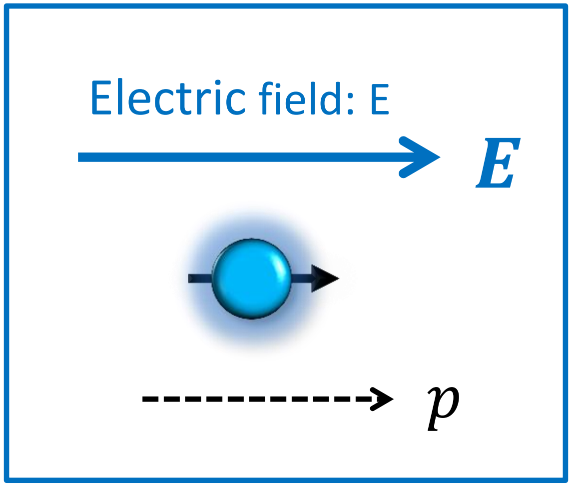


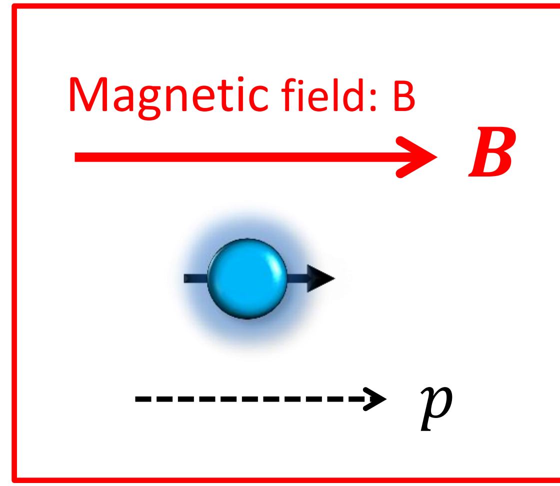
$$j_e \propto \boldsymbol{B} \;?$$

$\rightarrow$ **YES !**

``Chiral Magnetic Effect (CME)''

SURPRISING ?





$$\boldsymbol{j}_e \propto \boldsymbol{E} \qquad \boldsymbol{j}_e \propto \boldsymbol{B}$$

[odd] = [odd] Parity $\underbrace{\text{[odd]} \neq \text{[even]}}_{}$

Contradict ?

→ No problem: *I explain later*

Main Purpose

In general, highly technical knowledge and tedious calculations are required to derive the CME

[K. Fukushima, D. E. Kharzeev, and H. J. Warringa, The chiral magnetic effect, Phys. Rev. D **78**, 074033 (2008).]

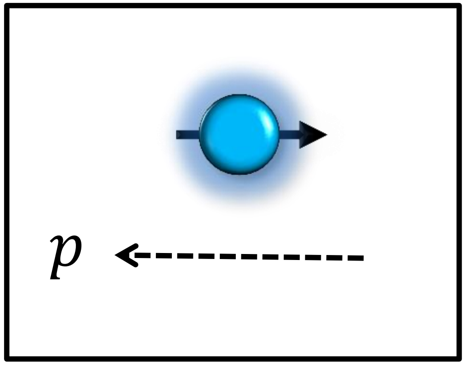
To show you an example of CME in condensed matter physics based on a qualitative discussion

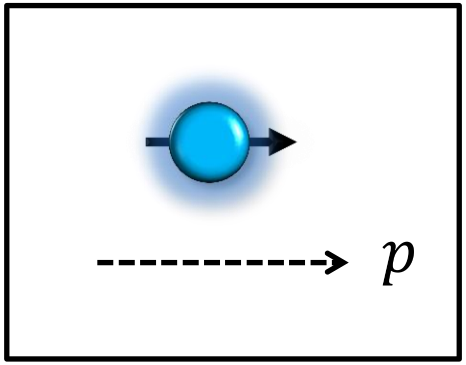
Those currents flow in
`**ground state**' or `**nonequilibrium steady state**' ?

■ QUICK REVIEW

Chirality

→ Relativistically moving fermions





✓ **Chirality: Left-handed**

Chemical potential: $\boldsymbol{\mu}_{\mathbf{L}}$

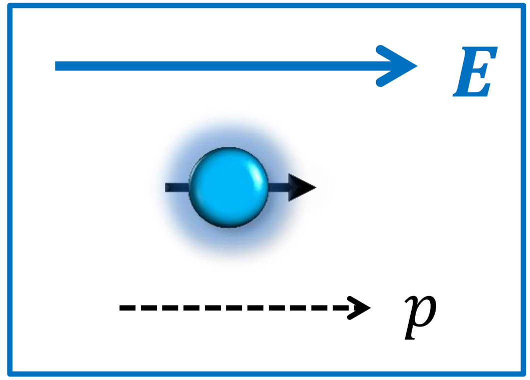
✓ **Chirality: Right-handed**

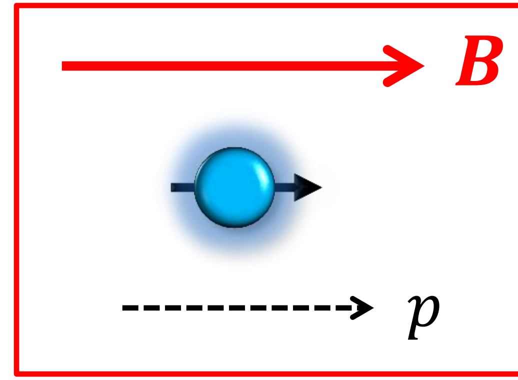
Chemical potential: $\boldsymbol{\mu}_{\mathbf{R}}$

