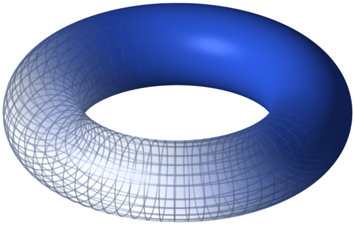
Majorana Fermion Surface Code for Fault-Tolerant Quantum Computation

by Sagar Vijay and Liang Fu (arXiv:1509.08134)



Constantin Schrade

University of Basel

Outline

Fermionic $\mathbb{Z}_2$ Topological Order

Physical realization

Stabilizer measurement

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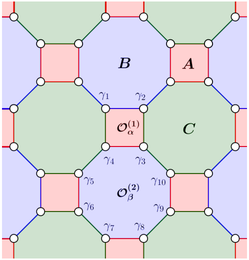

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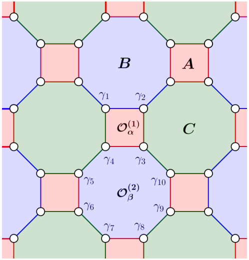

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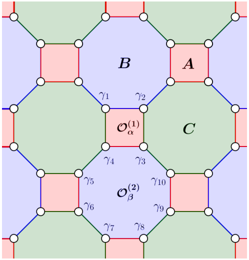

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Hamiltonian: $H = -u_1 \sum_\alpha \mathcal{O}^{(1)}_\alpha - u_2 \sum_\beta \mathcal{O}^{(2)}_\beta$

- Plaquette operators mutually commute
- Eigenvalues $\mathcal{O}^{(1,2)}_{\alpha,\beta} = \pm 1$
- Ground state: $\mathcal{O}^{(1,2)}_{\alpha,\beta} = +1 \ \forall \alpha, \beta$
- Excited states: $\mathcal{O}^{(1,2)}_{\alpha,\beta} = -1$ for some α, β

Ground State Degeneracy

Total fermion parity Γ is conserved.

$$\Gamma = (i\gamma_1\gamma_2)...(i\gamma_{N-1}\gamma_N) = i^{N/2}\prod_n \gamma_n \quad \text{with} \quad [\Gamma, H] = 0$$

We study the system in a sector of fixed fermion parity Γ.

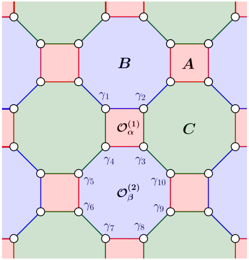
- Ground state is defined by $\mathcal{O}^{(1,2)}_{\alpha,\beta} = +1\ \forall \alpha, \beta$
- Each constraint reduced Hilbert space dimension by 1/2

Total Hilbert space dimension → $2^{\frac{N}{2}}$; Total fermion parity constraint → -1

$$GSD = \frac{2^{\frac{N}{2}-1}}{2^{N_{\text{constraint}}}}$$

Ground State Degeneracy

We devide the system into three types of plaquettes with different color:



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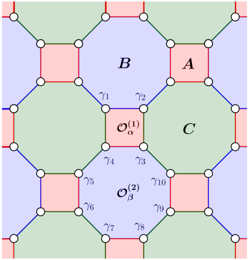

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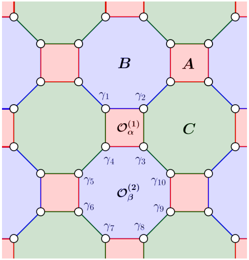

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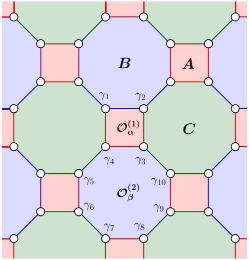

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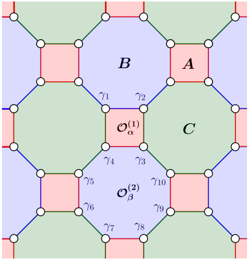

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$$\text{GSD} = \frac{2^{\frac{N}{2}-1}}{2^{(\frac{N}{4}-1)+2(\frac{N}{8}-1)}} = 4$$

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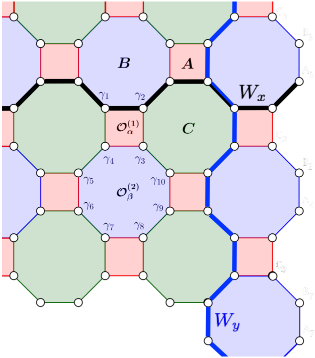
The GSD is of topological nature, i.e. GS can be labeled by eigenvalues of non-local Wilson loop operators.