✓ **Chiral chemical potential:** $\mu_5 \equiv \frac{\mu_{\mathrm{R}} - \mu_{\mathrm{L}}}{2}$

Parity

$$p:\ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \mapsto \begin{pmatrix} -x \\ -y \\ -z \end{pmatrix}$$





	Electric field: E	Magnetic field: B
Parity: p	E: odd	B; even
Charge current $\boldsymbol{j}_e$	$\boldsymbol{j}_e \propto \boldsymbol{E}$ [odd] = [odd] → **OK !**	$\boldsymbol{j}_e \propto \boldsymbol{B}$ [odd] $\neq$ [even] → **Contradict ?**

Chiral magnetic effect

$$\boldsymbol{j}_e \propto (\mu_{\mathrm{R}} - \mu_{\mathrm{L}})\boldsymbol{B}$$

→ [odd] = [odd] × [even]

Chiral Magnetic Effect (CME)

✓ *(To the best of my knowledge)*
The Chiral Magnetic Effect has been originally proposed in the context of **QCD** (i.e., the quark-gluon plasm physics)

[K. Fukushima, D. E. Kharzeev, and H. J. Warringa, The chiral magnetic effect, Phys. Rev. D **78**, 074033 (2008).]

$$\boxed{\boldsymbol{j}_e = e^2(\mu_\mathrm{R} - \mu_\mathrm{L})\boldsymbol{B}/(4\pi^2\hbar^2 c) \propto \boldsymbol{B}}$$

↓

✓ These chiral transport phenomena **are expected to appear in a wide area of physics** from ***condensed matter physics*** and nuclear physics to cosmology and astrophysics.

→ An example of **`CME' in condensed mater physics (CME)** is introduced without using the specific knowledge of quantum field theory (anomaly *etc.*).

`CME' in CMP

✓ Noninteracting **massless Dirac fermions** (right- and left-handed massless chiral fermions) in **a torus with the cross section S** whose inside is pierced by **a homogeneous magnetic field B.**

1. Vary the magnetic flux in the torus adiabatically by one quantum unit

$$\delta\Phi \equiv \oint \delta A \cdot dl = \frac{2\pi\hbar}{e}.$$

→ Trivial Aharonov-Bohm phase : $\exp\left(-ie\delta\Phi/\hbar\right) = 1$

2. The chemical potential difference between two Fermi surfaces $\boldsymbol{\mu_R - \mu_L \neq 0}$

→ Transfer of massless fermions N_B from the Fermi surface of left-handed to right-handed

→ This transfer requires the energy $\boldsymbol{N_B(\mu_R - \mu_L)}$

$$\delta E = \int d^3x\, j \cdot \delta A - N_{\rm B}(\mu_{\rm R} - \mu_{\rm L}) = I\left(\frac{2\pi\hbar}{e}\right) - N_{\rm B}(\mu_{\rm R} - \mu_{\rm L}), \quad = 0$$

Since the system comes back to the original state

$$I = \frac{N_{\rm B} e}{2\pi\hbar}(\mu_{\rm R} - \mu_{\rm L})$$

- Magnetic field B → Landau levels for Dirac fermions.
- Those in the lowest Landau level are massless

→ The degeneracy per unit transverse area is $eB/(2\pi\hbar c)$

$$j = \frac{e^2\mu_5}{2\pi^2\hbar^2 c}B,$$

`CME'

QUESTION 2

Those currents due to CME flow in

Ground state ?

or

nonequilibrium steady state ?

✓ **Nonequilibrium steady state**

Why ?

→ *We discuss with generalizing the Bloch's theorem*

Generalized Bloch-type no-go theorem

1. **The ground state $|\Omega >$ gives the lowest energy state $\langle H \rangle = E_{\mathrm{min}}$**
2. ***Assume*** the ***nonozero*** total particle number current $J = \int d^3x\, j(x)$ by $j = \frac{\partial \mathcal{L}}{\partial(\nabla\psi)} \frac{\delta\psi}{\delta\theta} + \mathrm{H.c.}$

   (→ The Noether current associated with the global U(1) particle number symmetry)
3. Consider the **trial state** defined by $|\Omega'\rangle = e^{i\delta p \cdot x}|\Omega\rangle$

$$\rightarrow \boxed{\langle \Omega' | H(\psi) | \Omega' \rangle = \langle \Omega | H(\psi') | \Omega \rangle}$$

where $\psi'(x) = e^{i\theta(x)}\psi(x)$ with $\theta(x) = \delta p \cdot x$

= A `local gauge transformation'