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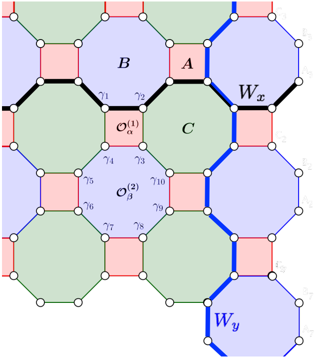

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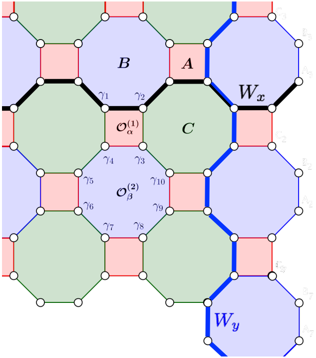

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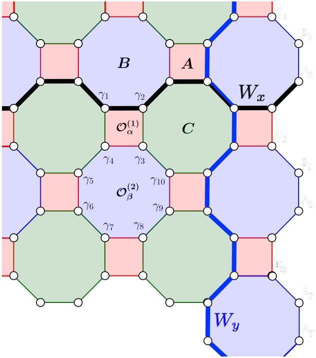

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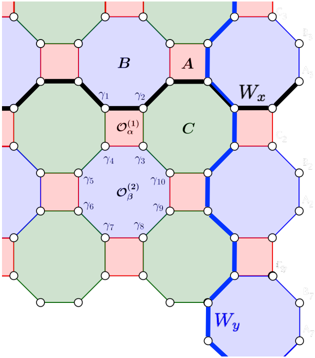

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- $[W_x, W_y] = [W_x, H] = [W_y, H] = 0$
- The 4 degenerate GS are distinguished by their eigenvalues under W_x and W_y.

Excitations

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Braiding statistics:

	1	A	B	C	AB	BC	AC	ABC
1	$+1$	$+1$	$+1$	$+1$	$+1$	$+1$	$+1$	$+1$
A	$+1$	$+1$	-1	-1	-1	$+1$	-1	$+1$
B	$+1$	-1	$+1$	-1	-1	-1	$+1$	$+1$
C	$+1$	-1	-1	$+1$	$+1$	-1	-1	$+1$
AB	$+1$	-1	-1	$+1$	-1	-1	-1	$+1$
BC	$+1$	$+1$	-1	-1	-1	-1	-1	$+1$
AC	$+1$	-1	$+1$	-1	-1	-1	-1	$+1$
ABC	$+1$	$+1$	$+1$	$+1$	$+1$	$+1$	$+1$	-1

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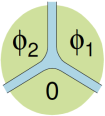
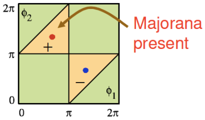
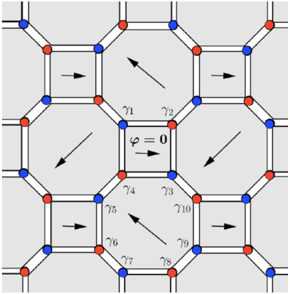
Physical realization







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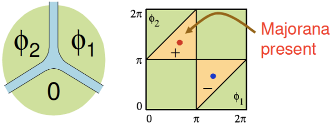

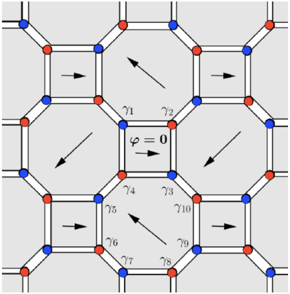

Fu, Kane(2008):

- Trijunction of SCs on top of TI
- MF exists at crossing point for the yellow regions of the phase diagram

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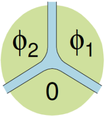

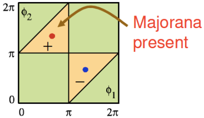

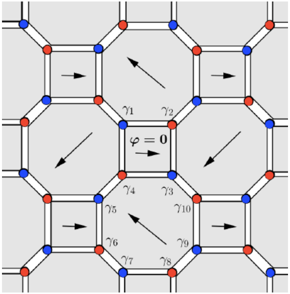

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Setup:

- Array of SC islands on top of TI
- SC phases are fixed by external magnetic field to $\phi = 0, \pm\frac{2\pi}{3}$
- "Vortex" ("Antivortex"): Phase winds by $(-)2\pi$ around trijunction

Physical realization

Hamiltonian:

"offset charge" tunable by external electric fields

number operator for transfered Cooper pairs

"offset phase" chosen so that minimum Josephson energy is at $0, \pm\frac{2\pi}{3}$

$$H_\alpha(n_g) = E_c\left(-i\frac{\partial}{\partial\varphi_\alpha} - n_g\right)^2 - E_J\sum_{\langle\alpha,\,\beta\rangle}\cos(\varphi_\alpha - \varphi_\beta - a_{\alpha\beta})$$

"charging energy" to transfer a Cooper pair

"Josephson energy"

The charging energy term and the Josephson energy term do not commute and so the superconducting phase is a quantum mechanical variable.

We will study the effective Hamiltonian in the limit $E_J \gg E_c$.

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How do the Majorana fermions enter the story?

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The effect of any two MFs $\gamma_{1,2}$ is that one fermion $d = \gamma_1 + i\gamma_2$ can be added to the ground state at no energy cost:

$$|\phi, e\rangle = \sum_n e^{i\phi n}|2n\rangle$$
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- On the level of states we have $|\phi + 2\pi, e/o\rangle = \pm|\phi, e/o\rangle$

Physical realization

How do the Majorana fermions enter the story?

The constraint on the states is removed by a gauge transformation

$$|\widetilde{\Psi}\rangle = \Omega^\dagger |\Psi\rangle \quad \text{with} \quad \Omega = \exp\left(i(1 - i\gamma_1\gamma_2)\frac{\phi}{4}\right)$$

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Charging energy couples indirectly to the MFs via a constraint on the eigenstates.

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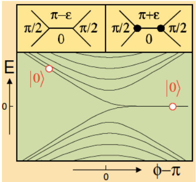

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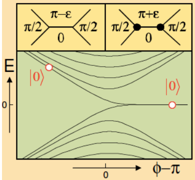

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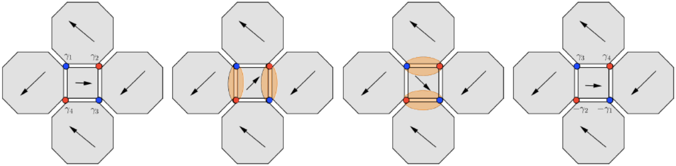

$\varphi = 0$ $\varphi = \pi/3$ $\varphi = 5\pi/3$ $\varphi = 2\pi$

The braiding in a 2π phase slip is implemented by: $U = \frac{1+\gamma_1\gamma_3}{\sqrt{2}} \frac{1+\gamma_2\gamma_4}{\sqrt{2}}$

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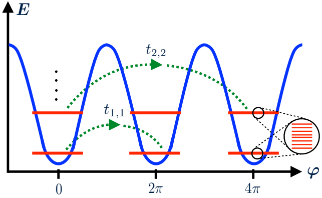

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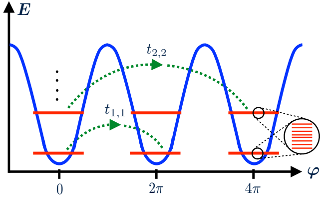

$$H_\alpha(n_g) = \epsilon_0 + \left(t_\alpha \, \hat{U} e^{2\pi i n_g} + \text{h.c.}\right)$$

$$\hat{U} = \frac{1+\gamma_1\gamma_3}{\sqrt{2}} \frac{1+\gamma_2\gamma_4}{\sqrt{2}} \quad , \quad t_\alpha \propto e^{-\sqrt{2E_J/E_c}}$$

Four body interaction term

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When we insert $\hat{U}$ into $H_\alpha(n_g)$ we find

$$H_\alpha(n_g) = \epsilon_0 - t_\alpha \cos(2\pi n_g)\gamma_1\gamma_2\gamma_3\gamma_4 + t_\alpha \sin(2\pi n_g)(i\gamma_1\gamma_3 + i\gamma_2\gamma_4)$$

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$$H_\alpha(n_g) = \epsilon_0 + \left(t_\alpha\ \hat{U} e^{2\pi i n_g} + \mathrm{h.c.}\right) \quad , \quad \hat{U} = \tfrac{1+\gamma_1\gamma_3}{\sqrt{2}}\tfrac{1+\gamma_2\gamma_4}{\sqrt{2}}$$

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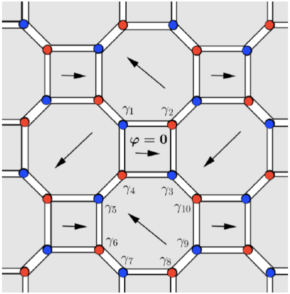
$$H_\alpha(n_g) = \epsilon_0 - t_\alpha \cos(2\pi n_g)\gamma_1\gamma_2\gamma_3\gamma_4 + t_\alpha \sin(2\pi n_g)(i\gamma_1\gamma_3 + i\gamma_2\gamma_4)$$

Set $2n_g \in \mathbb{N}$ and precisely obtain the fourbody interaction $\mathcal{O}_\alpha^{(1)}$.