See textbook by M. D. Schwartz
[Quantum Field Theory and the Standard Model (2013)]

taking the expectation value
with respect to $|\Omega >$ & $H(\psi')$

$$\delta\mathcal{H} = \nabla\theta \cdot j \longrightarrow \boxed{\delta E = \delta p \cdot \langle J \rangle + O(\delta p^2)} \leq 0$$

(← Can lower than E_{min})

→ Can contradict with the original **condition 1**

Generalized Bloch-type no-go theorem

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✓ **No currents in the ground states**

= The current due to CME flows in the nonequilibrium steady state

Remark

Using the same approach,
it can be shown that **the quantum time crystal** proposed by Wilcheck cannot exist.

← *James introduced [Phys. Rev. Lett. 114, 251603 (2015)]*

Quantum time crystal

[F. Wilczek, Quantum Time Crystals, Phys. Rev. Lett. 109, 160401 (2012)]

A hypothetical state of matter that spontaneously breaks the continuous translational symmetry in time, analogously to the usual crystals that spontaneously breaks the continuous translational symmetry in space.

CONCLUSION

✓ **Chiral Magnetic Effect in CMP**

$$\boxed{j_e = e^2(\mu_{\mathrm{R}} - \mu_{\mathrm{L}})\boldsymbol{B}/(4\pi^2\hbar^2 c) \propto \boldsymbol{B}}$$

→ Generated in a **nonequilibrium steady state**

(The parity is consistent)

An example: Hamiltonian

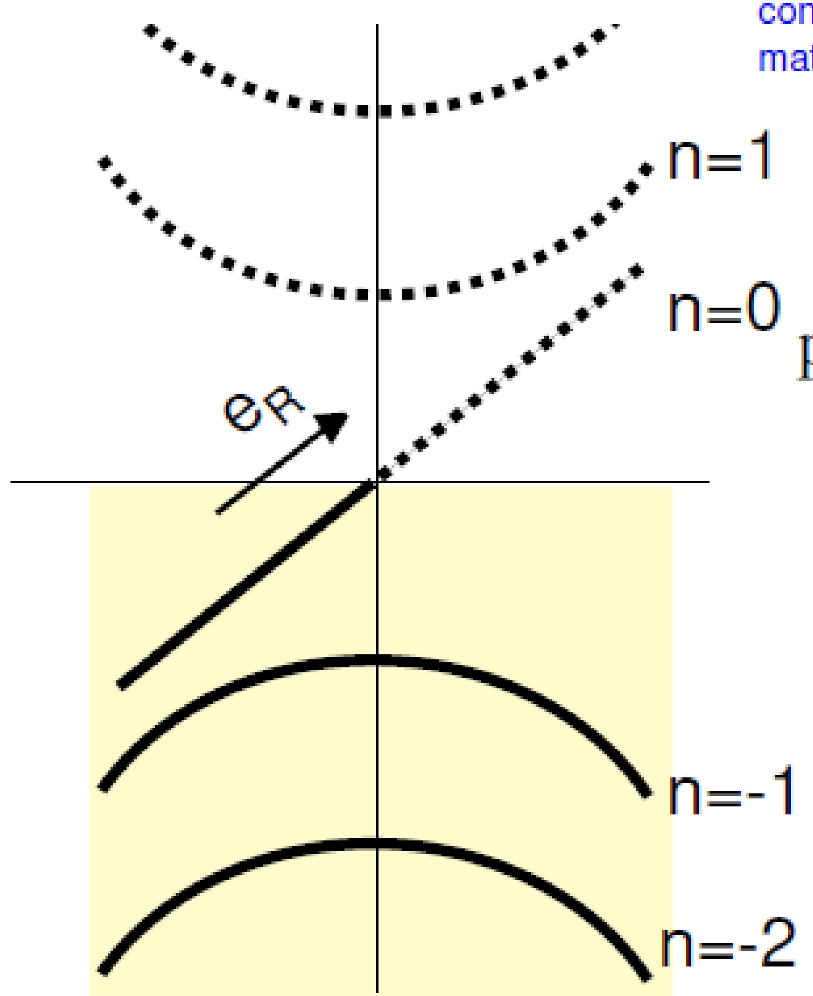
Landau levels: $E^2 = p_z^2 + e|B|(2n+1) + 2e\vec{B}\cdot\vec{s}$

G.E. Volovik, cond-mat/9802091

- Only particles with $(\vec{B}\cdot\vec{s} < 0)$ have massless branches:

$$E = \begin{cases} -p_z & \text{left branch} \quad (\vec{p}\cdot\vec{s}) > 0 \\ p_z & \text{right branch} \quad (\vec{p}\cdot\vec{s}) < 0 \end{cases}$$

- Electric field $\vec{E} = E\hat{z}$ creates right particle (because $p_z(t) = p_z(0) + eEt$)
- Electric field destroys left particles
- Total number does not change
- Difference of left minus right appears – chiral anomaly!



Torus