Eight body interaction term

Idea: Introduce tunnel couplings between adjacent square islands



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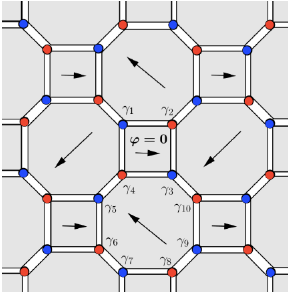

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$$H = \epsilon_0 + t_\alpha(i\gamma_1\gamma_2)(i\gamma_3\gamma_4) + \delta(i\gamma_4\gamma_5)$$

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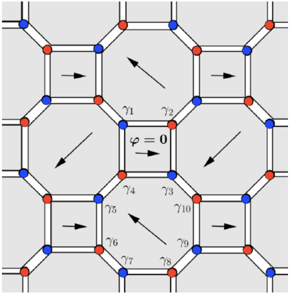

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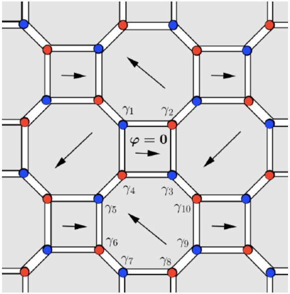

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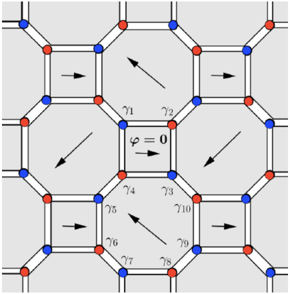

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A 4th order process $(\gamma_4\gamma_5), (\gamma_6\gamma_7), (\gamma_8\gamma_9), (\gamma_{10}\gamma_7)$ brings the system back to the GS manifold! This gives terms:

$$H_\beta = -\frac{5\delta^4}{16t_\alpha^3}\mathcal{O}_\beta^{(2)}$$

Outline

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How do we measure the eigenvalue of the 8-body operator $\mathcal{O}_\beta^{(2)}$?

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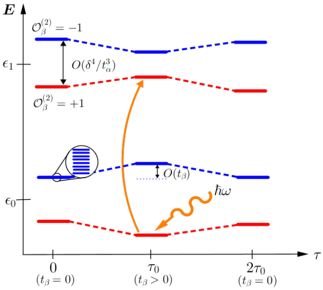

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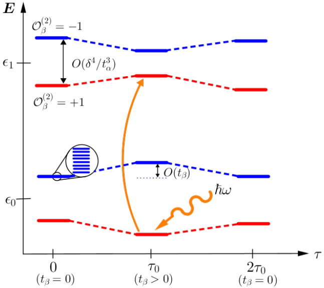

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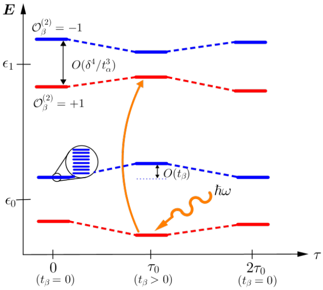

1) Prepare the system in a stabilizer eigenstate

2) Adiabatically ($\tau \gg t_\alpha^3/\delta^4$) turn on the charging energy on the octagon island:

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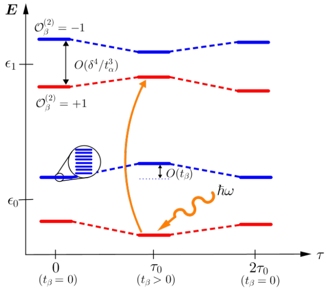

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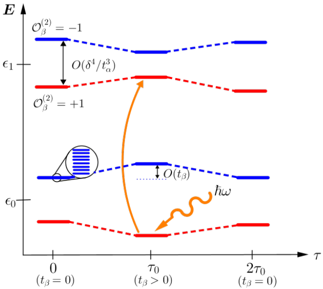

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4) Adiabatically decrease the charging energy to return to the stabilizer eigenstate